
ECE 661 Homework #5

Name: Haoyu Chen
Email: chen1562@purdue.edu
ID Number: 00271-72202

5 Oct 2020

1 Theory Questions

1.1 Conceptually speaking, how do we differentiate between the inliers and the outliers when using RANSAC for solving the homography estimation problem using the interest points extracted from two different photos of the same scene?

In RANSAC algorithm, we take several (N) random experiments. For each of these experiment, we will compute a estimated homography based on only a few points. Using that homography, we will check each point correspondence and see if the computed coordinate lies within acceptable range of the actual coordinate (i.e., distance between computed coordinate and actual coordinate is maller than δ); if so, that point correspondence is considered an inlier. With enough iterations, we shall find a homography estimation with enough inliers and assume that this is a close estimation of the true homography between two images, and inliers are true correspondences between two images. Hence, a refined homography is computed based on all the inlier correspondences.

1.2 As you will see in Lecture 12, the Gradient-Descent (GD) is a reliable method for minimizing a cost function, but it can be excruciatingly slow. At the other extreme, we have the much faster Gauss-Newton (GN) method but it can be numerically unstable. Explain in your own words how the Levenberg-Marquardt (LM) algorithm combines the best of GD and GN to give us a method that is reasonably fast and numerically stable at the same time.

Gradient-Descent works best when the estimation is rather further from actual local minimum, but gets much slower near local minimum. Gauss-Newton, however, is rather unstable when further away from local minimum, but could provide fast one-step solution when the estimation is closer to local minimum. Levenberg-Marquardt (LM) combines Gradient-Descent (GD) and Gauss-Newton (GN) by adding a control factor μ . By adjusting μ with iterative trials, one essentially shifts between Gradient-Descent (GD) and Gauss-Newton (GN). When the estimation starts, μ is set to be large so that the step taken is closer to that of Gradient-Descent; then, as the estimation gets closer to actual local minimum, μ decreases and the step size and direction are closer to that of Gauss-Newton, therefore combining the best of two algorithm.

2 Algorithm Implementation

2.1 RANSAC

As briefly stated earlier, In RANSAC algorithm, we take N random experiments hoping to find a close estimation of the actual homography, and then refine the homography by using all inlier correspondence find through the estimated homography. In this homework, I choose the following parameters:

- $\delta = 3$ for the common assumption discussed in lecture, whereas $\sigma = 1$

- $p = 0.99$ for the probability that at least one trial is free of outliers
- For ϵ choice, I tried 0.25, 0.5, and 0.75, and counter-intuitively, 0.5 and 0.75 works better. This could be because the threshold was set too high when establishing correspondences using SIFT+SSD, and percentage of outliers are actually around or higher than 50%, especially in more complex scenes. Do note that even though the percentage of outliers are higher than 50%, they do not form a consistent homography, and the largest set of inliers that agree with an estimated homography would still be the actual inlier set.
- $n = 4$, for estimating homography from n random point pairs
- $N = \frac{\ln(1-p)}{\ln(1-(1-\epsilon)^n)}$; $N = 70$ for $\epsilon = 0.5$, $N = 1176$ for $\epsilon = 0.75$
- $M = n_{total}(1 - \epsilon)$

2.2 Least Square Estimation

Now that we have a group of inlier correspondences, we'll use Linear Least Square to estimate the homography.

As mentioned in hw2, given a homography \mathbf{H} that maps \mathbf{x} to \mathbf{x}' , we can write in homogeneous coordinates: $\mathbf{x}' = \mathbf{H}\mathbf{x}$. Let $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$, and $\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix}$ (since homogeneous coordinates

is all about ratios, we can set $x_3 = 1$ and $h_{33} = 1$). We have $\mathbf{x}' = \begin{bmatrix} x'_1 \\ x'_2 \\ x'_3 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$.

Now, if we divide x'_1 and x'_2 by $x'_3 = h_{31}x_1 + h_{32}x_2 + 1$, we now have $\mathbf{x}' = \begin{bmatrix} x'_1 \\ x'_2 \\ 1 \end{bmatrix}$, where

$$x'_1 = \frac{h_{11}x_1 + h_{12}x_2 + h_{13}}{h_{31}x_1 + h_{32}x_2 + 1}$$

$$x'_2 = \frac{h_{21}x_1 + h_{22}x_2 + h_{23}}{h_{31}x_1 + h_{32}x_2 + 1}$$

i.e., $h_{31}x'_1x_1 + h_{32}x'_1x_2 + x'_1 = h_{11}x_1 + h_{12}x_2 + h_{13} \Rightarrow x'_1 = h_{11}x_1 + h_{12}x_2 + h_{13} - h_{31}x'_1x_1 - h_{32}x'_1x_2$.

Hence, for each point pair, we could acquire two equations for solving \mathbf{H} :

$$\begin{bmatrix} x'_1 \\ x'_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & 1 & 0 & 0 & 0 & -x_1x'_1 & -x_2x'_1 \\ 0 & 0 & 0 & x_1 & x_2 & 1 & -x_1x'_2 & -x_2x'_2 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix}$$

With more than four points, we have essentially an over-determined system, and the linear least-square estimation can be computed using pseudo-inverse

$$\hat{\mathbf{h}} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b}$$

2.3 Levenberg-Marquardt

Now that we have an estimation for \mathbf{H} , we'll use an off-the-shelf implementation of Levenberg-Marquardt algorithm provided by SciPy, to refine the estimation. In this case, the cost function is given by the error between computed coordinates using \mathbf{H} and actual coordinates in range space.

2.4 Pixel Mapping

For pixel mapping, I used the value of nearest pixel to computed coordinate. Also, when two images overlap, I choose not to replace the previous layer (i.e., if a pixel already has value assigned to it, the pixel mapping function won't replace it).

