Optimal pulse width modulation for sinusoidal fringe generation with projector defocusing

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Recently, a study showed that generating sinusoidal fringe patterns by properly defocusing binary ones can significantly simplify three-dimensional shape measurement system development and drastically improve its speed. However, when the fringe stripes are very wide, it is very difficult for this technique to achieve high-quality measurement. This Letter presents a method to improve this technique by selectively eliminating high-frequency harmonics induced by a squared binary pattern. As a result, better sinusoidal fringe patterns can be generated with a small degree of defocusing even for wide fringe stripes. Simulation and experiments will be presented to verify the performance of this proposed technique. © 2010 Optical Society of America

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In recent years, digital fringe projection techniques have received significant attention for noncontact three-dimensional (3D) shape measurement because of its high speed, flexibility, low cost, and potentially high accuracy [1]. To achieve high measurement accuracy, ideal sinusoidal fringe patterns have to be generated by the projector. This is not always easy, because the commercial video projector is a nonlinear device. Recently, we proposed a technique that is to generate sinusoidal fringe patterns by properly defocusing binary structured ones [2]. In this Letter, we called this technique squared binary method (SBM). This fringe generation method has numerous advantages over a traditional fringe generation method [3]: (1) it is less sensitive than the synchronization between the camera and the projector; (2) the nonlinear gamma of the projector does not affect the measurement; (3) the exposure time can be much shorter than the projector's single channel projection time of an off-the-shelf digital light processing (DLP) projector [4]; and (4) it enables a superfast 3D shape measurement technique [5].

However, this technology is not trouble-free. We found that it performs well only when the fringe stripe is narrow but shows significant problems if the fringe stripe is wide. This limits its measurement capability especially when a step height object is to be measured. Actually, the sinusoidal pulse width modulation method (SPWM) is a well-known technique of electrical engineering for the generation of sinusoidal waveforms using low-pass filtered structured binary signals [6]. Recently, the SPWM method was proposed to greatly improve the defocusing technique when the fringe stripe is wide [7]. In brief, SPWM modulates the squared binary patterns by comparing the desired sinusoidal pattern with a high-frequency triangular waveform. This means the high-frequency harmonics are shifted farther away from the fundamental frequency and thus are easier to be suppressed by a low-pass filter. Therefore, SPWM could generate a better sinusoidal pattern even with a small degree of defocusing. However, we found that SPWM outperforms the binary method only when the fringe stripe is within a certain range of pixel number. This may be caused by the discrete fringe generation nature of a digital fringe projection technique.

In this Letter, we propose a technique called optimal pulse width modulation (OPWM) to further improve the defocusing technique by selectively eliminating undesired harmonics. Because this technique can remove any harmonics that are affecting the measurement quality the most, it has the potential to generate high-quality sinusoidal fringe patterns. Both simulation and experimental results showed that this technique outperforms the SPWM or the SBM method with a very large range of fringe stripe size.

Phase-shifting methods are widely used in optical metrology because of their speed and accuracy [8]. We use a three-step phase-shifting algorithm to find the phase value and thereby the phase error. Three fringe images with a phase shift of $2\pi/3$ can be written as follows:

$$I_1(x,y) = I'(x,y) + I''(x,y)\cos(\phi - 2\pi/3), \quad (1)$$

$$I_2(x,y) = I'(x,y) + I''(x,y)\cos(\phi), \quad (2)$$

$$I_3(x,y) = I'(x,y) + I''(x,y)\cos(\phi + 2\pi/3), \quad (3)$$

where $I'(x,y)$ is the average intensity, $I''(x,y)$ is the intensity modulation, and $\phi(x,y)$ is the phase to be solved for. The phase can be solved for from these equations as

$$\phi(x,y) = \tan^{-1}\left[\sqrt{3}(I_1 - I_3)/(2I_2 - I_1 - I_3)\right]. \quad (4)$$

This equation provides the wrapped phase ranging from $-\pi$ to $+\pi$ with 2π discontinuities. These 2π phase jumps can be removed to obtain the continuous phase map by adopting a phase unwrapping algorithm [9].

In a digital fringe projection system, sinusoidal patterns can be projected directly using a digital video projector. Usually, gamma calibration is necessary and the projection speed is limited, because a commercial video projector is a nonlinear device and the digital fringe generation nature. To circumvent these problems, sinusoidal fringe pattern generation by properly defocusing binary ones is proposed [2]. SPWM also belongs to the binary method, but it improves the performance of the conventional SBM method. However, it still has limitations when the fringe stripes are wide.

