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Andrew M. Chambarel and Herve R. Bolvin

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Study of the TDR Signal in a Heterogeneous Polarizable Medium

André M. CHAMBAREL ⁽¹⁾ and Hervé R. BOLVIN ⁽¹⁾

¹ UMR “Climate, Soil, Environment”

University of Avignon, 33 rue Louis Pasteur, F-84000 Avignon;

tel. (33) 4 90 14 44 71; fax. (33) 4 90 14 44 09;

email: andre.chambarel@univ-avignon.fr

Abstract

With the improvement of TDR probes, we can hope to extract more information of the obtained signal. For that, it is necessary to better know the signal of probe TDR in complex media. The signal transports much information about the electrical properties of the concerned medium, particularly in a wet soil. So we hope to determine geophysical properties of the medium.

In a first approach we propose to find a realistic mathematical model for TDR signal interpretation. The electric permittivity is frequency dependent consequently this model is not appropriate in time domain of TDR probe. Moreover if we apply the Fourier Transforms to TDR signal, we notice that the scale of frequency corresponds to the domain of relaxation time of the polarizable medium. Consequently it is not possible to neglect the transient polarization of charged particles. In fact we must take into account the inertia of elementary charges and dielectric loss. For this reason we built a model based on transient electric polarization $\vec{P}$ of the medium. For that we couple Maxwell's equations with the motion of charged particles in an impulse of electric field. So we obtain a more realistic model of TDR signal simulation. Moreover we consider a random space distribution of properties of charged particles. The numerical model is solved in the time domain using the Finite Element Method. A special code is developed for time domain wave simulation. This code is developed in a modern context, particularly an efficient C++ Object-Oriented Programming. Moreover a parallel algorithm can be used for wave simulations.

This numerical work is associated with experimental studies which have for aim the validation of the models. Based on the Campbell probe, the experiments are developed in various heterogeneous media. Numerical profiles of electromagnetic energy are presented and at last we compare the numerical results with the experiments. We notice a good agreement.

Key Words: Electric polarization, Maxwell's equations, Time Domain Reflectometry, Finite Element method.

1 Introduction

Generally the electromagnetic properties of the matter are frequency dependent [1]. Many papers concern this context [2, 3]. In practice for fast transient electromagnetic waves we have not typical frequency for the determination of the electromagnetic properties particularly the electric permittivity. Moreover, the frequency model is not valid because the inertia of charged particles can not be neglected. We propose a numerical model in this case. We use

the basic formulation in first order of Maxwell's equations. The theoretical approach is formulated using 2D Maxwell's equations in TM mode associated with a macroscopic model of polarization. The partial differential system is solved in space by using of the Finite Element Method [4, 5] and in time domain we use an efficient multi step semi implicit method, so we do not develop numerical oscillations. Moreover, a specific code is developed with an efficient C++ Object-Oriented Programming for the Finite Element. This code is called FAFEMO (Fast Adaptive Finite Element Modular Object). This constitutes a general Finite Element solver for the Maxwell equations [5].

To illustrate this model, we use a TDR probe constituted by a wave guide composed of two or three parallel metallic rods stuck in a polarizable medium. At initial time, a step of electric field is applied between the electrodes and the intensity is measured at the entrance of the wave guide [6]. This measurement contains information about the polarizable medium. This work is a contribution to the interpretation of the TDR (Time Domain Reflectometry) probe signals.

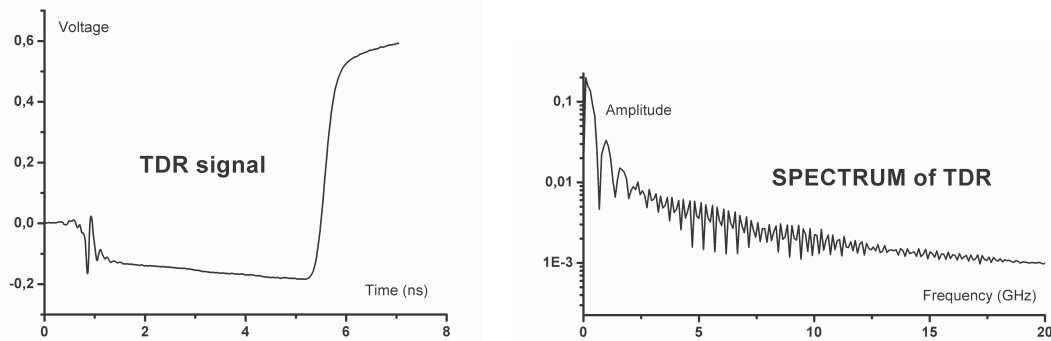


Fig. 1. Signal and spectrum of TDR

Fig. 1 shows an experimental signal of TDR and his FFT. We notice that the characteristic frequencies of TDR signal are similar those of permittivity evolution. In this case, it is not possible to consider this parameter as a constant value or frequency dependent in time domain because the transient polarization. According to the characteristic times we are in the case of a polarization of electric dipoles, particularly in wet medium. Consequently the electric polarization vector is not a linear function of the electric field.. So we couple the Maxwell's equations with a model of motion of electric dipoles.

Moreover the permittivity according to the moisture of the medium is given by the non linear Topp law [7].