3 Images and Results

3.1 Inputs

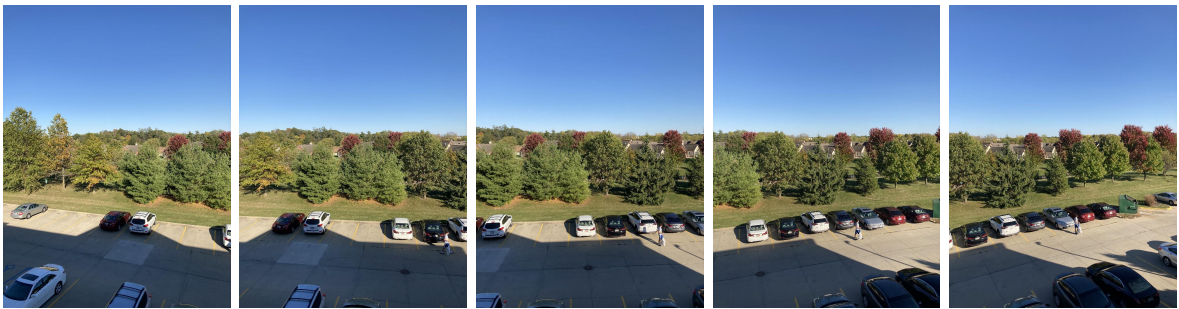


Figure 1: 5 input images

3.2 Correspondences, Inliers, and Outliers

3.2.1 Image 1 and 2

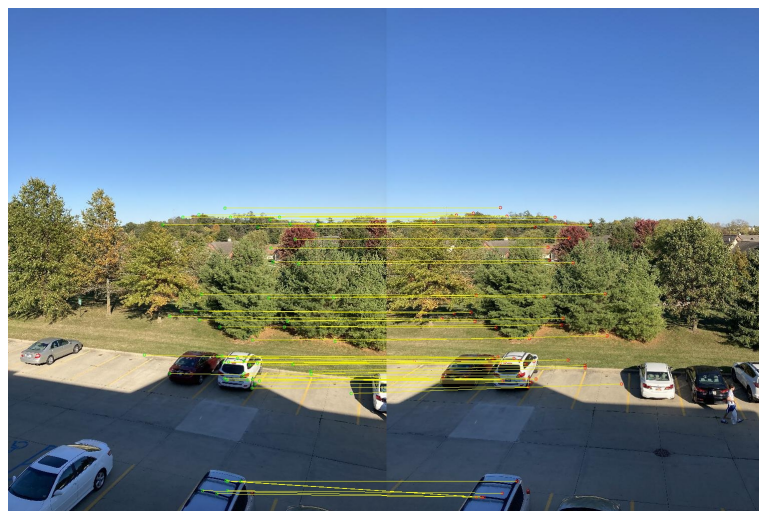


Figure 2: All SIFT correspondences between image 1 and 2

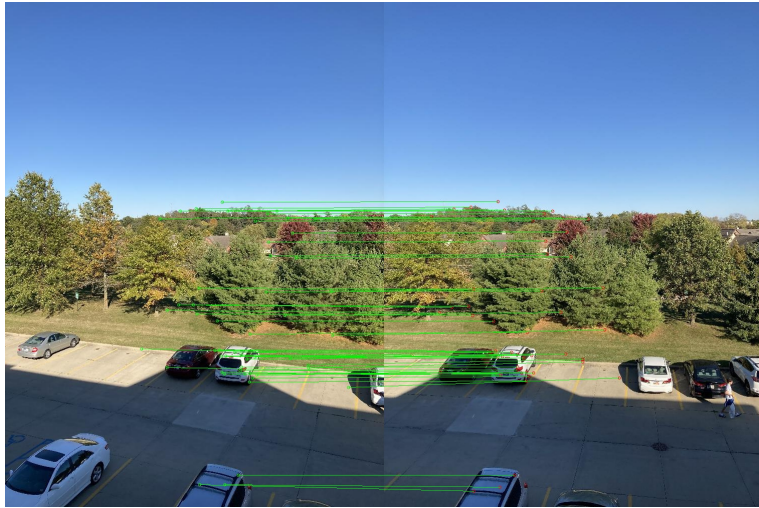


Figure 3: Inliers determined by RANSAC



Figure 4: Outliers rejected by RANSAC

3.2.2 Image 2 and 3



Figure 5: All SIFT correspondences between image 2 and 3

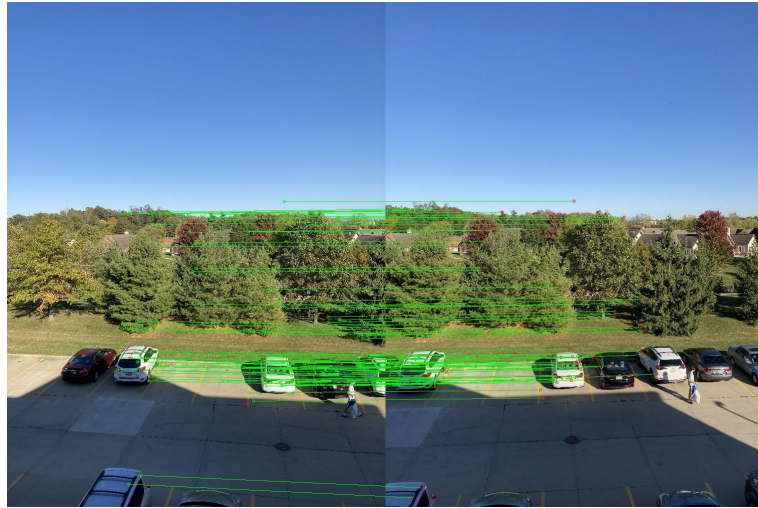


Figure 6: Inliers determined by RANSAC



Figure 7: Outliers rejected by RANSAC

3.2.3 Image 3 and 4

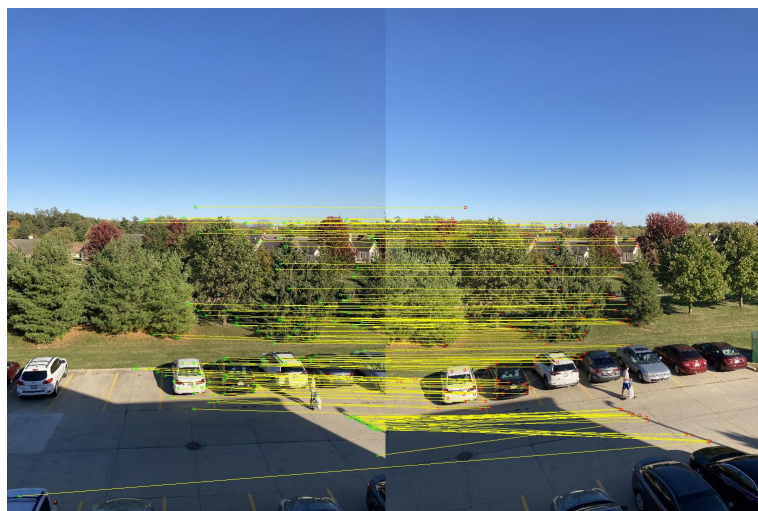


