TDR 2006

Purdue University

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Ref.: Huisman, J. A., Lambot, S. and Vereecken, H., "Determining Soil Water Content Variation Along the TDR Probe with Inverse Modeling: Theory, Practice, and Challenges", Proc. TDR 2006, Purdue University, West Lafayette, USA, Sept. 2006, Paper ID 28, 10 p., https://engineering.purdue.edu/TDR/Papers

Determining Soil Water Content Variation Along the TDR Probe with Inverse Modelling: Theory, Practice and Challenges

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Abstract

Recent studies have shown that inverse modelling of TDR waveforms allows the determination of detailed soil water content variation along the wires of the TDR probe. Before inverse modelling can successfully be applied for soil water profile reconstruction under field conditions, the following requirements need to be fulfilled: valid TDR wave propagation model, appropriate reference level for the input signal, appropriate choices of optimisation strategy, measurement device and probe type. In this review paper, we will first describe a commonly used TDR wave propagation model. Second, we will review the merits and limitations of different choices for the input signal reference levels. Such practical issues as input signal variation due to temperature changes will also be discussed. Then, we will review different optimisation strategies. The discussion on optimisation strategies will deal with appropriate optimisation algorithms, strategies to constrain the optimization problem, and frequency versus time domain optimisation approaches. Finally, we will review different measurement devices and probe types. We hope that this review provides an overview of current practice and challenges in determining the water content profile from a single TDR measurement made in the field.

Keywords: inverse modelling, water content profile reconstruction

Introduction

In vadose zone hydrology, there is a need for information on the vertical distribution of soil water content. When using time domain reflectometry (TDR), this information is typically obtained by either installing horizontal TDR probes at several depths or by installing vertical TDR probes of different length. Such arrays of TDR probes require a large number of TDR sensors and might not provide the spatial resolution required for some applications.

Recent studies have shown that inverse modelling of TDR waveforms allows the determination of detailed soil water content variation along the wires of the TDR probe[1-6]. In TDR inverse modelling, the parameters of a model describing TDR wave propagation are adapted until a satisfying fit between measured and modelled TDR waveforms is achieved. If the reconstruction of the vertical water content profile is accurate, this information would, for example, be very helpful for validation of remote sensing methods, such as ground penetrating radar, active radar and passive radiometry. These remote sensing methods inherently have a variable and shallow measurement depth and detailed information on the vertical water content profile of the upper soil would be valuable in assessing the sampling depth of these methods. Other applications where detailed information on the vertical water content profile might be useful are irrigation scheduling, root water uptake studies and infiltration studies, among others.

Before inverse modelling can successfully be applied for soil water profile reconstruction under field conditions, a number of choices have to be made regarding the 1) forward model, 2) input signal, 3) optimization strategy, 4) measurement device and 5) probe type. In the following, we will first present some theoretical background. Then we will separately discuss each of the abovementioned choices in detail. We hope that this review provides an overview of current practice and challenges in determining the water content profile from a single TDR measurement.

TDR Wave Propagation Model

Two model approaches are used to model TDR wave propagation in the vadose zone hydrology community. The first approach is based on a time domain formulation of TDR wave propagation[1-2,4-6] and the second approach is based on a frequency domain formulation[3,7-8]. It can be shown that these two approaches are formally equivalent (Oswald, personal communication). In this paper, we use the frequency domain formulation out of convenience, although we want to note here that this modeling approach requires special attention with regard to measurement acquisition to avoid unwanted artifacts when using the required Fourier transforms.

In the time domain, the measured TDR waveform $r(t)$ is related to the input signal $v_0(t)$ and the system function $s(t)$ by

$$r(t) = \int_{-\infty}^{\infty} v_0(t-\tau)s(\tau)d\tau \quad [1]$$

where τ is an integration variable. The equivalent frequency domain formulation is

$$R(f) = V_0(f)S(f) \quad [2]$$

in which f is the frequency [Hz], $R(f)$ is the measured TDR waveform, $V_0(f)$ is the input signal and $S(f)$ is the system response function. $R(f)$, $V_0(f)$ and $S(f)$ are related to $r(t)$, $v_0(t)$ and $s(t)$ by Fourier transforms. In the case of TDR measurements, $S(f)$ is calculated from the ratio of the discrete Fourier transformation (DFT) of the backward difference of the reflected and input waveform $r'(t)$ and $v_0'(t)$