2 General presentation of the model

The 2D wave guide (Fig.2) is represented by two parallel plate electrodes and the electric conductivity of these can be considered as infinite according to the used frequencies (Fig. 1). In 2D model we must choose a polarized electromagnetic wave. The electromagnetic wave polarization is the transverse magnetic mode (TM) for a realistic 2D model. We apply a step of voltage at the entrance of the wave guide. The obtained signal is the intensity at the entrance of the line. It is interpreted as a TDR signal according to the characteristic of the voltage generator.

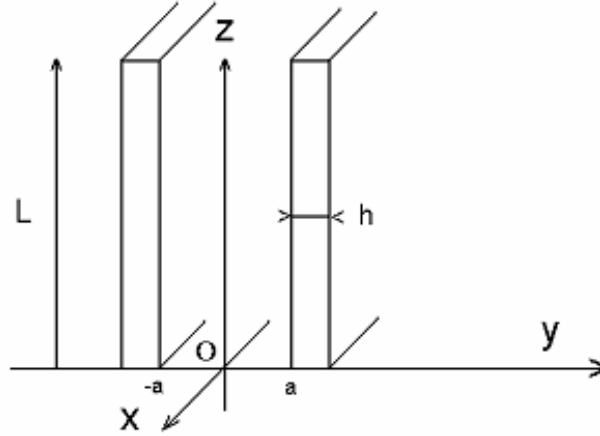


Fig. 2. The geometrical model

In our example we study the Transverse Magnetic mode. The components of the electromagnetic field are:

$$\vec{H} = \begin{Bmatrix} H_x \\ 0 \\ 0 \end{Bmatrix} \quad \text{and} \quad \vec{E} = \begin{Bmatrix} 0 \\ E_y \\ E_z \end{Bmatrix}$$

Considering the usual notations, the general Maxwell's equations are formulated as follows [8]:

$$\begin{cases} \overrightarrow{\text{curl}} \vec{E} = -\frac{\partial \vec{H}}{\partial t} & \text{div} \vec{B} = 0 \\ \overrightarrow{\text{curl}} \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}_c & \text{div} \vec{D} = \rho \end{cases} \quad (1)$$

We introduce in this general formulation the constitutive laws of the medium. We consider an non magnetic and dielectric medium and the constitutive laws can be written:

$$\begin{cases} \vec{B} = \mu_0 \vec{H} \\ \vec{D} = \epsilon_0 \vec{E} + \vec{P} \end{cases} \quad (2)$$

Under theses hypothesis Maxwell's equations become:

$$\begin{cases} \overrightarrow{\text{curl}} \vec{E} = -\frac{\partial \vec{H}}{\partial t} & \text{div} \mu_0 \vec{H} = 0 \\ \overrightarrow{\text{curl}} \vec{H} = \epsilon_0 \cdot \frac{\partial \vec{E}}{\partial t} + \frac{\partial \vec{P}}{\partial t} + \gamma \cdot \vec{E} & \text{div} (\epsilon_0 \vec{E} + \vec{P}) = \rho \end{cases} \quad (3)$$

We consider the polarization vector as unknown in this problem. So we must define a specific model. By introducing a conduction current associated to charged particle motion, we obtain:

$$\vec{J}_c = \frac{\partial \vec{P}}{\partial t} + \gamma \cdot \vec{E} = \frac{dq}{dv} \cdot \vec{V} + \gamma \cdot \vec{E} \quad (4)$$

A classical model is used in which an electric dipole, obeying the following dynamic equations for one particle. We complete the Debye model by introduction of the inertia of dipoles. Here dielectric loss is defined by a macroscopic viscous model [8]:

$$m \cdot \frac{d\vec{V}}{dt} = -a \cdot \vec{V} - k \cdot \vec{X} + q \cdot \vec{E} \quad \text{and} \quad \frac{d\vec{X}}{dt} = \vec{V} \quad (5)$$

If q is the electric charge of the particle, the coefficient a denotes the dielectric loss and k the elasticity constant. If we generalize this model dedicated to one particle in a continuous model for unit volume, we obtain:

$$\frac{dm}{dv} \cdot \frac{d\vec{V}}{dt} + \frac{da}{dv} \cdot \vec{V} + \frac{dk}{dv} \cdot \vec{X} = \frac{dq}{dv} \cdot \vec{E} \quad \text{with} \quad \frac{d\vec{X}}{dt} = \vec{V} \quad (6)$$

Each volumetric parameter d/dv can be a random space distribution. So we formulate a local equation and the mathematical model can be written:

$$\bar{m} \cdot \frac{d\vec{V}}{dt} + \bar{a} \cdot \vec{V} + \bar{k} \cdot \vec{X} = \bar{q} \cdot \vec{E} \Rightarrow \frac{d\vec{V}}{dt} + 2\alpha \cdot \vec{V} + \omega_0^2 \cdot \vec{X} = \frac{\bar{q}}{m} \cdot \vec{E} \quad (7)$$

The water coefficients of this differential equation can be determined by the properties of the concerned medium. If we consider ω -harmonic solutions for this differential equation, we obtain in complex formulation:

$$(\omega_0^2 - \omega^2 + 2i\omega\alpha) \cdot \vec{X} = \frac{\bar{q}}{m} \cdot \vec{E}_0 \cdot e^{i\omega t} \quad \text{or} \quad \vec{V} = \frac{i\omega \frac{\bar{q}}{m} \cdot \vec{E}_0}{(\omega_0^2 - \omega^2 + 2i\omega\alpha)} \cdot e^{i\omega t} \quad (8)$$