To further improve the defocusing technique, we proposed this OPWM method. This technique is to selectively eliminate undesired frequency components to generate the ideal sinusoidal fringe pattern by projector defocusing. Figure 1 illustrates a general quarter-wave symmetric OPWM pattern. The square wave is chopped n times per half-cycle. For a periodic waveform with a period of 2π, the Fourier series coefficients are

$$a_0 = \frac{1}{2\pi}\int_{\theta=0}^{2\pi} f(\theta)\mathrm{d}\theta, \quad (5)$$

$$a_k = \frac{1}{\pi}\int_{\theta=0}^{2\pi} f(\theta)\cos(k\theta)\mathrm{d}\theta, \quad (6)$$

$$b_k = \frac{1}{\pi}\int_{\theta=0}^{2\pi} f(\theta)\sin(k\theta)\mathrm{d}\theta. \quad (7)$$

Because of the half-cycle symmetry of an OPWM wave, only odd-order harmonics exist, i.e., $a_0 = 0$ and $a_k = 0$. Furthermore, b_k can be simplified as

$$b_k = \frac{4}{\pi}\int_{\theta=0}^{\pi/2} f(\theta)\sin(k\theta)\mathrm{d}\theta. \quad (8)$$

For the binary OPWM waveform, $f(\theta)$, we have

$$b_k = \frac{4}{\pi}\int_{0}^{\alpha_1}\sin(k\theta)\mathrm{d}\theta - \frac{4}{\pi}\int_{\alpha_1}^{\alpha_2}\sin(k\theta)\mathrm{d}\theta + \frac{4}{\pi}\int_{\alpha_2}^{\alpha_3}\sin(k\theta)\mathrm{d}\theta \cdots \frac{4}{\pi}\int_{\alpha_n}^{\pi/2}\sin(k\theta)\mathrm{d}\theta. \quad (9)$$

The n chops in the waveform afford n degrees of freedom. It is easy to eliminate $n-1$ selected harmonics while keeping the fundamental frequency component within a certain magnitude. To do this, one can set the corresponding coefficients in the above equation to be desired values (0 for the $n-1$ harmonics to be eliminated and the desired magnitude for the fundamental frequency) and solve for the angles for all notches [10]. Some research has been conducted to solve for transcendental equations [11]. Because of the ability to eliminate undesired high-order harmonics, the OPWM waveform could become sinusoidal after applying a low-pass filter, which is similar to a small degree of defocusing. Therefore, this technique can be realized by a real measurement system.

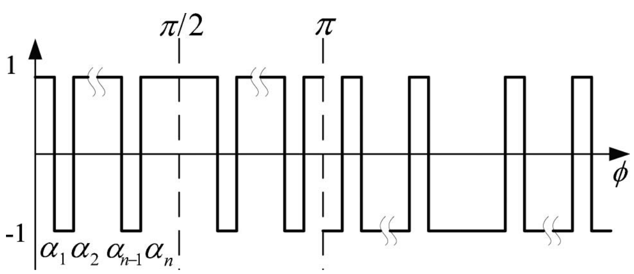

Fig. 1. Quarter-wave symmetric OPWM waveform.

Simulation was conducted to verify the effectiveness of the proposed method. A Gaussian filter is used to approximate the defocusing effect. In this research, a Gaussian filter with size of 5 and standard deviation of 2 pixels is applied twice for all fringe patterns generated by different methods. From the above analysis, there are numerous options for OPWM patterns having different numbers of notches for fringe patterns with a certain fringe pitch P, the number of pixels per fringe stripe. Therefore, it is interesting to find the best OPWM pattern to optimize the measurement quality. The best OPWM pattern was defined, as it will have the minimum phase error if it is used to perform a measurement. In this research, the phase is calculated by using a three-step phase-shifting algorithm, and the phase error is determined by comparing against the phase result from ideal sinusoidal fringe patterns with the same number of fringe pitch. A simulation was performed to compare the phase error within different methods under exactly the same degree of defocusing. Figure 2(a) shows the cross sections of the SBM, SPWM, and OPWM patterns. And in Fig. 2(b) the results after defocusing or smoothing by a Gaussian filter are shown. It should be noted that the SPWM was also optimized to minimize the phase error by selecting the proper modulation frequency.

Figure 2(c) shows the rms errors for different methods when the fringe pitch varies from 18 to 150 pixels. This simulation result shows that the OPWM gives the smallest phase error in almost all ranges and SPWM performs better than SBM only when the period is large enough. This also indicates that when the fringe stripe is narrow, such as 18 pixels or less in pitch, SBM performs the best. This is because the fringe pattern is generated discretely, and it does not have sufficient pixels to set proper notches for further improvement.

Experiments were also performed to verify the proposed approach. We developed a 3D shape measurement system that includes a DLP projector (Samsung P310) and a digital USB CCD camera (The Imaging Source DMK 21BU04). The camera is attached with a 12 mm focal length megapixel lens (Computar M1214-MP). The resolution of the camera is 640×480, with a maximum frame rate of 60 frames/s. The projector has a resolution of 858×600 with a lens of $F/2.0$ and $f = 16.67$ mm.