$$S(f) = \frac{DFT[r'(t)]}{DFT[v_0'(t)]} \quad [3]$$

Feng et al.[8] presented a multi-section scatter function to model the TDR waveform measured by a cable tester. In this paper, we adopt the common numbering convention[3,8] where section 1 is the section farthest away from the cable tester (i.e. the end of the wires of the TDR probe). To model a water content profile accurately, the TDR wire length should be divided in a sufficiently large number of sections. The corresponding multi-section scatter function measured by the cable tester is then given by:

$$S_{11}^k = \frac{\rho_s^k(f) + S_{11}^{k-1}(f)\exp(-2\gamma_k(f)L_{l,k})}{1+\rho_s^k(f)S_{11}^{k-1}(f)\exp(-2\gamma_k(f)L_{l,k})} \quad [4]$$

with

$$\rho_s^k(f) = \frac{Z_{k-1}(f) - Z_k(f)}{Z_{k-1}(f) + Z_k(f)} \quad [5]$$

where ρ_s is the reflection coefficient between the different sections k, $Z_k(f)$ the impedance and $\gamma_k(f)$ the propagation coefficient of section k and $L_{l,k}$ the length of a section. Eq. 5 is solved by iterating through all sections, starting with section 1. The frequency dependent impedance of a section, $Z(f)$, can be calculated by

$$Z(f) = \frac{Z_0}{[\varepsilon_r^*(f)]^{1/2}} \quad [6]$$

in Z0 is the probe impedance in air and $\varepsilon_r*(f)$ is the complex frequency dependent dielectric permittivity including a DC electrical conductivity term

$$\varepsilon_r^*(f) = \varepsilon_r(f) - \frac{i\sigma_{dc}}{2\pi f \varepsilon_0} \quad [7]$$

Finally, the propagation coefficient is given by

$$\gamma = \frac{i2\pi f [\varepsilon_r^*(f)]^{1/2}}{c} \quad [8]$$

in which c is the speed of light (3.0×10^8 ms^{-1}).

To solve Eqs. [4-8] it is required to define a reflection coefficient at the end of the probe. Typically, it has been assumed that there is a perfect reflection (ρ_s=1). Recently, it has been suggested that this might not be the case[2,5,6]. Using the time domain formulation, Leidenberger et al.[6] presented several alternative expressions: a resistive termination, a resistive voltage source termination and a parallel resistive capacitive termination. Since an accurate forward model is the key to a successful retrieval of the profile water content, a detailed analysis of these different probe terminations in media with known permittivity should be performed in the future. It should be noted that such a study requires the use of the time domain formulation, since it is not apparent how these terminations can be implemented in the frequency domain formulation used here.

Definition of Input Signal

There is a wide range of options to define an input signal, V_0. Heimovaara[7] used a custom-made 7-wire probe where the inner wire could be removed from the probe head. With such a probe, the reference level is at the beginning of the TDR wires, and the wave propagation model only needs to describe propagation along the wires of the probe. The effects of the cable and the probe head are included in the input signal and do not have to be modelled explicitly. Others[e.g. 8] have used a probe where the probe could be disconnected from the cable. With such a set-up, the effect of the probe head needs to be included in the wave propagation model. Heimovaara et al.[3] suggested to explicitly model the effects of the cable tester-cable transition, the cable itself and the probe head and to approximate the signal generated by the cable tester with an analytical function.

The potential advantage of modelling the entire system is that it does not require custom-made probes with detachable cables or removable inner wires. The disadvantage of modelling the cable tester – cable transition and the cable itself is the large number of parameters that needs to be calibrated. For example, Heimovaara et al.[3] required 14 parameters to calibrate the cable-probe system and 38 parameters to adequately model the entire TDR system. The additional measurements required for this calibration were a measurement with a 50Ω load directly attached to the cable tester to determine the two parameters of the analytical function describing the generated input function and three measurements with coaxial cables of different lengths to determine the transmission line parameters of the cable tester – cable transition and the coaxial cable.