The current density can be written:

$$\vec{J}_c = \bar{q} \cdot \vec{V} = \frac{\bar{q}^2}{m} \cdot \frac{\partial \vec{E}}{\partial t} = \frac{\epsilon_0 \cdot \omega_p^2}{(\omega_0^2 - \omega^2 + 2i\omega\alpha)} \cdot \frac{\partial \vec{E}}{\partial t} \quad (9)$$

Under these conditions, with equations (2) and (4) it appears the frequency dependent permittivity of the matter:

$$\epsilon = \epsilon' + i\epsilon'' = \epsilon_0 \cdot \left(1 + \frac{\omega_p^2 (\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2) + 4\alpha^2 \omega^2} \right) + i\epsilon_0 \cdot \frac{-2\omega\alpha\omega_p^2}{(\omega_0^2 - \omega^2) + 4\alpha^2 \omega^2} \quad (10)$$

The frequency dependent electromagnetic properties 18

[6] of the matter allows the determination of the coefficients of differential equation (7). In fact, if we know the permittivity at low, middle and high frequency we obtain the parameters of the differential equation. Under these conditions it is possible to obtain coupling equations between Maxwell's equations and the matter electric properties.

Before numerical resolution we prefer to formulate this problem in dimensionless equations. For that we choose the following reference quantities:

$$\begin{aligned} L & \text{length of wave guide} \\ c & \text{celerity of electromagnetic waves} \\ t_0 = \frac{L}{c} & \text{characteristic time} \\ E_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} H_0 & \text{characteristic fields} \\ R_m = \gamma \cdot \mu_0 \cdot L \cdot c & \text{magnetic Reynolds number} \end{aligned}$$

$$t^* = \frac{t}{t_0} \quad x_i^* = \frac{x_i}{L} \quad H^* = \frac{H}{H_0} \quad E^* = \frac{E}{E_0} \quad \text{with} \quad H_0 = E_0 \cdot \sqrt{\frac{\mu_0}{\epsilon_0}}$$

The dimensionless Maxwell's equations become respectively:

$$\begin{cases} \overrightarrow{\text{curl}} \vec{E} = -\frac{\partial \vec{H}}{\partial t} & \text{div} \vec{H} = 0 \\ \overrightarrow{\text{curl}} \vec{H} = \frac{\partial \vec{E}}{\partial t} + N \cdot \vec{V} + R_m \cdot \vec{E} & \text{div}(\vec{E} + \vec{P}) = \rho^* \end{cases} \quad (11)$$

The Maxwell's equations are associated with the equations of particle motion, on the form:

$$\begin{cases} \frac{d\vec{V}}{dt} = -\frac{\tau}{\tau_0} \cdot \vec{V} - \omega^2 \cdot \tau^2 \cdot \vec{X} + \vec{E} \\ \frac{d\vec{X}}{dt} = \vec{V} \end{cases} \quad (12)$$

τ and (or) ω correspond to a random space distribution.

The boundary conditions on the electric conductor - i.e. electrodes - can be written:

$$\vec{E} \times \vec{n} = \vec{0} \quad (13)$$

At initial time, we apply a constant electric potential difference between the electrodes. Moreover, the electromagnetic field is zero in the domain and the charged particles are motionless.

In all cases of TDR procedure, the electromagnetic wave never reaches that boundary because we only study the first reflection in the wave guide for the detection of electric properties. On the electrodes, we can calculate line current I by the Ampere theorem with the superficial current's density [8].

3 Finite Element Formulation

The Galerkin Finite Element method is applied to Maxwell's equations (11), and to equation (12). We use vector $(\vec{\delta H}, \vec{\delta E}, \vec{\delta V}, \vec{\delta X})$ and the weighted residual method can be written:

$$\left\{ \begin{array}{l} \int_{(\Omega)} \vec{\delta H} \cdot \frac{\partial \vec{H}}{\partial t} d\Omega = - \int_{(\Omega)} \vec{\delta H} \cdot \overrightarrow{curl} \vec{E} d\Omega \\ \int_{(\Omega)} \vec{\delta E} \cdot \frac{\partial \vec{E}}{\partial t} d\Omega = \int_{(\Omega)} \vec{\delta E} \cdot \overrightarrow{curl} \vec{H} d\Omega - \int_{(\Omega)} \vec{\delta E} \cdot N \cdot \vec{V} d\Omega - \int_{(\Omega)} \vec{\delta E} \cdot R_m \cdot \vec{E} d\Omega \\ \int_{(\Omega)} \vec{\delta V} \cdot \frac{\partial \vec{V}}{\partial t} d\Omega = \int_{(\Omega)} \vec{\delta V} \cdot \vec{E} d\Omega - \int_{(\Omega)} \vec{\delta V} \cdot \frac{\tau}{\tau_0} \cdot \vec{V} d\Omega - \int_{(\Omega)} \vec{\delta V} \cdot \omega^2 \cdot \tau^2 \cdot \vec{X} d\Omega \\ \int_{(\Omega)} \vec{\delta X} \cdot \frac{\partial \vec{X}}{\partial t} d\Omega = \int_{(\Omega)} \vec{\delta X} \cdot \vec{V} d\Omega \end{array} \right. \quad (14)$$