Figure 8: All SIFT correspondences between image 3 and 4

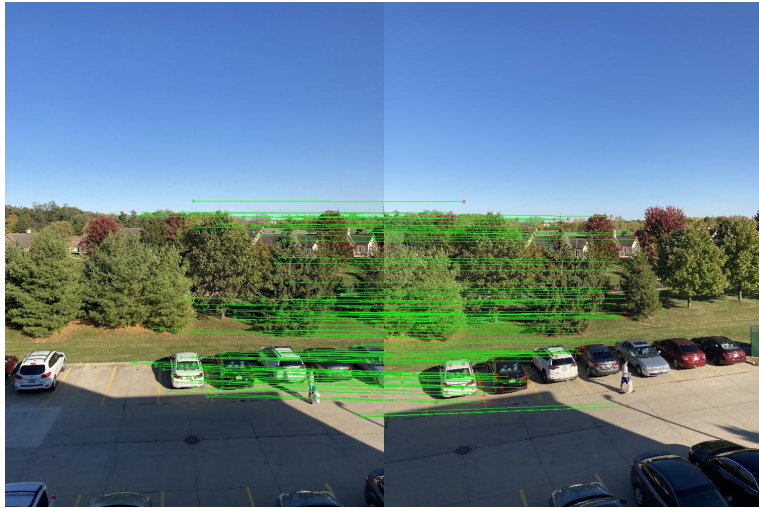


Figure 9: Inliers determined by RANSAC



Figure 10: Outliers rejected by RANSAC

3.2.4 Image 4 and 5

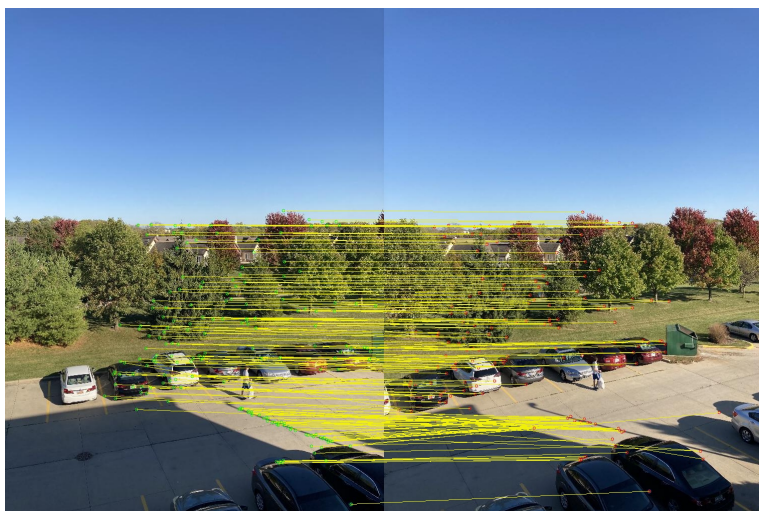


Figure 11: All SIFT correspondences between image 4 and 5

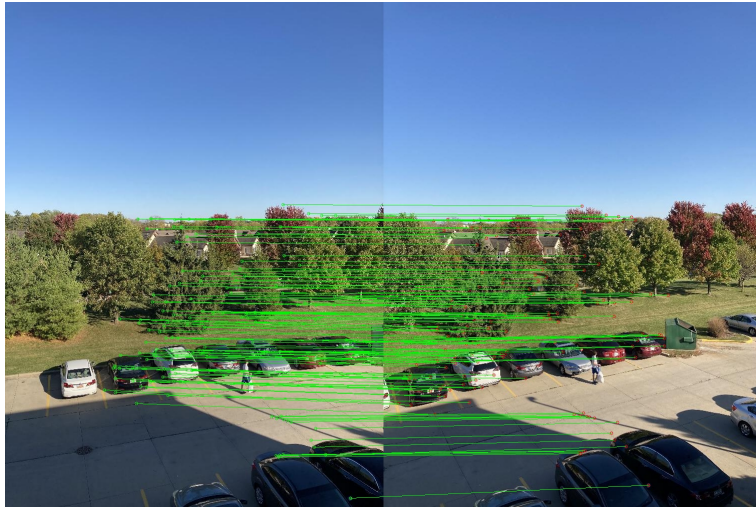


Figure 12: Inliers determined by RANSAC



Figure 13: Outliers rejected by RANSAC

3.3 Output



Figure 14: Output panorama image, without LM refinement



Figure 15: Output panorama image, with LM refinement

4 Remarks

Although the output image from above scene seems pretty good, I did observe a few problems with other scenes, especially indoor scenes.