After this extensive calibration, the model of Heimovaara et al.[3] was able to accurately describe TDR measurements made under laboratory conditions. However, under field conditions, cable properties (and therefore the input signal) might not be stable because of temperature variations. To test the influence of temperature on the input signal, a 24 meter long RG58 coaxial cable was inserted in a water bath and TDR measurements were made at four temperatures (5, 15, 25 and 35 °C). Fig. 1 shows these four measurements. Clearly, the permittivity of the coaxial cable (non-linearly) decreases with increasing temperature because the reflections arrive earlier at higher temperatures. Also, the temperature has affected the frequency content of the input signal because the rise time of the signal has changed.

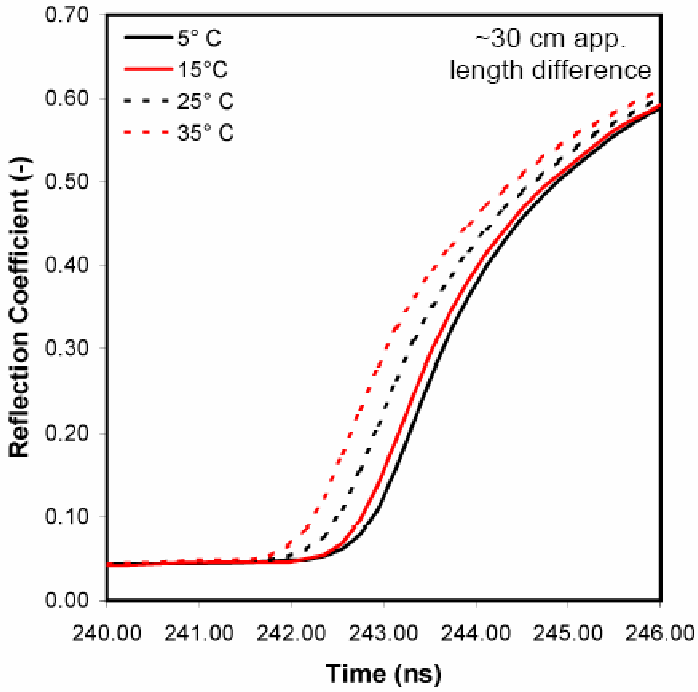

Fig. 1. Impact of Temperature on Cable Properties.

This short overview leads us to the conclusion that currently there is no way to accurately determine the input function for a field application with permanently installed sensors and automatic logging. Probes with removable inner wires or detachable cables do not allow automatic measurement of soil water content profiles. Currently, forward models describing the entire TDR system do not account for temperature effects on wave propagation, and are therefore not yet suited for use in the field. A possible solution to these problems is the development of a temperature dependent cable model. Accurate measurements with a range of cable lengths and temperatures should allow the determination of the temperature dependence of the capacitance, resistance and the shunt conductance of the coaxial cables. Another solution would be to add a remote diode short in the probe head. With this solution, an input signal could be obtained just before the actual TDR measurement used for the profile reconstruction by making a TDR measurement with an activated shorting diode. The TDR measurement made with an activated shorting diode should be closely related to the actual input signal. Finally, the accuracy of a pragmatic solution in which the input signal is temperature corrected by time shifting should also be evaluated. However, it should be noted that an error in the frequency content of the input signal will translate into an artificial change in soil water content because the model errors are compensated by a wrong model parameterization.

Choice of Optimization Strategy

Determining the water content profile from a TDR measurement certainly is a formidable optimization problem. Most studies have used powerful optimization tools, such as a genetic algorithm[2,6], or the Shuffled Complex Evolution Metropolis algorithm[3]. These algorithms have in common that they are able to find the global minimum of the objective function in a complex error landscape. Despite the use of these powerful optimization algorithms, there is a need for improved optimization constraints. This will be discussed in the next section. After that, attention will be focussed on the choice of an appropriate objective function.

Improved optimization constraints

Determining the vertical water content profile might require the optimization of a large number of parameters. For example, Oswald et al.[2] discretized a 1.0 m probe in 46 layers. The permittivity of each of these layers was optimized separately by a genetic algorithm. To stabilize the solution, they imposed a smoothness constraint on the resulting permittivity profile (regularization). Despite regularization, the resulting permittivity profile showed considerable fluctuations, unlikely to have been caused by variation within the soil sample. In a follow-up study, Leidenberger et al.[6] used a hierarchical optimization, which starts with a coarse spatial resolution which is increased as the convergence rate decreases. This hierarchical approach resulted in smoother and more realistic soil water content profiles that matched reference measurements reasonably well. Nevertheless, computation efforts were high in both studies with computation times of several hours per TDR measurement.

