# TDR 2006
## Purdue University

### Calibration of Column-Attaching TDR Probe Based on Dielectric Mixing Model

H. Miyamoto and J. Chikushi

Ref.: Miyamoto, H. and Chikushi, J., "Calibration of Column-Attaching TDR Probe Based on Dielectric Mixing Model", Proc. TDR 2006, Purdue University, West Lafayette, USA, Sept. 2006, Paper ID 23, 7 p., https://engineering.purdue.edu/TDR/Papers

# Calibration of Column-Attaching TDR Probe Based on Dielectric Mixing Model

H. Miyamoto [1] and J. Chikushi [2]

[1] Postdoctoral Research Fellow, Biotron Institute, Kyushu University, 6-10-1 Hakozaki, Higashi-ku, Fukuoka 812-8581, JAPAN; PH +81-92-642-3066; FAX +81-92-642-3069; email: miyamotoh@agr.kyushu-u.ac.jp

[2] Professor, Biotron Institute, Kyushu University, 6-10-1 Hakozaki, Higashi-ku, Fukuoka 812-8581, JAPAN; PH +81-92-642-3066; FAX +81-92-642-3069; email: chiku2@agr.kyushu-u.ac.jp

## Abstract

Besides conventional time domain reflectometry (TDR) probe having parallel electrodes, various types of downsized TDR probes, such as coil probe and noninvasive surface probe, have been designed to measure soil-water content ($\theta$). We also designed a column-attaching TDR probe (CA probe) to minimize disturbance of soil matrix by the presence of TDR probe. The CA probe was consisted of a couple of copper foils (length = 15 cm and width = 2 mm) adhered along inner wall of acrylic plastic ring, and was calibrated with sand for various water contents. Relative dielectric constants for sand increased with water content and reached to 12.7 at saturated condition ($\theta$ = 0.43). Compared with the referential relative dielectric constant measured by conventional probe, measured values for the CA probe were smaller for almost all $\theta$ range. The relative dielectric constant measured by the CA probe should be composite dielectric constant ($\varepsilon_T$) of 2 components, medium ($\varepsilon_m$) and acrylic part ($\varepsilon_{ac}$). To evaluate the effect of acrylic part on $\varepsilon_T$, $\varepsilon_T$ was measured with several fluid media, and held in 2-phase dielectric mixing model ($\varepsilon_T = (w\varepsilon_m^{\alpha} + (1-w)\varepsilon_{ac}^{\alpha})^{1/\alpha}$ ; where $w$ and $\alpha$ are fitting parameters). Combined model, by substituting $\varepsilon_m$ in the Topp equation by the estimate of $\varepsilon_m$ from the mixing model ($w$ = 0.412 and $\alpha$ = 1.07), generally agreed with measured $\theta$. The determination of the fitting parameters ($w$ and $\alpha$) with fluid media could make it easier to calibrate relative dielectric constant vs. $\theta$ than that with soil.

**Key Words** column-attaching probe, calibration, Topp equation , dielectric mixing model

## Introduction

Time domain reflectometry (TDR) is widely used as non-destructive method to measure soil-water content ($\theta$). TDR determines relative dielectric constant ($\varepsilon_m$) of soil by measuring the propagation velocity of an electromagnetic wave guided along electrodes. Empirical relationship proposed by Topp et al. (1980)[1]:

$$q = 4.3\times10^{-6}\varepsilon_m^{\ 3} - 5.5\times10^{-4}\varepsilon_m^{\ 2} + 2.92\times10^{-2}\varepsilon_m - 5.3\times10^{-2} \quad (1)$$

provides adequate description for many soils in the range $q < 0.5$ with an error of 0.013 $m^3$ $m^{-3}$ (Topp et al.,1980[1]), except for organic soil and clayey soil (Bridge et al., 1996[2]; Miyamoto et al., 2001[3]).

Parallel TDR probes, consisting of 2 or 3 parallel electrodes, are conventional in previous studies. TDR probes have been downsized for practical use in situ and laboratory experiments. In case of 2-wire probes, probe length can be shortened to 2.5 cm using a high bandwidth TDR instrument (Kelly et al., 1995[4]). In addition to the short probe, various types of downsized TDR probes, having unique shapes, such as coil probe and noninvasive surface probe, have also been designed (Nissen et al., 1998[5]; Selker et al., 1993[6]).