Alternative scene 1 (Section 5.1) shows example with uneven plane. As the left record and middle one are well-aligned, the right one seems to be a bit uneven, and the result did show a slight discrepancy on the right-most side, with or without LM refinement.

More significantly, in alternative scene 2 (Section 5.2), the two main object (monitor and poster) are on two different plane. Hence, the homography for the two planes across 5 images are different, and the RANSAC algorithm chooses the more dominant plane—monitor. As a result, the panorama image shows a rather consistent monitor plane, but completely fails to connect the poster.

5 Supplemental: alternative scenes and results

5.1 Alternative 1

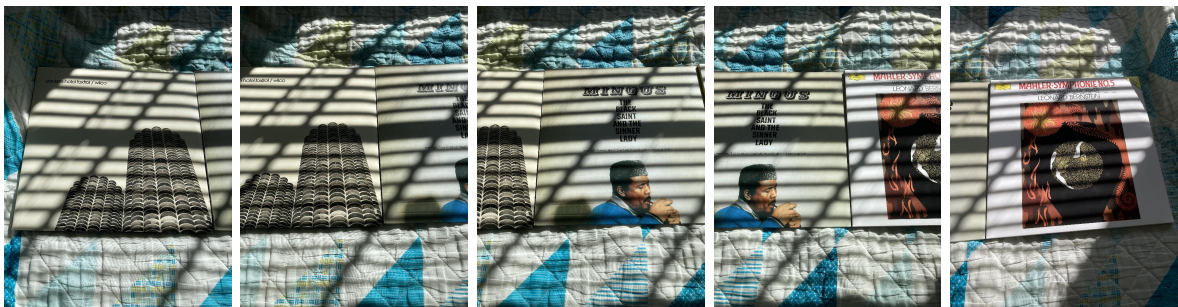


Figure 16: 5 input images



Figure 17: Sample SIFT correspondences between image 4 and 5

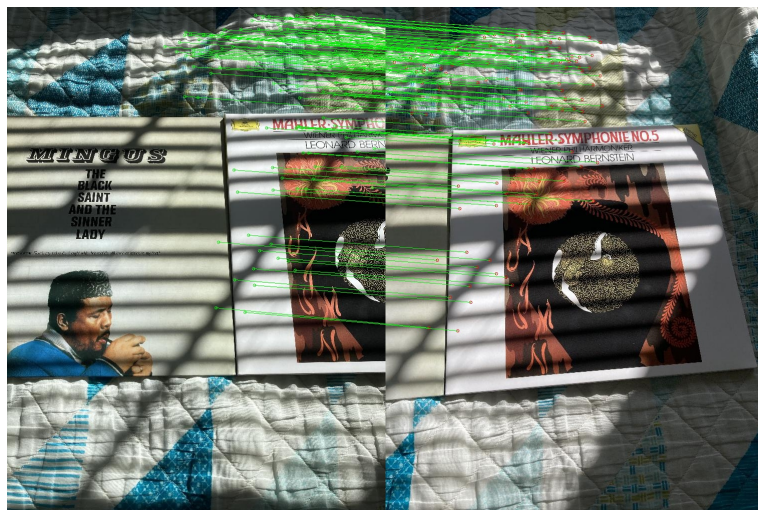


Figure 18: Inliers determined by RANSAC



Figure 19: Outliers rejected by RANSAC



Figure 20: Result panorama image, without LM refinement

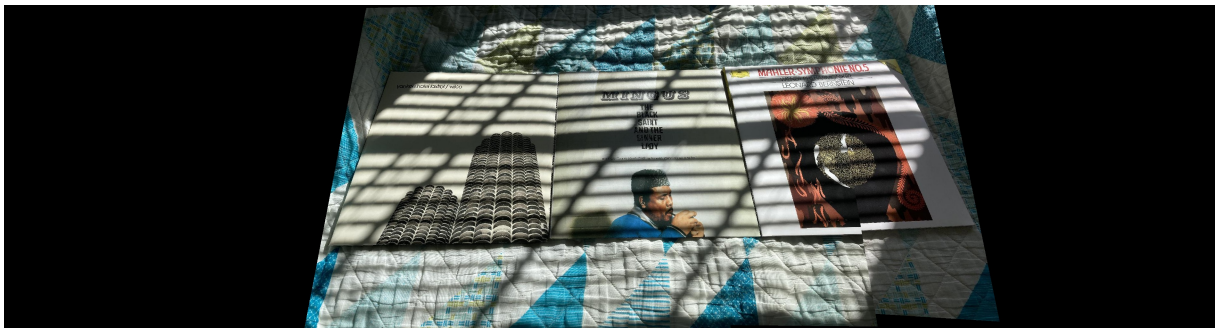


Figure 21: Result panorama image, with LM refinement

5.2 Alternative 2

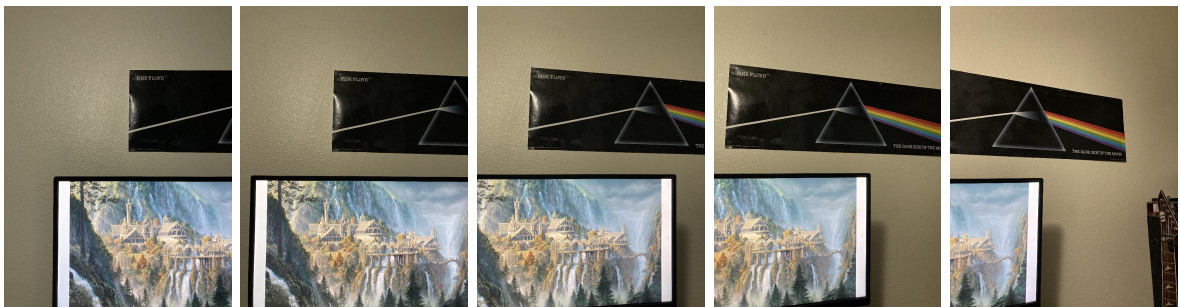


Figure 22: 5 input images

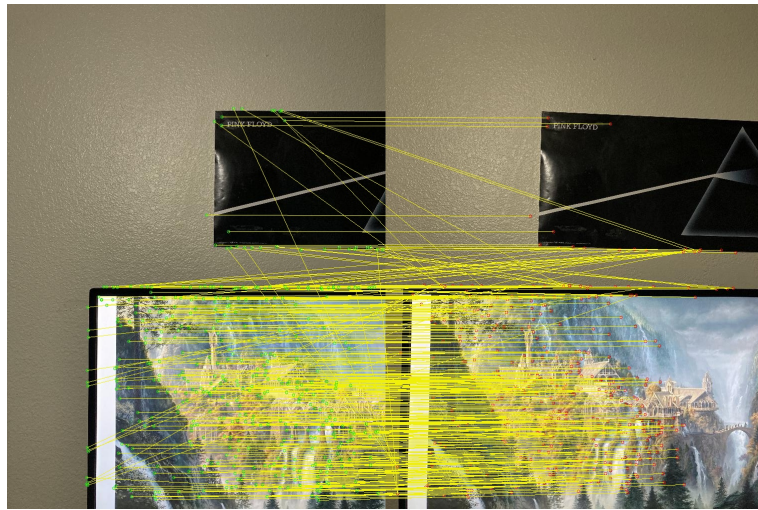


Figure 23: Sample SIFT correspondences between image 1 and 2

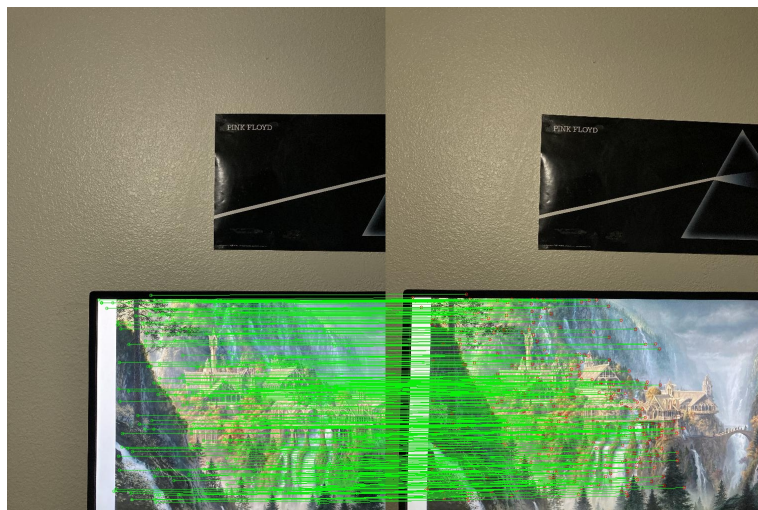


Figure 24: Inliers determined by RANSAC

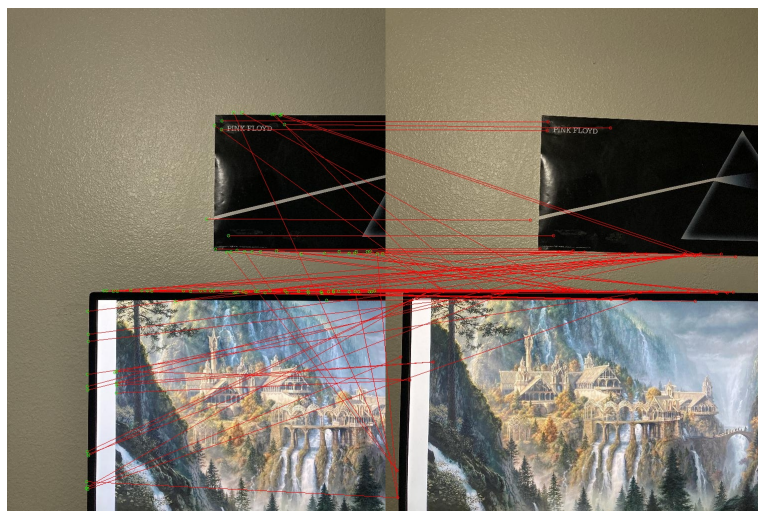


Figure 25: Outliers rejected by RANSAC

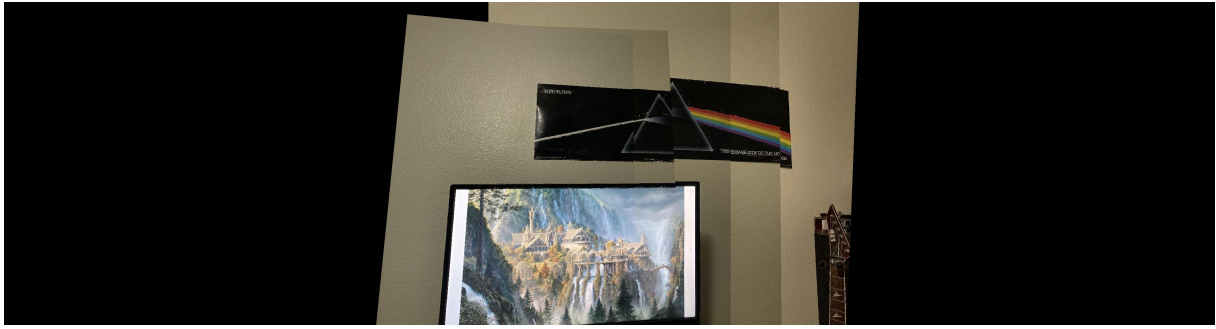


Figure 26: Result panorama image

6 Source Codes

```

1
2 import cv2
3 import numpy as np
4 import math
5 import pdb
6 from scipy.ndimage import map_coordinates
7 from scipy.optimize import least_squares
8
9