After discretization of domain Ω with ne elements Ω_e and using Lagrange's polynomial base we obtain the following matricial structure:

$$\sum_{ne} \langle \delta u_e \rangle \cdot [m_e] \left\{ \frac{du_e}{dt} \right\} + [k_e] \{u_e\} - \{f_e\} = 0 \quad (15)$$

where $u_e = (H_x^{(e)}, E_y^{(e)}, E_z^{(e)}, V_y^{(e)}, V_z^{(e)}, X_y^{(e)}, X_z^{(e)})$

- $[m_e]$ is called the mass matrix
- $[k_e]$ is the electric stiffness matrix
- $\{f_e\}$ the elementary electric load.

By choice of the numerical integration each block in the mass matrix is diagonal. The elementary mass matrix is also diagonal. Consequently the global mass matrix is diagonal, it is an absolute necessity to run the algorithm describe in paragraph 4.1.

With:

$$[m_e] = \begin{bmatrix} \{n\}\langle n \rangle & [0] & [0] & [0] & [0] & [0] & [0] \\ [0] & \{n\}\langle n \rangle & [0] & [0] & [0] & [0] & [0] \\ [0] & [0] & \{n\}\langle n \rangle & [0] & [0] & [0] & [0] \\ [0] & [0] & [0] & \{n\}\langle n \rangle & [0] & [0] & [0] \\ [0] & [0] & [0] & [0] & \{n\}\langle n \rangle & [0] & [0] \\ [0] & [0] & [0] & [0] & [0] & \{n\}\langle n \rangle & [0] \\ [0] & [0] & [0] & [0] & [0] & [0] & \{n\}\langle n \rangle \end{bmatrix}$$

and

$$[k_e] = \begin{bmatrix} [0] & \{n\} \left\langle \frac{\partial n}{\partial z} \right\rangle & -\{n\} \left\langle \frac{\partial n}{\partial y} \right\rangle & [0] & [0] & [0] & [0] \\ -\{n\} \left\langle \frac{\partial n}{\partial z} \right\rangle & R_m \cdot \{n\} \langle n \rangle & [0] & \{n\} \langle n \rangle & [0] & [0] & [0] \\ \{n\} \left\langle \frac{\partial n}{\partial y} \right\rangle & [0] & R_m \cdot \{n\} \langle n \rangle & \{n\} \langle n \rangle & [0] & [0] & [0] \\ [0] & -\{n\} \langle n \rangle & [0] & \frac{\tau}{\tau_0} \cdot \{n\} \langle n \rangle & [0] & \omega^2 \cdot \tau^2 \cdot \{n\} \langle n \rangle & [0] \\ [0] & [0] & -\{n\} \langle n \rangle & [0] & \frac{\tau}{\tau_0} \cdot \{n\} \langle n \rangle & [0] & \omega^2 \cdot \tau^2 \cdot \{n\} \langle n \rangle \\ [0] & [0] & [0] & -\{n\} \langle n \rangle & [0] & [0] & [0] \\ [0] & [0] & [0] & [0] & -\{n\} \langle n \rangle & [0] & [0] \end{bmatrix}$$

In this formulation (15), the physical parameters appear as elementary properties. Their values are distributed to each element.

After a classical assembling operation, the differential system is as follows [5]:

$$[M] \cdot \frac{d}{dt} \{U\} = \{F\} - [K] \cdot \{U\}$$

$$\text{or} \quad \frac{d}{dt} \{U\} = [M]^{-1} \cdot \{\Psi(U, t)\} \quad \text{where} \quad \{\Psi(U, t)\} = \{F\} - [K] \cdot \{U\} \quad (16)$$

4 Numerical resolution

4.1 Semi-implicit Method

The corresponding k order algorithm is as follows:

$$\begin{aligned} & t_n = 0 \\ & \text{while } (t_n \leq t_{\max}) \\ & \quad \left\{ \begin{aligned} & \left\{ \Delta U_n^i \right\} = \Delta t \cdot \beta \cdot [M]_{n+\theta}^{-1} \cdot \psi_{n+\theta} + \sum_{i=1}^p \alpha_i \cdot [M]_{k+1-i}^{-1} \cdot \psi_{k+1-i} \\ & i = 1, 2, \dots \quad \text{until} \quad \left\| \Delta U_n^i - \Delta U_n^{i-1} \right\| \leq \text{tolerance} \end{aligned} \right\} \\ & \quad \{U_{n+1}\} = \{U_n\} + \{\Delta U_n\} \\ & \quad t_{n+1} = t_n + \Delta t_n \\ & \text{end while} \end{aligned}$$

where θ is a upward time-parameter.

4.2 The Code and the Automatic Multigrid System (AMS)

We use efficient C++ Object-Oriented Programming for the Finite Element code called FAFEMO (Fast Adaptive Finite Element Modular Object) developed by A. Chambarel [6]. In

this context, our numerical calculus uses a technique called the AMS. For all iterative or step-by-step processes, an expert system chooses the unknown degrees of freedom for the update of the solution, and the size of the unknown vector is optimized.

