ECE 661: Homework 1

Fall 2020

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Problem 1

What are all the points in the representational space $\mathbb{R}^3$ that are the homogeneous coordinates of the origin in the physical space $\mathbb{R}^2$?

Since an arbitrary homogeneous vector representative of a point is of the form $(x_1, x_2, x_3)^\top$, representing the point $(x_1/x_3, x_2/x_3)^\top$ in $\mathbb{R}^2$.

The points in the representational space $\mathbb{R}^3$ that are the homogeneous coordinates of the origin $(0/k, 0/k)^\top$ in the physical space $\mathbb{R}^2$ is,

$$\begin{pmatrix} 0 \\ 0 \\ k \end{pmatrix}, k \in \mathbb{R}, k \neq 0 \tag{1}$$

Problem 2

Are all points at infinity in the physical plane $\mathbb{R}^2$ the same? Justify your answer.

No. Points at infinity, or Ideal points are points with last coordinate $x_3 = 0$.

Both $(x_1, y_1, 0)$ and $(x_2, y_2, 0)$ are Ideal points. In the physical plane $\mathbb{R}^2$, both of them represent points at infinity. However, if (x_1, y_1) is not equal to (x_2, y_2) and (x_1/y_1) is not equal to (x_2/y_2), these points will reach infinity in different directions.

Problem 3

Argue that the matrix rank of a degenerate conic can never exceed 2.

A degenerated conics can be represented by summation of 2 outer products between 2 lines:

$$C = lm^T + ml^T$$

Since rows of each outer product matrix are linearly dependent, its rank must be 1. Therefore, the matrix of a degenerated conics, which is an summation of two rank 1 matrices, cannot exceed rank 2.

Problem 4

Derive in just 3 steps the intersection of two lines l_1 and l_2 with l_1 passing through the points $(0;0)$ and $(3;5)$, and with l_2 passing through the points $(-3;4)$ and $(-7;5)$. How many steps would take you if the second line passed through $(-7;-5)$ and $(7;5)$?

(1)

$$\left(\begin{pmatrix}0\\0\\1\end{pmatrix}\times\begin{pmatrix}3\\5\\1\end{pmatrix}\right)\times\left(\begin{pmatrix}-3\\4\\1\end{pmatrix}\times\begin{pmatrix}-7\\5\\1\end{pmatrix}\right)$$
$$=\left(\begin{pmatrix}-5\\3\\0\end{pmatrix}\times\begin{pmatrix}-1\\-4\\13\end{pmatrix}\right)$$
$$=\begin{pmatrix}39\\65\\23\end{pmatrix}=\begin{pmatrix}\frac{39}{23}\\\frac{65}{23}\\1\end{pmatrix}$$

First step, calculate l_1; second step, calculate l_2; Third step, calculate the intersection of two lines.

(2)

$$l_2=\begin{pmatrix}-7\\-5\\1\end{pmatrix}\times\begin{pmatrix}7\\5\\1\end{pmatrix}=\begin{pmatrix}-10\\14\\0\end{pmatrix}$$

This implies that both l_1 and l_2 pass through the origin $[0,0,1]^T$. Therefore, the point of intersection is the origin (0, 0) in $\mathbb{R}^2$. So it will take 2 steps to find the values of L1 and L2.

Problem 5

Let l_1 be the line passing through points (0; 0) and (5;-3) and l_2 be the line passing through points (-5; 0) and (0;-3). Find the intersection between these two lines. Comment on your answer.

$$l_1=\begin{pmatrix}0\\0\\1\end{pmatrix}\times\begin{pmatrix}5\\-3\\1\end{pmatrix}=\begin{pmatrix}3\\5\\0\end{pmatrix}$$

$$l_2=\begin{pmatrix}-5\\0\\1\end{pmatrix}\times\begin{pmatrix}0\\-3\\1\end{pmatrix}=\begin{pmatrix}3\\5\\15\end{pmatrix}$$

$$P_{1,2}=\begin{pmatrix}3\\5\\0\end{pmatrix}\times\begin{pmatrix}3\\5\\15\end{pmatrix}=\begin{pmatrix}75\\-45\\0\end{pmatrix}$$

$P_{1,2}$ is an ideal point, which implies that the lines l_1 and l_2 are parallel.