10 def compute_distance(F1, F2, mode = 'SSD'):
11     # input: feature from point 1 and 2
12     # same as the one used in HW4
13
14     if mode == 'SSD':
15         diff = F1 - F2
16         distance = np.sum(diff ** 2)
17     elif mode == 'NCC':
18         mu1 = np.mean(F1)
19         mu2 = np.mean(F2)
20         num = np.sum((F1-mu1)*(F2-mu2))
21         denom = np.sqrt(np.sum((F1-mu1)**2) * np.sum((F2-mu2)**2))
22         distance = 1 - (num / denom)
23         # NCC's range is [-1,1], with 1 being the closest match
24         # in order to apply the same "smaller distance = closer match" logic
25         # I use d = 1-NCC, hence smaller = better
26
27     return distance
28
29 def Homography(domain_coord, range_coord):
30     # computing homography using only 4 point pairs
31     # same as the one used in HW2
32     A = np.zeros((8,8))
33     b = range_coord.flatten()
34     for i in range(0,4):
35         A[i*2,0:3] = [domain_coord[i][0], domain_coord[i][1], 1]
36         A[i*2,3:6] = [0,0,0]
37         A[i*2,6:8] = [(-1*domain_coord[i][0]*range_coord[i][0]), (-1*domain_coord[i][1]*
38             range_coord[i][0])]
39         A[i*2+1,0:3] = [0,0,0]
40         A[i*2+1,3:6] = [domain_coord[i][0], domain_coord[i][1], 1]
41         A[i*2+1,6:8] = [(-1*domain_coord[i][0]*range_coord[i][1]), (-1*domain_coord[i]
42             ][1]*range_coord[i][1])]
43     # print(A)
44     h=np.dot(np.linalg.pinv(A),b)
45     H= np.zeros((3,3))
46     H[0]= h[0:3]
47     H[1]= h[3:6]

```

```

46 H[2][0:2]= h[6:8]
47 H[2][2]= 1
48 # print(H)
49 return H
50
51
52 def Homography_LS(domain_coord , range_coord ):
53 # computing homography for an over-determined system
54 # (more than 4 point pairs)
55 # using least square estimation
56 # same as the one used in HW2