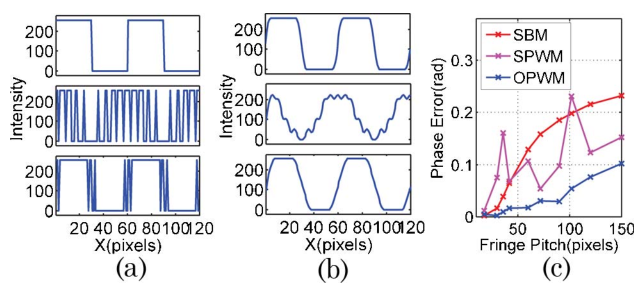

Fig. 2. (Color online) (a) Cross sections of the SBM, SPWM, and OPWM patterns; (b) corresponding defocused results; (c) rms errors with different fringe pitches.

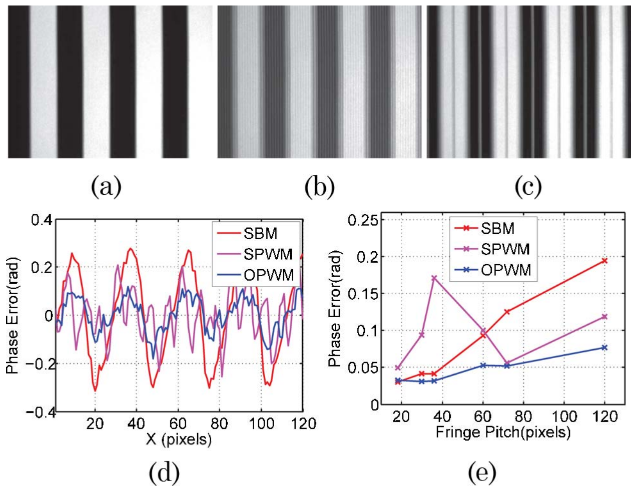

Fig. 3. (Color online) First row shows fringe patterns when $P = 120$ pixels. (a) SBM pattern, (b) SPWM pattern, (c) OPWM pattern, (d) cross sections of phase errors when $P = 120$ pixels, (f) rms errors for different fringe pitches.

We first evaluate the phase error by measuring a uniform white flat surface. Phase-shifted fringe patterns with exactly the same fringe pitch generated are captured to obtain the phase value for each of them. Figures 3(a)–3(c) show the fringe patterns with a pitch of 120 pixels. In addition, the ideal sinusoidal fringe patterns are also used to obtain the grand truth phase values to compare against. The phase errors shown in Fig. 3(e) are obtained by getting the phase differences between the associated method with the sinusoidal method. It should be noted that the projector's nonlinear gamma was corrected to generate the ideal sinusoidal fringe patterns.

Figure 3(e) clearly shows that when the fringe pitch number is small (e.g., 30 pixels), the SPWM cannot perform as well as either the SBM or OPWM method. However, when fringe pitch increases to 120 pixels, the OPWM gives the best result. Figure 3(d) shows the cross sections of the phase errors for each method when the pitch is 120 pixels.

To further compare the measurement quality using these methods, a complicated 3D sculpture was measured. Figure 4 shows the measurement results. It can be seen that when the fringe stripes are narrow, the SBM and the OPWM methods outperform the SPWM, and the former two give a similar quality of measurement. However, when the fringe pitch increases, the OPWM produces the best result.

This Letter has introduced an OPWM technique to generate high-quality sinusoidal fringe patterns by slightly defocusing binary ones. This technique is to selectively eliminate undesired high-frequency high harmonics of standard squared binary patterns. Both simulation and experiment results have verified that the method indeed could significantly improve the measurement quality if the fringe stripes are more than 18 pixels.

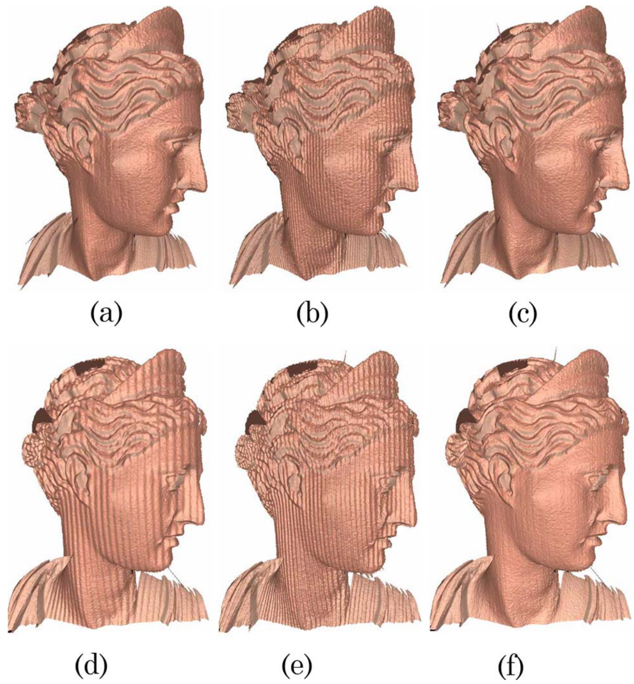

Fig. 4. (Color online) 3D shape measurement of a sculpture. The first row shows the results when $P = 30$ pixels, and the second row shows the results when $P = 60$ pixels.

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