To reduce calibration times and the number of calibrated parameters, Greco[5] suggested a flexible 4-parameter functional equation that can describe the key features of a variety of soil water content profiles. To test the Greco[5] approach, a 10 cm 3-wire TDR sensor was installed vertically in column filled with a dry sandy soil and water was allowed to infiltrate. After some time, a TDR measurement was made. The wave propagation model was discretized in 20 layers (~0.5 cm). Prior to the experiment, the input function was determined by detaching the cable from the probe head. The probe head properties, the probe impedance and the electromagnetic probe length were calibrated with measurements in water and air. Fig. 2 shows the measured and modelled TDR waveforms and the permittivity profile obtained with the Greco[5] approach. The permittivity profile nicely shows that the infiltration front reached a depth of ~6 cm at the time of the measurement. It should be noted that an analysis of this TDR measurement with the traditional tangent line method would most likely have resulted in an erroneous water content estimate.

Fig. 2 also shows that there are still some apparent mismatches between measurement and model, especially in the simulated reflection coefficients between 30 and 40 ns. In this case, the mismatches are most likely related to the use of a 3-wire probe with an inferior head design. Huisman et al.[9] achieved a better match between measured and modelled TDR waveforms with a custom-made TDR probe (e.g. see Fig. 5).

It can be concluded that the functional equation approach is an interesting alternative to reduce the optimization problem to manageable proportions. The number of function evaluation was two orders of magnitude lower compared to the other methods. An approach that uses the final result of the functional equation approach as a starting point in the hierarchical approach seems a worthwhile direction to explore.

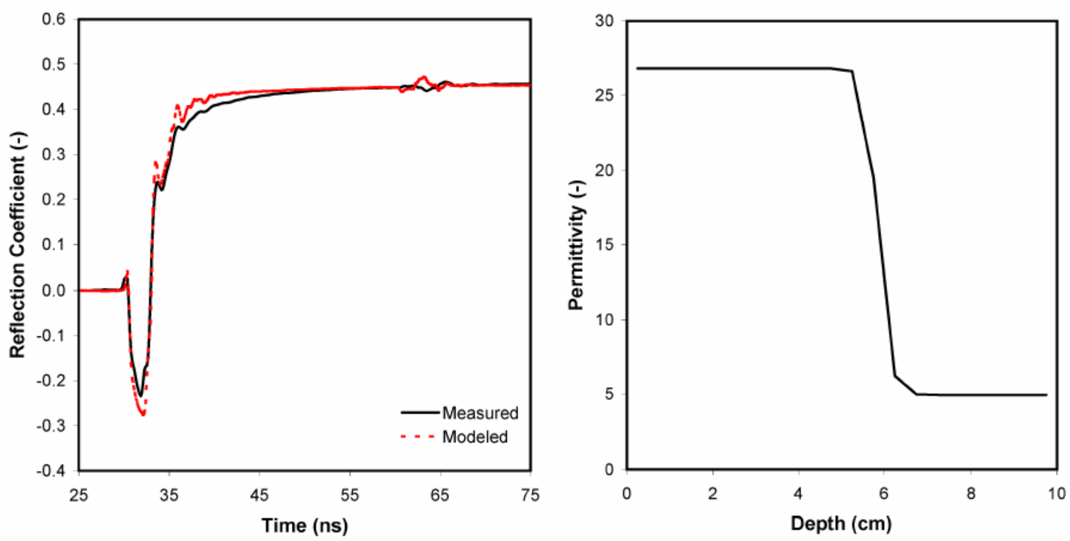

Fig.2. Measured and Modelled TDR Waveforms and Retrieved Permittivity Profile for Water Infiltrating in a Sandy Soil (10 cm Long 3-Wire Probe).