4.3 Space discretization

The 2D model is presented in Fig. 2. We discretized the domain with triangular linear elements. They are described in literature references [9]. We refined the meshing near the electrodes (Fig. 3), so we have 13,985 triangular elements and 7,300 nodes. 51,100 differential equations are generated by the above Finite Element process. For numerical quadrature points, the nodes of the elements are chosen. So mass matrix $[M]$ (19) is diagonal. Then, inversion is an easy procedure. Under these conditions we have tested a semi-implicit method. We have used a matrix-free technique: the mass matrix and the stiffness matrix are never built; only the elementary matrices are calculated.

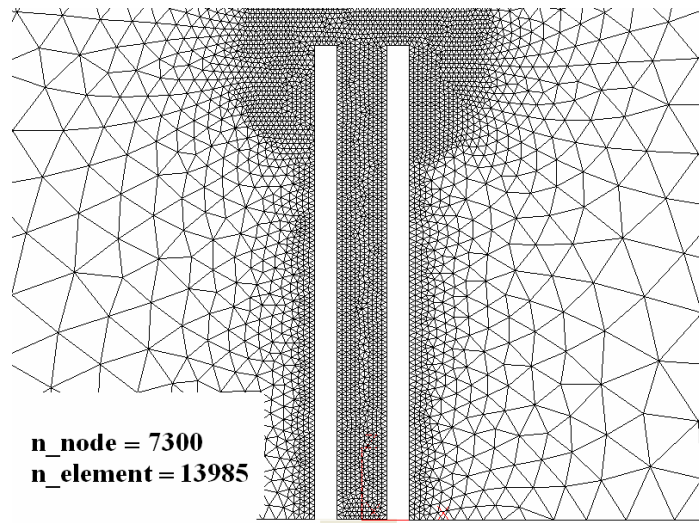


Fig.3. Local meshing

4.4 Numerical Parameters.

The harmonic mode allows the determination of the coefficients of differential equation (12) because the frequency dependent laws are known for water (Table 1).

Table 1. Coefficients of differential equation (12)

N	$\omega^2 \cdot \tau^2$	τ / τ_0
10^6	1.0×10^5	2500
10^7 (random)	4.0×10^5	//

Electric charge density N allows the determination of a relative permittivity of the medium at low frequency. So we can accede to the soil moisture with Topp formula [7].

On Fig. 4 we can see the propagation of the magnetic field in the wave guide, for growing values of electric charge density N for the same non-dimensional time. We notice a decrease in the celerity of the electromagnetic wave in the guide if N increases. This decrease of the celerity is coherent with an apparent value of permittivity. In a medium with a density of

charged particles of 10^6 we build a random space-distribution a percentage (5% $\rightarrow$ 25%) a density of 10^7 . In fact it is a heterogeneous wet.

Moreover we consider a medium with two components, in our physical model it corresponds at two different frequencies. In our example we choose ω_0 and $2\omega_0$ with a random space distribution with the ratio 25% to 75%.

5 Numerical results

First of all we consider a homogeneous medium (Fig. 4a) to compare with the heterogeneous medium. Next we present the case of a heterogeneous medium to simulate a wet ground with space random distribution of properties. At end we present numerical results applied to a mixture of two components with different properties of polarization. In according to the polarization model, we notice that the celerity of electromagnetic waves is strongly modified.