57 n = domain_coord.shape[0]
58 A = np.zeros((n*2,8))
59 b = range_coord.flatten()
60 for i in range(0,n):
61     A[i*2,0:3] = [domain_coord[i][0], domain_coord[i][1], 1]
62     A[i*2,3:6] = [0,0,0]
63     A[i*2,6:8] = [(-1*domain_coord[i][0]*range_coord[i][0]), (-1*domain_coord[i][1]*
64         range_coord[i][0])]
65     A[i*2+1,0:3] = [0,0,0]
66     A[i*2+1,3:6] = [domain_coord[i][0], domain_coord[i][1], 1]
67     A[i*2+1,6:8] = [(-1*domain_coord[i][0]*range_coord[i][1]), (-1*domain_coord[i]
68         ][1]*range_coord[i][1])]
69 # print(A)
70 tmp = np.linalg.pinv(np.matmul(A.T,A))
71 h=np.dot(np.matmul(tmp,A.T),b)
72 H= np.zeros((3,3))
73 H[0]= h[0:3]
74 H[1]= h[3:6]
75 H[2][0:2]= h[6:8]
76 H[2][2]= 1
77 # print(H)
78 return H
79
80 def sift_pair(img1_raw , img2_raw , name):
81 # need to use older version to bypass patent issues
82 # opencv-contrib-python == 3.4.2.16
83 if len(img1_raw.shape) == 3:
84     img1 = cv2.cvtColor(img1_raw , cv2.COLOR_BGR2GRAY)
85 else:
86     img1 = img1_raw
87 if len(img2_raw.shape) == 3:
88     img2 = cv2.cvtColor(img2_raw , cv2.COLOR_BGR2GRAY)
89 else:
90     img2 = img2_raw
91
92 sift = cv2.xfeatures2d.SIFT_create()
93 kp1, des1 = sift.detectAndCompute(img1, None)
94 kp2, des2 = sift.detectAndCompute(img2, None)
95
96 # OTS brute force matcher by OpenCV
97 # bf = cv2.BFMatcher()
98 # matches = bf.knnMatch(des1, des2, k=2)
99 # good = []
100 # for m,n in matches:
101 #     if m.distance < 0.75*n.distance:
102 #         good.append([m])
103 # comb = np.concatenate((img1_raw , img2_raw), axis=1)
104 # cv2.drawMatchesKnn(img1_raw, kp1, img2_raw, kp2, good[0:100], comb, flags=2)
105
106 w = img1_raw.shape[1]
107 comb = np.concatenate((img1_raw , img2_raw), axis=1)
108 domain_coord = np.array([[0, 0]])