Adequate size of soil sample is required to install TDR probe. For laboratory experiments using cylindrical soil-column, internal diameter of cylindrical container should be larger than the length of TDR probe; the size of soil-column depends on the length of TDR probe. However, such limitation is not practical to conduct small-scaled laboratory experiment. Furthermore, the presence of TDR probe embedded in soil may disturb soil matrix, and affect flow condition of soil-water, regardless of its length and shape. Especially in laboratory experiment using cylindrical soil column, even small-scale disturbance of soil matrix may result in causing error for the experimental result. Although installing TDR probe with simultaneous soil packing could reduce such disturbance, the existence of embedded electrodes often complicates packing soil uniformly.

To conduct non-destructive and small-scaled $q$ measurement, a column-attaching TDR probe (CA probe), adhered along inner wall of acrylic ring, was designed. The CA probe was calibrated for sand at various water contents, and applicability of Topp equation (Eq. 1) to the observed data was investigated. Dielectric characteristics of the CA probe were examined by measuring relative dielectric constant of ethanol having different concentrations, and were compared with those of standard TDR probe. To reduce the additional efforts due to calibration experiment under specific condition, the Topp equation was improved by considering the dielectric characteristics of the CA probe, and the efficiency of the modified equation was investigated.

## Methods

### Probe characterization with fluid media

Figure 1 shows schematic diagram of the CA probe. The CA probe is consisted of a couple of long and thin copper foils (length = 15 cm, width = 0.2 cm, and thickness = 0.008 cm), adhered along inner wall of acrylic plastic ring (internal diameter = 5 cm) at the intermediate height. To evaluate the dielectric characteristics of the probe, relative dielectric constants for several fluid media, such as air, distilled water (DW), and ethanol solutions having different concentrations

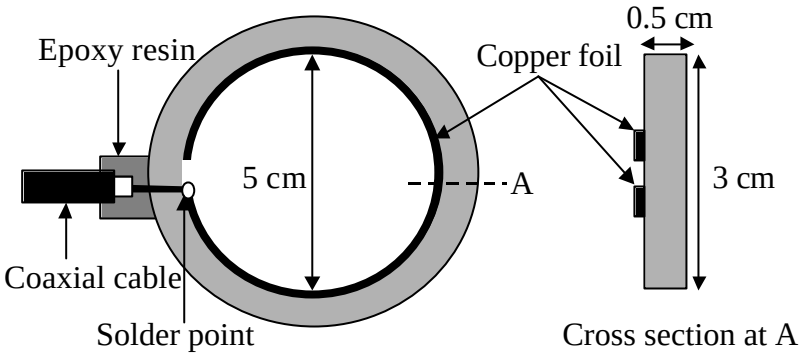


Fig. 1 Schematic diagram of a CA probe.

(0.2, 0.4, 0.6, 0.8, and 1.0 kg $kg^{-1}$), were measured by using Tektronix 1502C cable tester.

**Calibration with sand**

A calibration experiment of the CA probe was carried out with Hakozaki sand, sampled from Fukuoka city, Japan. Air-dried sand ($q$ = 0.002 $m^3$ $m^{-3}$) was segmented, and mixed with distilled water (DW) in plastic bags. The $q$ of sands was adjusted to 0.002, 0.04, 0.09, 0.14, 0.18, 0.23, 0.27, 0.29, 0.33, 0.34, 0.37, 0.39, 0.40, and 0.43 $m^3$ $m^{-3}$ by changing the mixing ratio of the volume of distilled water. These soil samples were packed into the column with the CA probe uniformly (bulk density = 1.5 Mg $m^{-3}$), and relative dielectric constants for each sample were measured. Relative dielectric constants for the same media were also measured with the 3-wire probe (length = 10 cm), which consists of parallel stainless steel rods.

## Result and Discussion

### Probe characteristics

Table 1 shows relative dielectric constants of distilled water (DW), ethanol solutions, and air, measured by the 3-wire probe and the CA probe. For the 3-wire probe, measured relative dielectric constant decreased with increase in ethanol concentration. Although similar tendency for ethanol concentration was observed for the CA probe, the measured values for the CA probe for each media were almost half of those for the 3-wire probe due to the presence of a low-permittivity backing acrylic plastic.

Assuming the relative dielectric constants of fluid media by measuring with the 3-wire probe (Table 1) are generally close to the real relative dielectric constant of the media ($e_m$), we can directly relate $e_m$ with relative dielectric constant measured by the CA probe ($e_T$) as shown in Fig. 2. Except for air, the measured $e_T$ values were lowered below the line of $e_T = e_m$.