Appropriate objective function

The success of inverse modelling depends on the sensitivity of the model towards the parameters that are being optimized. Huisman et al.[9] already showed that some model parameters could be more accurately determined when the TDR waveform is converted to the frequency domain in case of the determination of the complex frequency dependent dielectric permittivity from TDR waveforms. To investigate whether this is also true for the determination of vertical water content profiles from TDR waveforms, two TDR waveforms were simulated. The first waveform represents a TDR measurement in a homogeneous wet soil and the second measurement represents a measurement in a soil with a wet upper layer above a dry soil (e.g. infiltration front as in Fig. 2). Fig. 3 shows these two simulated TDR waveforms in the time domain and frequency domain. Clearly, the sensitivity of the model towards a change in the model parameters is much higher in the frequency domain. Potentially, this means that optimization should preferably be performed in the frequency domain. However, the periodic nature of the scatter function in the frequency domain makes the optimization problem very susceptible to local minima in the objective function. Therefore, a sequential approach where the time domain representation of the TDR waveform is used for a first estimate of the permittivity profile and where the frequency domain representation is used to further improve the permittivity profile seems to be promising.

Choice of Measurement Device

The choice between optimization in the time domain or in the frequency domain is closely related to the choice of measurement device. Huisman et al.[9] compared the use of vector network analyzer (frequency domain) and cable tester (time domain) measurements. That study showed that cable tester measurements are very noisy above ~1 GHz in the frequency domain. This is illustrated in Fig. 4, which shows the magnitude of a scatter function after a numerically generated TDR waveform with added realistic Gaussian noise is converted to the frequency domain. Network analyzer measurements provided much better results in the frequency domain. Fig. 5 shows measured and modelled measurements in the frequency domain and in the time domain. The measurements were made with a 7-wire probe with a specially designed 50Ω head and a network analyzer. Clearly, these measurements show much less noise, which is evident when the frequency domain representations of Fig. 4 and 5 are compared. It can be concluded that network analyzer measurements should be preferred over cable tester measurements when the higher sensitivity of the scatter function towards permittivity changes is to be used in the optimization.

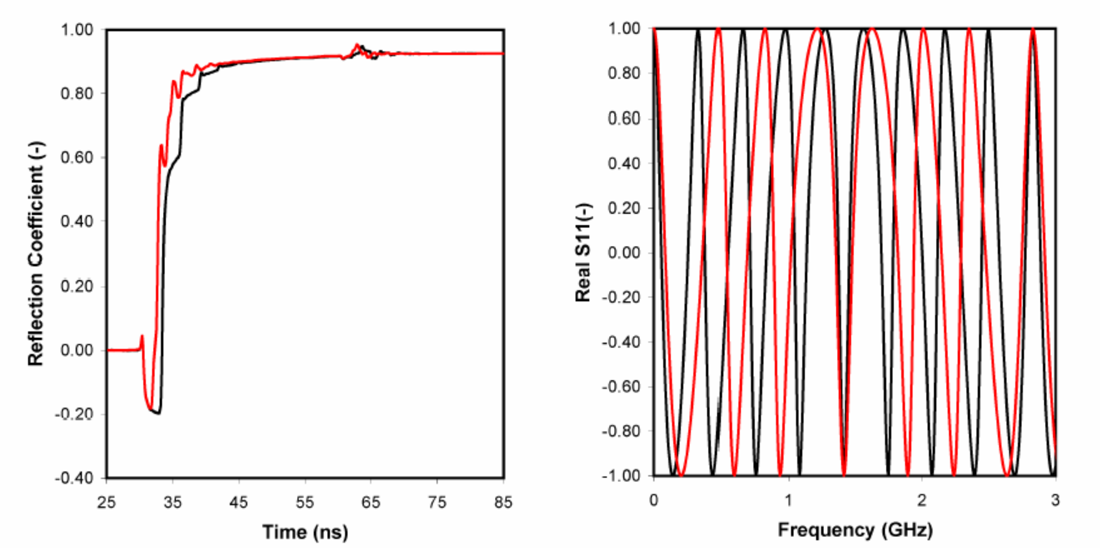

Fig. 3. Simulated TDR Waveforms for a Wet Soil (Black) and a Soil with a Wet Upper Layer and a Dry Lower Layer. Left Plot shows the TDR Waveforms in the Time Domain and the Right Plot Shows the Measurements in the Frequency Domain.