```

```

108 # domain_coord = []
109 range_coord = np.array([[0,0]])
110 # range_coord = []
111 print(len(kp1))
112 for i in range(len(kp1)):
113     d_tmp = []
114     domain_coord_tmp = list(map(int, kp1[i].pt))
115     F1 = des1[i]
116     # domain_coord.append(list(kp1[i].pt))
117     for j in range(len(kp2)):
118         F2 = des2[j]
119         distance = compute_distance(F1,F2)
120         d_tmp.append(distance)
121     # pdb.set_trace()
122     best_match = list(kp2[np.argsort(d_tmp)[0]].pt)
123     best_match = list(map(int, best_match))
124
125     # if i%100 == 0:
126     #     print(i)
127     # tracking progress
128
129     # enroll the correspondence if the distance is within reasonable range
130     if np.min(d_tmp) < 10000:
131         domain_coord = np.append(domain_coord, [domain_coord_tmp], axis=0)
132         range_coord = np.append(range_coord, [best_match], axis=0)
133         # pdb.set_trace()
134         pt1 = tuple(domain_coord_tmp)
135         pt2 = (best_match[0]+w, best_match[1])
136
137         cv2.circle(comb, pt1, 3, (10,240,10), 1)
138         cv2.circle(comb, pt2, 3, (10,10,240), 1)
139         cv2.line(comb, pt1, pt2, (10,240,240), 1)
140         # visualized
141
142     domain_coord = np.delete(domain_coord, 0, axis=0)
143     range_coord = np.delete(range_coord, 0, axis=0)
144     # using numpy array rather than list for faster computation
145     # to preserve dimensions, i did not initialize them as empty arrays
146     # but use a placeholder which would be deleted in the end
147
148     cv2.imwrite(name+'.jpg',comb)
149     return domain_coord, range_coord
150
151
152 def ransac_find_H(domain_coord, range_coord, LM):
153     # empirical parameters:
154     n = 4
155     # 4 <= n <= 10
156     epsilon = 0.75
157     # key parameter, if epsilon = 0.25, N = 12
158     # if epsilon = 0.5, N = 70
159     # if epsilon = 0.75, N = 1176
160     p = 0.99
161     delta = 3.0
162     N = int(math.log(1 - p) / math.log(1 - (1 - epsilon) ** n))
163     # number of trials
164     n_total = domain_coord.shape[0]
165     # total correspondences
166     M = int((1 - epsilon) * n_total)
167     # expected number of inliers
168     print('N:',N)
169     print('M:',M)
170
171     best_inlier_indices = []

```

```

172 inlier_pct = 0
173 # inlier percentage
174 all_indices = list(range(n_total))
175
176 for i in range(N):
177     rand_idx = np.random.choice(all_indices, n)
178     domain_coord_rand = domain_coord[rand_idx]
179     range_coord_rand = range_coord[rand_idx]
180     H_tmp = Homography(domain_coord_rand, range_coord_rand)
181     inliers_idx = find_inliers(H_tmp, domain_coord, range_coord, delta)
182     num_inliers = inliers_idx.shape[0]
183     print(num_inliers)
184     if num_inliers >= M and num_inliers/n_total > inlier_pct:
185         # if num_inliers/n_total > inlier_pct:
186             inlier_pct = num_inliers/n_total
187             best_inlier_indices = inliers_idx
188
189 H = Homography_LS(domain_coord[best_inlier_indices], range_coord[
190     best_inlier_indices])
191 if LM == True:
192     h = H.flatten()
193     res = least_squares(cost_func_H, h, args=(domain_coord[best_inlier_indices],
194     range_coord[best_inlier_indices]), method="lm")
195     h_new = res.x
196     H = h_new.reshape((3,3))
197
198 return H, best_inlier_indices
199
200 def cost_func_H(h, domain_coord, range_coord):
201     # h: flattened H for optimization
202     H = h.reshape((3,3))
203
204     domain_coord_homo = np.insert(domain_coord, 2, 1, axis=1)
205     range_coord_homo = np.insert(range_coord, 2, 1, axis=1)
206     range_coord_computed = np.matmul(H, domain_coord_homo.T).T
207     range_coord_computed = range_coord_computed.T / range_coord_computed.T[2, :]
208     range_coord_computed = range_coord_computed.T
209     error = np.abs(range_coord_computed - range_coord_homo)
210     return (error**2)
211
212 def find_inliers(H, domain_coord, range_coord, delta):
213     domain_coord_homo = np.insert(domain_coord, 2, 1, axis=1)
214     range_coord_homo = np.insert(range_coord, 2, 1, axis=1)
215     range_coord_computed = np.matmul(H, domain_coord_homo.T).T
216     range_coord_computed = range_coord_computed.T / range_coord_computed.T[2, :]
217     range_coord_computed = range_coord_computed.T
218     error = np.abs(range_coord_computed - range_coord_homo)
219
220     d = np.sum(error**2, axis=1)
221     # pdb.set_trace()
222     ind = np.where(d <= delta ** 2)[0]
223
224     return ind
225
226 def visualize_inliers(ind, domain_coord, range_coord, img1_raw, img2_raw, name):
227     comb_in = np.concatenate((img1_raw, img2_raw), axis=1)
228     comb_out = np.concatenate((img1_raw, img2_raw), axis=1)
229     w = img1_raw.shape[1]
230
231     for i in range(domain_coord.shape[0]):
232         if i in ind:
233             # inlier