Composite dielectric constant of multiphase mixture can be functionally related to the relative

Table 1 Relative dielectric constants of distilled water (DW), ethanol having different concentrations (0.2, 0.4, 0.6, 0.8, 1.0 kg kg$^{-1}$), and air, measured by 3-wire probe and CA probe.

| | Measured relative dielectric constant | |
|---|---|---|
| $C$ (kg kg$^{-1}$)* | 3-wire probe | CA probe |
| 0 (DW**) | 81.0 | 36.4 |
| 0.2 | 67.3 | 30.5 |
| 0.4 | 56.3 | 25.8 |
| 0.6 | 41.6 | 19.6 |
| 0.8 | 29.6 | 14.5 |
| 1.0 | 19.0 | 10.0 |
| Air | 1.5 | 3.1 |

* Ethanol / (ethanol + water) [kg kg$^{-1}$], ** Distilled water

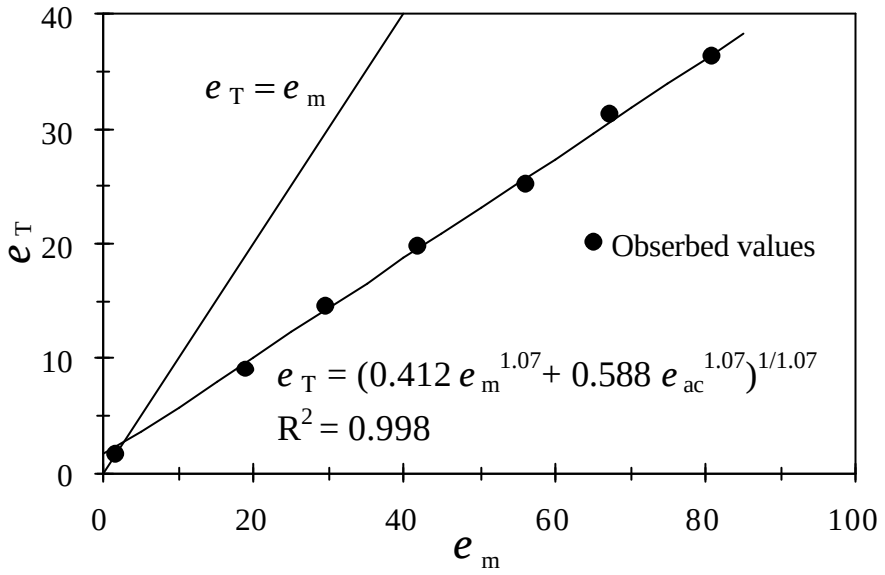


Fig. 2 Relative dielectric constant measured by the CA probe ($e_T$) for relative dielectric constant of fluid media ($e_m$).

dielectric constant and volumetric fractions of materials around TDR probe according to dielectric mixing model (Roth et al., 1990[7]). Since the backing acrylic plastic had enough thickness, air part outside the plastic did not affect on the measured relative dielectric constant. Assuming $e_T$ should be the integration of 2 components, that is inner medium ($e_m$) and acrylic part ($e_{ac}$ = 2.7), we can express the $e_T$ by the following equation, based on 2-phase dielectric mixing model:

$$\varepsilon_T = \left\{ w\varepsilon_m^{\alpha} + (1-w)\varepsilon_{ac}^{\alpha} \right\}^{1/\alpha} \quad (2)$$

where $w$ is weighting factor, and $\alpha$ is geometry exponent. Both $w$ and $\alpha$ depend on the dielectric characteristics of the CA probe. For the determination of both parameters, non-linear least square method was applied to fit Eq. 2 with the data sets of measured $\varepsilon_T$ corresponding to $\varepsilon_m$ of ethanol solutions. The optimized $w$ and $\alpha$ were 0.412 and 1.07, respectively (Eq. 2). The weighting factor for $\varepsilon_m$ was smaller than that for $\varepsilon_{ac}$ (0.588). This indicates that the presence of acrylic part largely affects on $\varepsilon_T$, and thus the CA probe become less sensitive to the change in $\varepsilon_m$ than the 3-wire probe.

**Probe performance with sand**

Figure 3 shows TDR waveforms for distilled water and sand ($\theta$ = 0.002, 0.09, 0.23, 0.33, and 0.43 $m^3$ $m^{-3}$) using the CA probe. Apparent distance ($L_a$), and thus the relative dielectric constant, for the CA probe increased with $\theta$ as well as the 3-wire probe (data is not shown).