Problem 6

As you know, when a point **p** is on a conic **C**, the tangent to the conic at that point is given by $\mathbf{l} = \mathbf{Cp}$. That raises the question as to what **Cp** would correspond to when **p** was outside the conic. As you'll see later in class, when **p** is outside the conic, **Cp** is the line that joins the two points of contact if you draw tangents to **C** from the point **p**. This line is referred to as the polar line. Now let our conic **C** be an ellipse that is centered at the coordinates (3, 2), with $a = 1$ and $b = 1/2$, where a and b, respectively, are the lengths of semi-major and semi-minor axes. For simplicity, assume that the major axis is parallel to x-axis and the minor axis is parallel to y-axis. Let **p** be the origin of the $\mathbb{R}^2$ physical plane. Find the intersections points of the polar line with x- and y-axes.

The ellipse function,

$$(x-3)^2 + 4(y-2)^2 = 1$$
$$x^2 + 4y^2 - 6x - 16y + 24 = 0$$

The conic coefficient matrix is given by

$$\mathrm{C} = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 4 & -8 \\ -3 & -8 & 24 \end{bmatrix}$$

The polar line

$$l = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 4 & -8 \\ -3 & -8 & 24 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ -8 \\ 24 \end{bmatrix}$$

The polar line function

$$-3x - 8y + 24 = 0$$

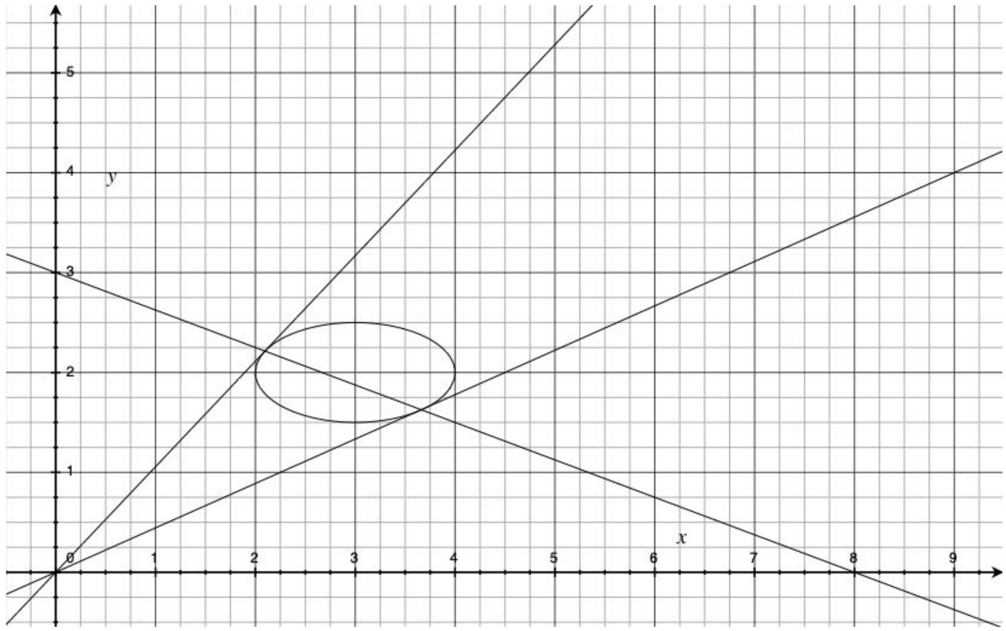
Finally, the intersection between l and $x - axis$ is:

$$\begin{bmatrix} -3 \\ -8 \\ 24 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -24 \\ 0 \\ -3 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 1 \end{bmatrix}$$

Similarly, the intersection between l and $y - axis$ is:

$$\begin{bmatrix} -3 \\ -8 \\ 24 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 24 \\ 8 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$$

The intersection point of the polar line with x-axis is (8,0); The intersection point of the polar line with y-axis is (0,3).



Problem 7

Find the intersection of two lines whose equations are given by $x = 1/2$ and $y = -1/3$

$$l_1 = \begin{pmatrix} 1 \\ 0 \\ -\frac{1}{2} \end{pmatrix}$$

$$l_2 = \begin{pmatrix} 0 \\ 1 \\ \frac{1}{3} \end{pmatrix}$$

$$P_{1,2} = \begin{pmatrix} 1 \\ 0 \\ -\frac{1}{2} \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ \frac{1}{3} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{3} \\ 1 \end{pmatrix}$$