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Mixed Regularization for Damage Localization Using Electrical Impedance Tomography in Three Dimensional Composite Materials

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ABSTRACT

Electrical impedance tomography (EIT) is a promising tool for nondestructive evaluation (NDE) of materials that exhibit stimulus-responsive electrical conductivity. Solution of the EIT problem requires regularization because it is ill-posed. While smoothness-promoting regularization methods are widely used in materials-based EIT, they are not ideal for characterizing localized damage. Here, we introduce a novel regularization method that combines a traditional smoothing regularization technique with one that promotes sparsity. The method is demonstrated on experimental data of two three-dimensional composite structures that were subjected to impact damage. The mixed regularization method is shown to outperform traditional smoothing regularization methods.

Keywords: Electrical impedance tomography, Bayesian methods, Composites

INTRODUCTION

Electrical impedance tomography (EIT) is a non-invasive method of spatially mapping the electrical conductivity distribution of a domain based on external voltage-current measurements. This modality has been investigated for damage detection, localization, and characterization in conductive composites for structural health monitoring (SHM). We refer to [1] for an extensive review of previous work.

The EIT inverse problem is mathematically ill-posed and therefore requires regularization to solve. Materials-focused practitioners of EIT commonly use smoothness-promoting methods such as Tikhonov regularization. This is limiting because much more advanced types of regularization exist and have potential to significantly improve EIT for material state awareness. Thus, in this work we propose a novel regularization technique for the EIT inverse problem. Specifically, we solve the inverse problem in the Bayesian framework and apply a mixed prior which combines a smoothness prior with a conditionally Gaussian prior that favors sparse solutions. The proposed mixed formulation is experimentally validated on two different three-dimensional composite structures: a carbon black (CB)-modified glass fiber/epoxy tube and a carbon fiber/epoxy laminate shaped as a representative NACA airfoil. Both specimens were subjected to low-velocity impact damage via a drop-tower rig. The mixed prior is shown to outperform the smoothness prior on its own.

EIT FORWARD AND INVERSE PROBLEMS

The EIT forward problem is to predict voltages on the boundary of the domain given the conductivity distribution inside the domain and a set of current injections. It starts with Laplace's equation for steady-state diffusion in the absence of internal sources,

$$\nabla \cdot \sigma \nabla \phi = 0 \qquad \textbf{(Eq. 1)}$$

where σ is the conductivity distribution and ϕ is the electric potential in the domain. This equation combined with the complete electrode model [2] constitutes the forward model for EIT. For our application, the forward problem is solved using finite element techniques.

The inverse problem is to predict the conductivity distribution inside the domain of interest based on voltage measurements on the boundary of the domain. In this work we utilize difference imaging, meaning we wish to find the change in conductivity between two different measurement times (i.e. before and after damage).

To mathematically formulate the inverse problem for difference imaging, we denote $y = V_m(t_2) - V_m(t_1)$ as the change between two sets of boundary voltage measurements recorded at times t_1 and t_2. Denote also σ and $\sigma + \delta\sigma$ as the conductivity at times t_1 and t_2, respectively, in the forward model. Using a truncated Taylor series expansion to linearize the forward model (see [1]), we can write

$$y \approx J\delta\sigma \qquad \textbf{(Eq. 2)}$$

where J is the Jacobian of the forward model computed with respect to σ. In the case of an isotropic material, $\delta\sigma$ is the spatially varying scalar conductivity change in the forward model. For anisotropic materials, the conductivity tensor is written as $\sigma = \kappa\bar{\sigma}$, where κ is selected such that $\det(\bar{\sigma}) = 1$. In that case, the EIT inverse problem seeks to find the spatially varying change in κ, $\delta\kappa$. For notational simplicity, we denote by γ either $\delta\kappa$ or $\delta\sigma$ depending on the material system (i.e. $\delta\sigma$ for the electrically isotropic tube and $\delta\kappa$ for the electrically anisotropic airfoil).

Thus, the goal of the inverse problem is to find the conductivity change γ such that

$$\gamma^* = \arg\min \|J\gamma - y\|^2 + \alpha\|R(\gamma)\|^2 \qquad \textbf{(Eq. 3)}$$

where R is a regularization operator and α is a weight that controls the contribution of the regularization term. This work introduces a novel method to formulate the regularization operator.

BAYESIAN MIXED REGULARIZATION

We solve the inverse problem in the Bayesian framework, so that the solution to the inverse problem is given by the posterior distribution of the unknown conditioned on the measured data. Recall that the posterior distribution is the product of the likelihood distribution of the data conditioned on the unknown and the prior distribution of the unknown. The likelihood distribution accounts for any discrepancy between the data and the forward model, while the prior distribution incorporates any information we have about the unknown before taking the data into account.

Here we assume our data have been corrupted by Gaussian measurement noise with standard deviation ω, resulting in a Gaussian likelihood distribution.

$$\pi(y \mid \gamma) \propto \exp\left(-\frac{1}{2\omega^2}\|J\gamma - y\|^2\right) \qquad \textbf{(Eq. 4)}$$

Traditionally, EIT makes use of a Gaussian smoothness prior (see [3]) of the form

$$\pi(\gamma) \propto \exp\left(-\frac{1}{2\rho^2}\|L\gamma\|^2\right), \quad \textbf{(Eq. 5)}$$

where L is the discrete Laplace operator and ρ controls the range of values allowable by the prior. While this prior is commonly used, it has a tendency to overestimate the size of the damage area and to underestimate the loss in conductivity due to damage. To mitigate these effects, we formulate a mixed prior which combines the traditional smoothness prior with a conditionally Gaussian prior (see [4]) that favors sparse solutions. The conditionally Gaussian prior assumes that the individual components of γ are independent and each follow a Gaussian distribution with mean zero and unknown variance. The variance of each component is estimated along with γ as part of the inverse problem.