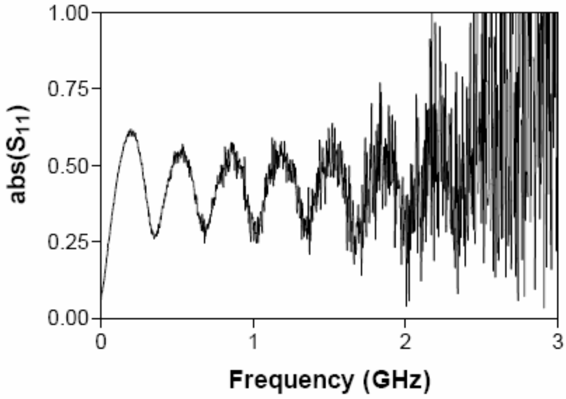

Fig. 4 (from Huisman et al.[9]). System Function Magnitude after Conversion of a Numerically Generated TDR Waveform with Added Noise to the Frequency Domain.

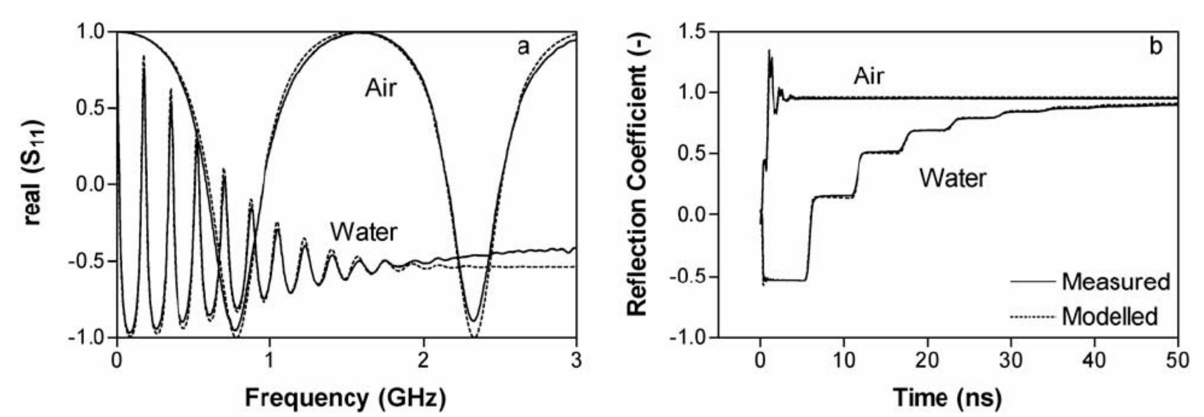

Fig. 5 (from Huisman et al.[9]). (a) Measured and Modelled Real Parts of the Scatter Function after Probe calibration in Water and Air and (b) Measured and Modelled Time Domain Waveforms.

At this point, it should be noted that automated field measurements with network analyzers are not yet feasible. However, contrary to cable tester technology, network analyzers are continuously being improved. Recently, portable network analyzers with sufficient accuracy have been developed, and they are available for an acceptable price (comparable to the Tektronix system). Although these portable systems are not yet rugged enough for field use and multiplexer systems are not yet available, these developments bring high quality frequency domain measurements in the field a step closer.

Choice of Probe Type

Appropriate probe design is important for accurate determination of water content profiles. Most studies have used either a 2-wire probe[2,6], 3-wire probe[3,5] or a 7-wire probe[7,9]. The probe length has varied from 0.10 to 1.0 m. The success of inverse modelling with these different probe designs depends on the appropriateness of the wave propagation model for that particular design. Little is known about this issue, also because work has mainly focussed on the time domain representation of TDR measurement in which these errors are much less apparent.