The observed $\varepsilon_m$ vs. $\theta$ for the 3-wire probe, and $\varepsilon_T$ vs. $\theta$ for the CA probe are shown in Fig. 4. The $\varepsilon_m$ vs. $\theta$ estimated by Topp equation (Eq. 1) is also drawn in Fig. 4. For the CA probe, sensitivity of $\varepsilon_T$ to $\theta$ was high at wet condition, but less at low $\theta$ range.

Good agreement was obtained between Topp equation and measured values for the 3-wire probe, while observed values for the CA probe disagreed with Topp equation due to the presence of backing acrylic plastic. This result coincided with the result for noninvasive surface probe (Selker et al., 1993[6]). The discrepancy between Topp equation and observed data was presumed as a common characteristic of unique shaped TDR probe.

Substituting $\varepsilon_m$ from Eq. 2 for Topp equation (Eq. 1), we can combine Topp equation with dielectric mixing model as:

$$\theta = 4.3\times10^{-6}\left(\frac{1}{w}\varepsilon_T^{\alpha} - \frac{1-w}{w}\varepsilon_{ac}^{\alpha}\right)^{3/\alpha} - 5.5\times10^{-4}\left(\frac{1}{w}\varepsilon_T^{\alpha} - \frac{1-w}{w}\varepsilon_{ac}^{\alpha}\right)^{2/\alpha} + 2.92\times10^{-2}\left(\frac{1}{w}\varepsilon_T^{\alpha} - \frac{1-w}{w}\varepsilon_{ac}^{\alpha}\right)^{1/\alpha} - 5.3\times10^{-2} \quad (3)$$

The estimated $\varepsilon_T$ vs. $\theta$ according to combined model (Eq. 3) is also drawn with broken line in Fig. 4. The combined model agreed with observed $\varepsilon_T$ vs. $\theta$ for the CA probe. This indicates that the estimation based on the combined model has enough adaptability in the calibration for particular shaped TDR probe.

Equation 2 expresses the relationship between measured relative dielectric constant using the TDR cable tester for the CA probe ($\varepsilon_T$) and the relative dielectric constant of medium ($\varepsilon_m$). In this case, medium is independent of material, that is Eq. 2 and Eq. 3 should be also allowed as calibration equations for sand as well as ethanol solutions. The calibration procedure may be easier for using ethanol solutions than for using soil. This suggests that calibration curve

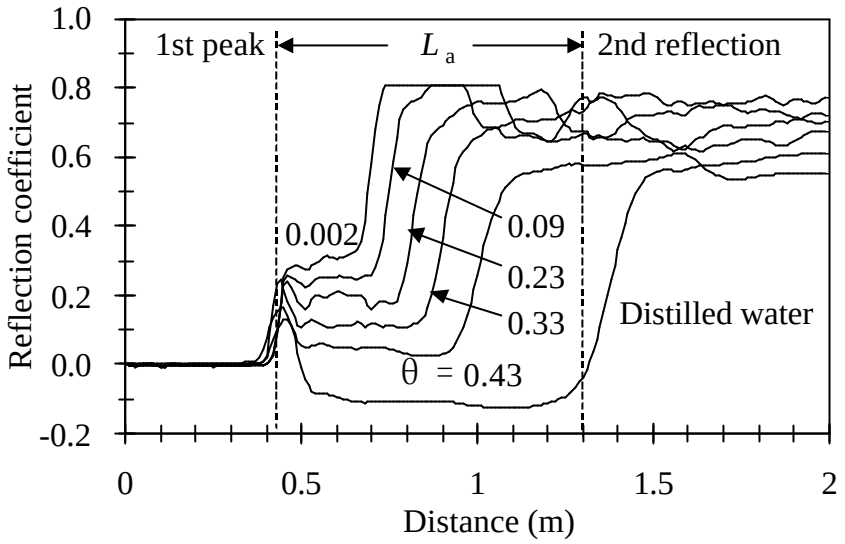


Fig. 3 TDR waveforms of water and sand, measured by CA probe.