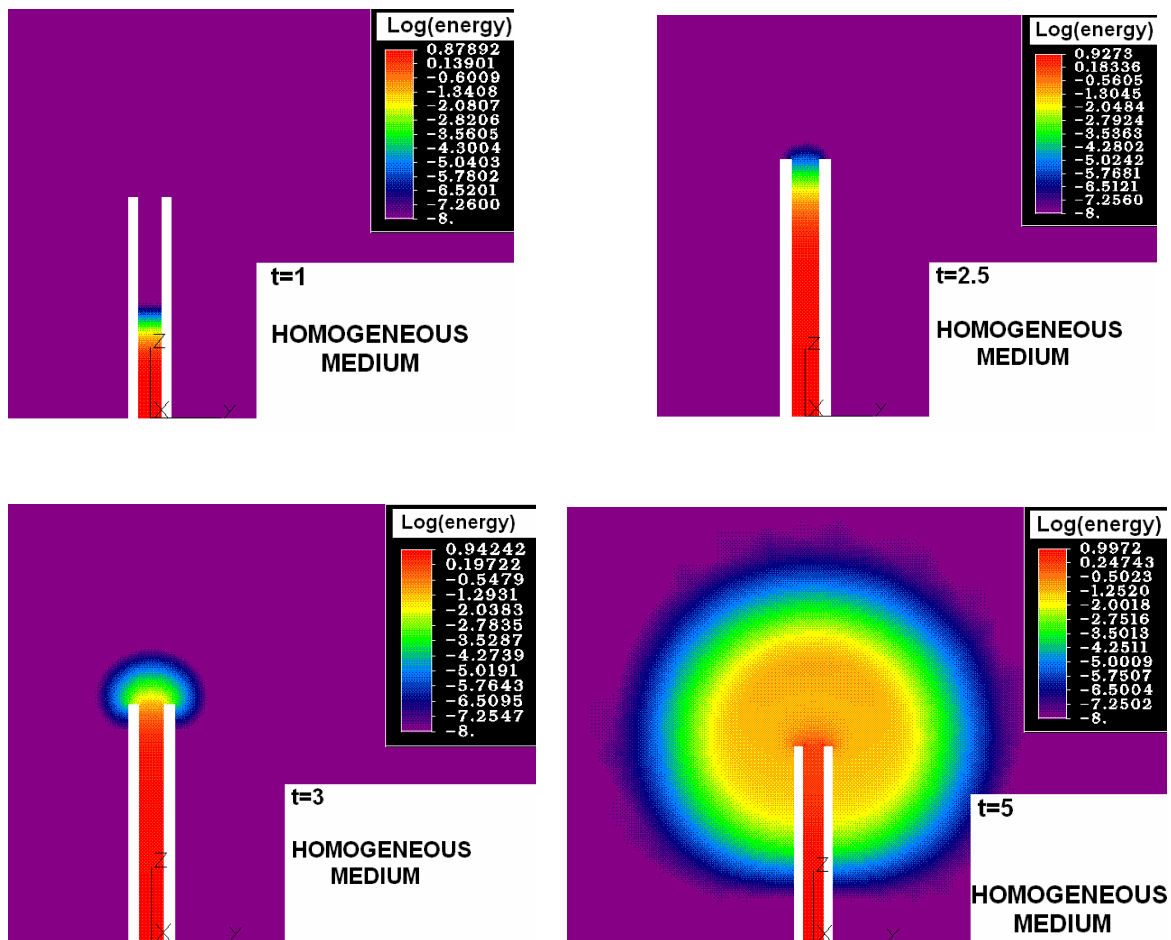


Fig. 4a. Electromagnetic is propagating in a homogeneous medium

In a homogeneous media, we consider an uniform distribution of the polarization properties. We notice a modification of the wave front profile according to the case of constant electric properties. In this case the transient polarization is active and gives information about the composition of the ground particularly the moisture.

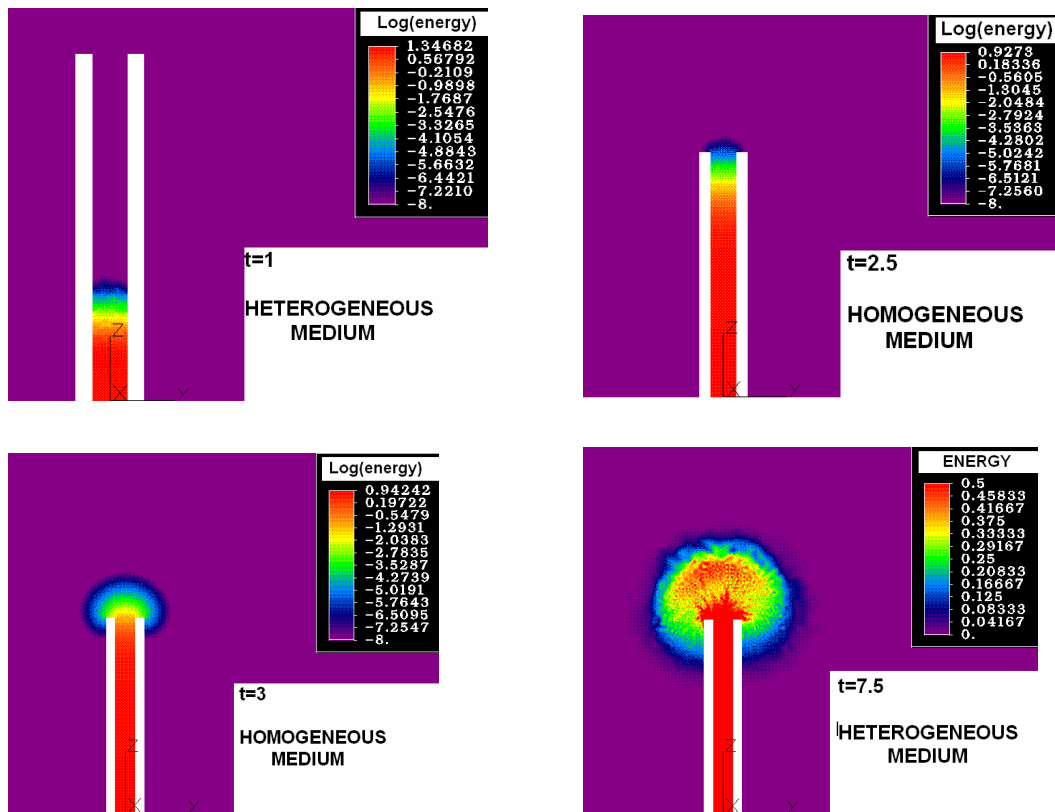


Fig. 4b. Electromagnetic is propagating in a heterogeneous medium.