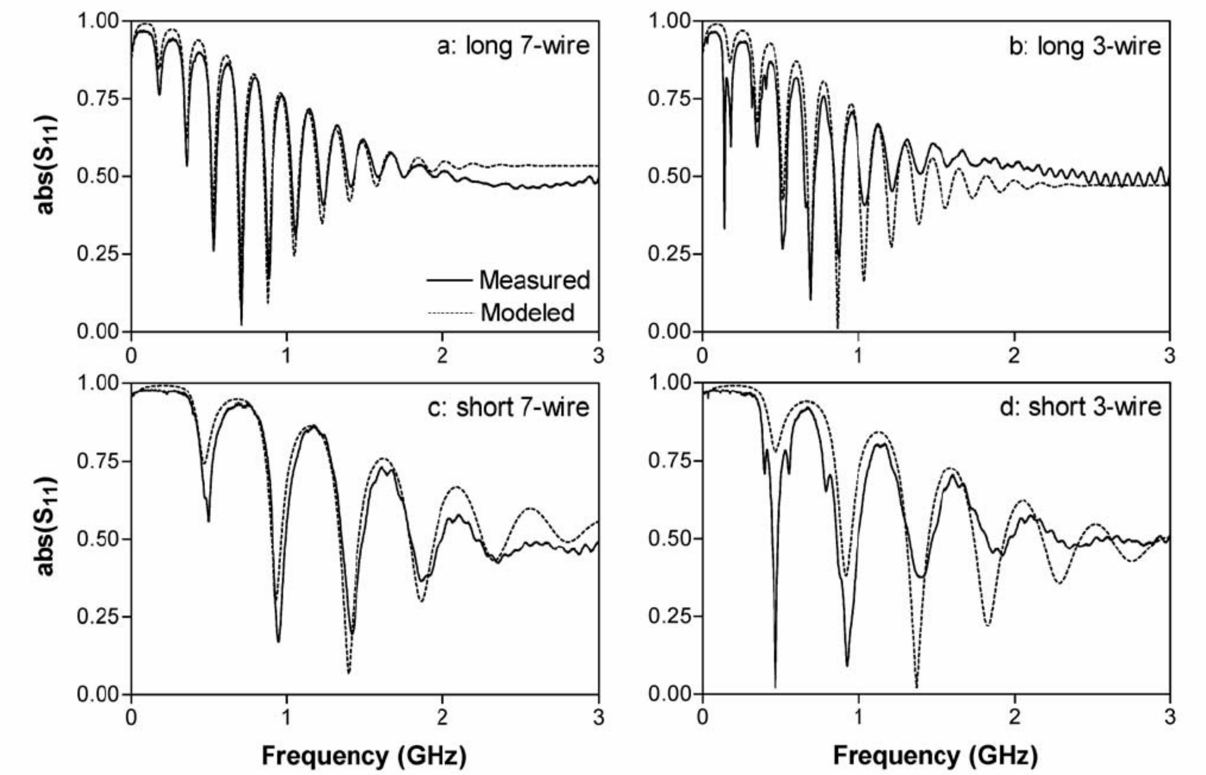

Fig. 6 (from Huisman et al. [9]). Measured and Modelled Magnitude of the Scatter Function for Water for Different Probe Types: (a) 9.6 cm Long 7-Wire Probe; (b) 9.6 cm Long 3-Wire Probe; (c) 3.7 cm Short 7-Wire Probe, and (d) 3.7 cm Short 3-Wire Probe.

Huisman et al.[9] compared 4 probe designs: a long (9.6 cm) and short (3.7 cm) 7-wire and 3-wire probe. The 3-wire probe was obtained after removing the appropriate wires from the 7-wire probe. Therefore, the probes are identical except for the number of wires. Fig. 6 shows frequency domain representations of TDR measurements made in demineralised water for these four probe designs. Clearly, the mismatch between model and measurement is lower for the 7-wire probe than for the 3-wire probe. Also, the wave propagation model seems to be more appropriate for longer probes. For water content profile retrieval, longer probes (up to

1.0 m) are of interest, but these have not yet been investigated in detail. Other aspects of probe design, such as the connection between cable and probe in the probe head, are also known to affect the modelling quality. For example, Nussberger[10] reported that the head transition from an unbalanced to a balanced signal on a 2-wire probe was difficult to model, partly because of a lack of suitable calibration measurements.

Final Remarks

We hope that this review provides an overview of the current state-of-art of inverse modelling of TDR waveforms and points to the future challenges that need to be resolved before TDR can be used to routinely monitor soil water content profiles in the field. Nevertheless, profile reconstruction methods have strongly improved the last years, and the method is ready for extensive testing under laboratory conditions.

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Bibliographic Information

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