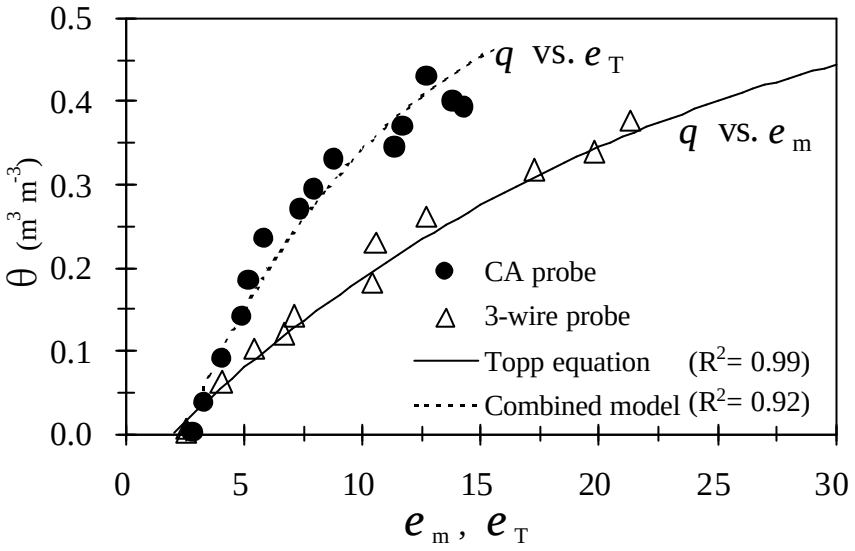


Fig. 4 Volumetric water content ($q$) for relative dielectric constant ($e_m$) and/or composite dielectric constant ($e_T$).

obtained by using ethanol solutions could be easily utilized for the curve for different soil materials.

**Conclusion**

To conduct non-destructive soil-water measurement, a column-attaching TDR probe (CA probe)

was calibrated with sand having different soil-water contents ($q$), and tested with several fluid media. The CA probe is less sensitive to the change in $q$ at relatively dry condition due to the presence of backing acrylic plastic. The probe has, however, enough sensitivity to evaluate $q$ of packed soil samples adequately, especially at relatively high $q$ range. Although Topp equation cannot be directly applied for the CA probe, the modified equation (Eq. 3) could be effective for estimating $q$ from measured composite dielectric constant $e_T$.

## References


[1] Topp, G. C., Davis, J. L., and Annan, A. P. (1980). Electromagnetic determination of soil water content: measurements in coaxial transmission lines. *Water Resour. Res.*, vol. 16, pp. 574-582.

[2] Bridge, B. J., Sabburg, J., Habash, K. O., Ball, J. A. R. and Hancock, N. H. (1996). The dielectric behavior of clay soils and its application to time domain reflectometry. *Aust. J. Soil Res.*, vol. 59, pp. 97-102.

[3] Miyamoto, T., Kobayashi, R., Annaka, T., and Chikushi, J. (2001). Applicability of multiple length TDR probes measure water distributions in an andisol under different tillage systems in Japan. *Soil & Tillage Res.*, 60: 91-99.

[4] Kelly, S. F., Selker, J. S., and Green, J. L. (1995). Using short soil moisture probe with high-bandwidth time domain reflectometry measurements. *Soil Sci. Soc. Am. J.*, vol. 59, pp. 97-102.

[5] Nissen, H. H., Moldrup, P., and Henriksen, K. (1998). High-resolution time domain reflectometry coil probe for measuring soil water content. *Soil Sci. Soc. Am. J.*, vol. 62, pp. 1203-1211.

[6] Selker, J. S., Graff, L., and Steenhuis, T. (1993). Noninvasive time domain reflectometry soil moisture measurement probe. *Soil Sci. Soc. Am. J.*, vol. 57, pp. 934-936.

[7] Roth, K., Schulin, R., Flühler, H., and Attinger, W. (1990). Calibration of time domain reflectometry for water content measurement using composite dielectric approach. *Water Resour. Res.*, vol. 26, pp. 2267-2273.


**Biographical Information**

Hideki Miyamoto obtained his B.S. and M.S. degrees in agricultural engineering, Saga University, and then completed his Ph.D. study in Kagoshima University in 2005. He works at Biotron Institute, Kyushu University as a postdoctoral research fellow.

Jiro Chikushi earned B.S., M.S., and Ph.D. degrees in agricultural engineering, Kyushu University. In 1979, he was on Agricultural Faculty of Tottori University as Assistant Professor. In 1986, he moved to back to Biotron Institute, Kyushu University, as Associate Professor. He promoted to Professor and took up as Director of the Institute in 2000.