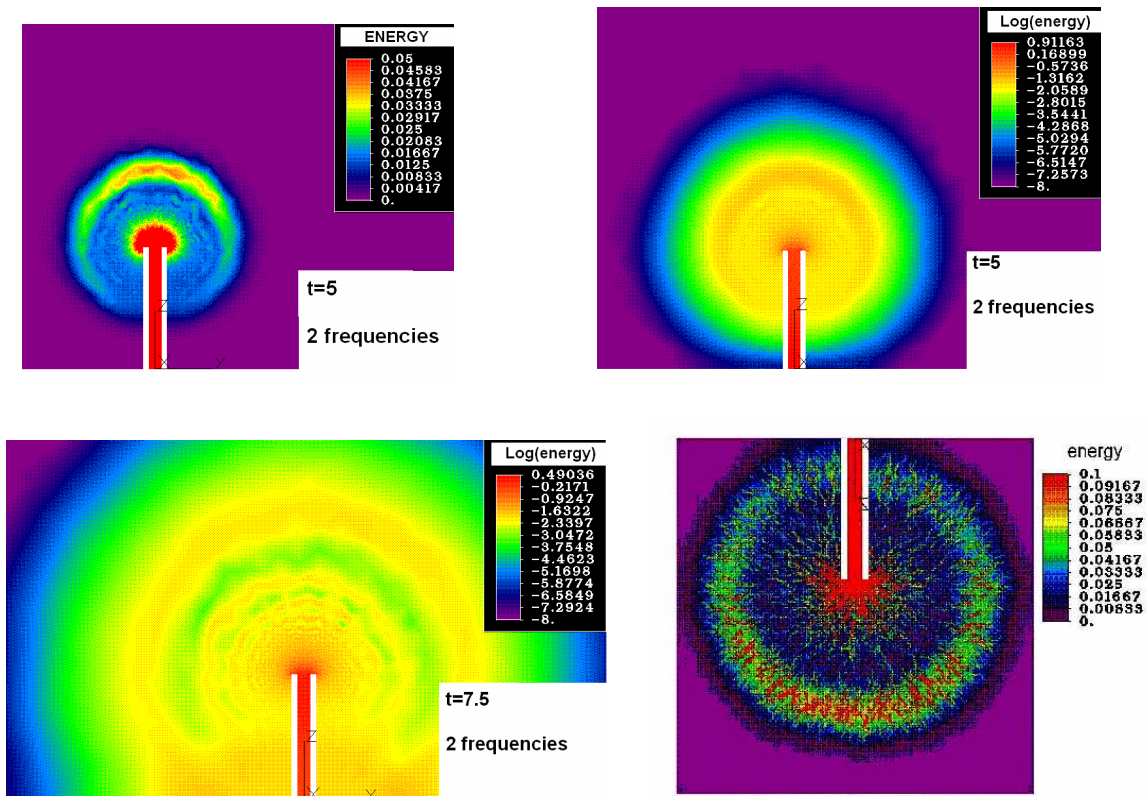


Fig. 4c. Two different polarizable media.

Figure 4b presents the case of a heterogeneous medium. The model concerns a random space distribution of the number of polarizable electric dipoles and appears as an elementary property of finite element grid.. Table 1 shows the numerical values of the density. The electric. Figure 4c present the case of a mixture of two components with a characteristic frequency random space distribution..

6 Comparison with experiment

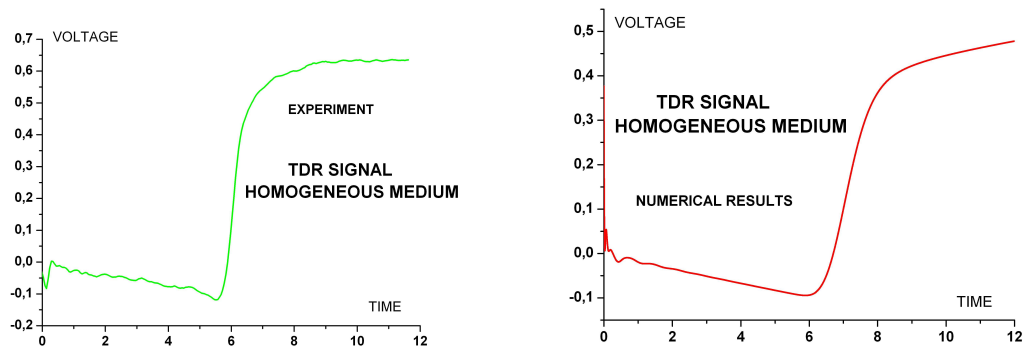


Fig. 5a. The homogeneous medium.

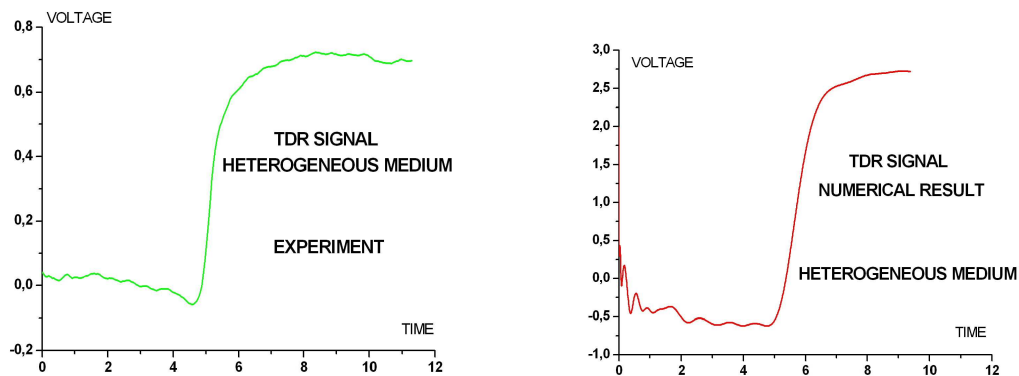


Fig. 5b. The heterogeneous medium.