In practice, we estimate the reciprocal of the variance of each component denoted by λ_k. Because it is unknown, we must assign λ_k a prior distribution; specifically, we assume that λ_k follows an exponential distribution with mean $1/\beta$. The full expression for the joint prior for γ and λ is then given by

$$\pi(\gamma, \lambda) \propto \exp\left(-\frac{1}{2}\|\Lambda\gamma\|^2 - \frac{\beta}{2}\sum_{k=1}^{n}\lambda_k + \frac{1}{2}\sum_{k=1}^{n}\log\lambda_k\right) \quad \textbf{(Eq. 6)}$$

where Λ is a diagonal matrix whose entries are given by $\sqrt{\lambda}$. The total prior for γ is the product of the smoothness and focal priors, resulting in the following posterior distribution:

$$\pi(\gamma, \lambda \mid y) \propto \exp\left(-\frac{1}{2\omega^2}\|J\gamma - y\|^2 - \frac{1}{2\rho^2}\|L\gamma\|^2 - \frac{1}{2}\|\Lambda\gamma\|^2 - \frac{\beta}{2}\sum_{k=1}^{n}\lambda_k + \frac{1}{2}\sum_{k=1}^{n}\log\lambda_k\right). \quad \textbf{(Eq. 7)}$$

The algorithm to find the MAP estimates of γ and λ is iterative. The details of the algorithm can be found in [4].

EXPERIMENTAL SETUP

The proposed method was experimentally validated on a carbon black (CB)-modified glass fiber/epoxy tube and a carbon fiber/epoxy laminate shaped as a NACA 4424 airfoil. Both the tube and airfoil were impacted twice using a CEAST 9340 drop tower. EIT measurements were collected on both specimens before and after each impact. Validation data for the tube was acquired from [5]; data for the airfoil was obtained from [6]. Further details on the construction of each specimen and the experimental setup can be found in [5] for the tube and [6] for the airfoil.

RESULTS

We applied the mixed prior to all combinations of specimen and number of impacts. Results are shown in Figures 1 for the airfoil and 2 for the tube. For comparison, we also solved the inverse problem assuming only the smoothness prior. These results are shown alongside the results using the mixed prior.

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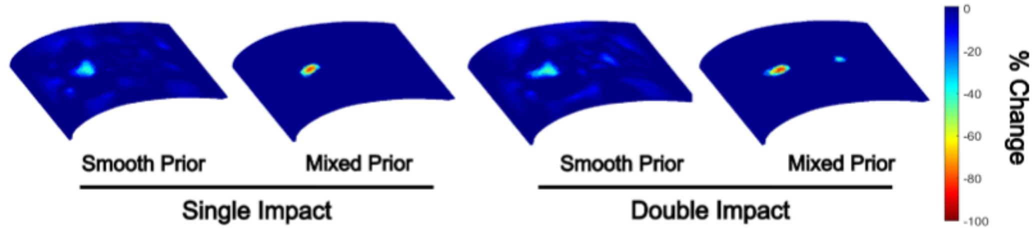

Figure 1. Results of applying the smoothness prior and the mixed prior to the airfoil data. The results for the airfoil after a single impact are shown on the left, and results after two impacts are shown on the right. The first impact was 15 J, and the second was 12 J.

The mixed prior is able to localize both impacts in the airfoil data and outperforms the smoothness prior. Note that while the smoothness prior is able to localize both impacts, it also identifies several other regions of conductivity change that do not correspond to real damage. The mixed prior is able to suppress these artifacts and localize only the impact damage. The mixed prior performs similarly well on the tube data and is able to localize both impacts. We note that some artifacts remain even with the use of the mixed prior; we hypothesize that this is due to noise in the tube data.

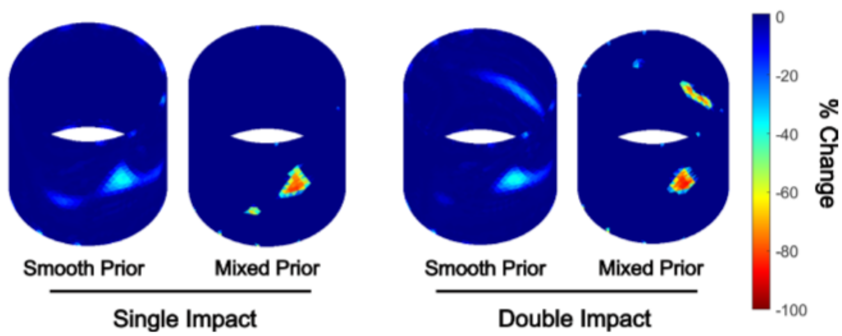

Figure 2. Results of applying the smooth and mixed priors to the tube data. The results for the single impact are shown on the left, and results for two impacts are shown on the right. The first impact was 14 J, and the second was 10 J.

CONCLUSIONS

We developed a novel regularization method to localize impact damage from EIT measurements. This method was successfully validated on experimental EIT data of three-dimensional composite structures with known damage and seems to outperform traditional smoothness-promoting regularization methods. This result is a key step to incorporate more sophisticated regularization techniques into EIT for NDE and transition EIT from a proof-of-concept phase into practice.

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