```

```

234     pt1 = tuple(domain_coord[i])
235     pt2 = range_coord[i]
236     pt2[0] = pt2[0] + w
237     pt2 = tuple(pt2)
238
239     cv2.circle(comb_in, pt1, 3, (10,240,10), 1)
240     cv2.circle(comb_in, pt2, 3, (10,10,240), 1)
241     cv2.line(comb_in, pt1, pt2, (10,240,10), 1)
242 else:
243     # outlier
244     pt1 = tuple(domain_coord[i])
245     pt2 = range_coord[i]
246     pt2[0] = pt2[0] + w
247     pt2 = tuple(pt2)
248
249     cv2.circle(comb_out, pt1, 3, (10,240,10), 1)
250     cv2.circle(comb_out, pt2, 3, (10,10,240), 1)
251     cv2.line(comb_out, pt1, pt2, (10,10,240), 1)
252
253 cv2.imwrite(name+'_inlier.jpg', comb_in)
254 cv2.imwrite(name+'_outlier.jpg', comb_out)
255
256 def pixel_mapping(comb, img, H):
257     h = img.shape[0]
258     w = img.shape[1]
259
260     h_total = comb.shape[0]
261     w_total = comb.shape[1]
262
263     H = np.linalg.inv(H)
264
265     X,Y = np.meshgrid(np.arange(0, w_total, 1), np.arange(0, h_total, 1))
266     pts = np.vstack((X.ravel(), Y.ravel())).T
267     # pts = np.flip(pts, axis=1)
268     pts = np.hstack((pts[:, 0:2], pts[:, 0:1]*0+1))
269
270     Hpts = np.dot(H, pts.T)
271     Hpts = Hpts / Hpts[2,:]
272     # Hpts = Hpts * (1.0 / np.tile(Hpts[:, 2], (3, 1))).T
273     Hpts = Hpts.T[:, 0:2].astype('int')
274     # Hpts = np.flip(Hpts, axis=1)
275     # pdb.set_trace()
276
277     # b_channel = img[:, :, 0]
278     # g_channel = img[:, :, 1]
279     # r_channel = img[:, :, 2]
280     # # pdb.set_trace()
281     # b_new = map_coordinates(b_channel, Hpts.T)
282     # g_new = map_coordinates(g_channel, Hpts.T)
283     # r_new = map_coordinates(r_channel, Hpts.T)
284
285     # comb[:, :, 0] = np.reshape(b_new, (h_total, w_total))
286     # comb[:, :, 1] = np.reshape(g_new, (h_total, w_total))
287     # comb[:, :, 2] = np.reshape(r_new, (h_total, w_total))
288     # hw3 stuff, not suitable
289
290     # only replace part of the (combined) image
291     valid_pts, valid_Hpts = find_valid_pts(pts, Hpts, w-1, h-1)
292     for i in range(valid_pts.shape[0]):
293         # pdb.set_trace()
294         if (comb[valid_pts[i,1], valid_pts[i,0]] != 0).all() == False:
295             comb[valid_pts[i,1], valid_pts[i,0]] = img[valid_Hpts[i,1], valid_Hpts[i,0]]
296
297

```



```

298     return comb
299
300
301 def find_valid_pts(pts, Hpts, w, h):
302     xmin = Hpts[:,0] >= 0
303     Hpts = Hpts[xmin,:]
304     pts = pts[xmin,:]
305
306     xmax = Hpts[:,0] <= w
307     Hpts = Hpts[xmax,:]
308     pts = pts[xmax,:]
309
310     ymin = Hpts[:,1] >= 0
311     Hpts = Hpts[ymin,:]
312     pts = pts[ymin,:]
313
314     ymax = Hpts[:,1] <= h
315     Hpts = Hpts[ymax,:]
316     pts = pts[ymax,:]
317
318     return pts, Hpts
319
320
321 def panorama(max_num = 5, LM = False):
322     # input: max number of pictures as input
323     mid_img = max_num // 2
324
325     H_all = []
326     for i in range(max_num-1):
327         path1 = 'input/' + str(i+1) + '.jpg'
328         path2 = 'input/' + str(i+2) + '.jpg'
329         img1 = cv2.imread(path1)
330         img2 = cv2.imread(path2)
331         img1_cp = img1.copy()
332         img2_cp = img2.copy()
333
334         domain_coord, range_coord = sift_pair(img1, img2, str(i+1)+'_and_'+str(i+2))
335         # compute SIFT features and construct correspondence
336
337         # np.save(str(i+1)+'_and_'+str(i+2)+'_domain', domain_coord)
338         # np.save(str(i+1)+'_and_'+str(i+2)+'_range', range_coord)
339
340         ## saving and loading point correspondences
341         ## so that I don't need to spend time repeating the process while debugging
342
343         # domain_coord = np.load(str(i+1)+'_and_'+str(i+2)+'_domain.npy')
344         # range_coord = np.load(str(i+1)+'_and_'+str(i+2)+'_range.npy')
345         # pdb.set_trace()
346         H, inliers = ransac_find_H(domain_coord, range_coord, LM)
347         np.save(str(i+1)+'_and_'+str(i+2)+'_inlier', inliers)
348
349         visualize_inliers(inliers, domain_coord, range_coord, img1_cp, img2_cp, str(i+1)
350         +'_and_'+str(i+2))
351
352         H_all.append(H)
353         np.save('H_all', H_all)
354
355     # H_all = list(np.load('H_all.npy'))
356
357     H_to_mid = np.eye(3)
358     for i in range(mid_img, len(H_all)):
359         H_to_mid = np.matmul(H_to_mid, np.linalg.inv(H_all[i]))
360         H_all[i] = H_to_mid

```

```

361 H_to_mid = np.eye(3)
362 for i in range(mid_img - 1, -1, -1):
363     H_to_mid = np.matmul(H_to_mid, H_all[i])
364     H_all[i] = H_to_mid
365 # set H to morph to a mid image
366
367 H_all.insert(mid_img, np.eye(3))
368 tx = 0
369 for i in range(mid_img):
370     path = 'input/' + str(i+1) + '.jpg'
371     img = cv2.imread(path)
372     tx = tx + img.shape[1]
373 H_t = np.array([[1, 0, tx], [0, 1, 0], [0, 0, 1]], dtype=float)
374 # pdb.set_trace()
375 # translational transformation
376 height = 0
377 width = 0
378 for i in range(max_num):
379     path = 'input/' + str(i+1) + '.jpg'
380     img = cv2.imread(path)
381     height = max(height, img.shape[0])
382     width = width + img.shape[1]
383
384 comb = np.zeros((height, width, 3), np.uint8)
385
386 for i in range(max_num):
387     path = 'input/' + str(i+1) + '.jpg'
388     img = cv2.imread(path)
389     H = np.matmul(H_t, H_all[i])
390     comb = pixel_mapping(comb, img, H)
391 cv2.imwrite('panorama.jpg', comb)
392
393 if __name__ == '__main__':
394     panorama(LM=True)

```