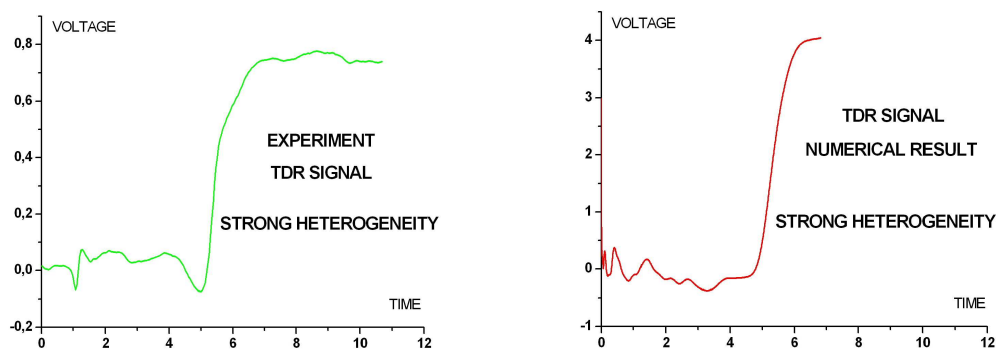


Fig. 5c. Strong heterogeneity

The numerical TDR signal is constructed according to the intensity at the entrance of TDR and the characteristics of the voltage supply. The figures 5a, 5b and 5c present different cases of an experiment and the numerical results. The experiments are performed with sand and water in several configurations of heterogeneity. It is very difficult to prepare experiments with an exact profile of moisture. The TDR probes are of type TEKTRONIX model 1502 C cable tester. In the case of strong heterogeneities we notice that the beginning signal is irregular because the dielectric loss. However we notice a coherent comparison between the numerical results and the experiments.

6 Conclusion

We present a general Finite Element formulation for 2D Maxwell's equations in the case of a propagation phenomenon in a TDR probe stuck in a complex medium as a soil. We present an efficient numerical tool adapted to these problems, particularly the numerical resolution of wave equations in complex cases in time domain. The aim is to contribute to the interpretation of the probe signals: we take into account the electric polarization of the dielectric medium. So we present a first level of polarization model and we validate the numerical method by a comparison with the experiment. This model can be adapted to other types of polarization or with conduction, according to soil characteristics.

References

- [1] Fletcher Armstrong C., Ligon J.T. and Thomson S.J., Calibration of Watermark model 200 soil moisture sensor. *Summer Meeting of American Society of Agricultural Engineers*, Michigan State University, 1985, 1-11.
- [2] Gaudu J.C., Chanzy A., Mohrath D., Mesure de l'humidité des sols par une méthode capacitive : analyse des facteurs influençant la mesure. *Agronomie (Paris)*, 1993, Vol. 13 (1), 57-73.
- [3] Spaans E.J.A. and Baker J., Examining the use of Time Domain Reflectometry for measuring liquid water-content in frozen soil. *Water Resour. Res.*, 1995, Vol. 31 (12), 2917-2925.
- [4] Assous F., Degond P., Heintze E., Raviard P.A. and Segre J., On a Finite Element Method for Solving the Three-Dimensional Maxwell Equations. *Journal of Computational Physics*, 1993, Vol. 109, 222-237.
- [5] Chambarel A. and Bolvin H., A general parallel computing approach using the Finite Element method and the Objects-Oriented programming by selected data technique. *Lecture Notes in Computer Science*, 2001, Vol. 2127, 421-429.
- [6] Heimovara T.J., Frequency domain modeling of TDR waveforms in order to obtain frequency dependent dielectric properties of soil sample: a theoretical approach. *Proceeding of the TDR 2001, Second International Symposium and Workshop on TDR for innovative geotechnical applications*, Northwestern University, Evanston, September 5-8, 2001, 9-21.

[7] Topp G.C., Davis J.L. and Annan A.P., Electromagnetic determination of soil water content : measurements in coaxial transmission lines. *Water Resour. Res.*, 1980, Vol. 16 (3), 574 -582.

[8] Feynman R., The Feynman Lectures on Physics, *Electromagnetism*, Addison-Wesley Publishing Co., Inc., Reading, MA, 1979.

[9] Jianming J., *The Finite Element Method in Electromagnetics*, 2nd edition. John Wiley & Sons Ed., New-York, 2002.

Authors

Andre CHAMBAREL, professor

Born in 1944

Work in TDR context since ten years.

Laboratory:

UMR “Climate, Soil, Environment”

University of Avignon - <http://www.univ-avignon.fr>

33 rue Louis Pasteur, F-84000 Avignon;

tel. (33) 4 90 14 44 71

e_mail: andre.chambarel@univ-avignon.fr

Herve Bolvin, lecturer

Born in 1947

Work in TDR context since seven years

Laboratory:

UMR “Climate, Soil, Environment”

University of Avignon - <http://www.univ-avignon.fr>

33 rue Louis Pasteur, F-84000 Avignon;

tel. (33) 4 90 14 44 71

e_mail : herve.bolvin@univ-avignon.fr