ECE 661: Homework 1

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Problem 1:

Let, $\begin{pmatrix} u \\ v \\ w \end{pmatrix}$ denote all points in the representational space R^3 that are homogeneous coordinates of the origin in the physical space R^2.

⇨ $x = \frac{u}{w} = 0$ and $y = \frac{v}{w} = 0$
⇨ $u = 0$ and $v = 0$
⇨ $\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ w \end{pmatrix} = w \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ for all $w \in R \setminus \{0\}$

∴ $w \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ for $w \in R \setminus \{0\}$ represents all points in the representational space R^3 that are homogeneous coordinates of the origin in the physical space R^2.

Problem 2:

No.

Let, $\begin{pmatrix} u \\ v \\ w \end{pmatrix} \in R^3$ be the homogeneous coordinates of point $\begin{pmatrix} x \\ y \end{pmatrix} \in R^2$.

⇨ $x = \frac{u}{w}$ and $y = \frac{v}{w}$

Now, as $w \to 0$, both x and y move away from the origin and approach infinity along a specific direction, which depends on the values of u and v. Therefore, all points at infinity in the physical space R^2 are not the same, although they lie on a single straight line.

Problem 3:

A degenerate conic is represented by $C = lm^T + ml^T$. Each of the two terms in this addition is a vector outer product having a rank 1. Now, we know that

$rank(A + B) \leq rank(A) + rank(B)$
⇨ $rank(C) = rank(lm^T + ml^T) \leq rank(lm^T) + rank(ml^T) = 1 + 1 = 2$
⇨ $rank(C) \leq 2$

Problem 4:

Case 1:

$$l_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix}$$

$$l_2 = \begin{pmatrix} -6 \\ 8 \\ 1 \end{pmatrix} \times \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 12 \end{pmatrix}$$

$$x = l_1 \times l_2 = \begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 6 \\ 3 \\ 12 \end{pmatrix} = \begin{pmatrix} 24 \\ 72 \\ -30 \end{pmatrix}$$

∴ The intersection point is $\begin{pmatrix} -24/30 \\ -72/30 \end{pmatrix}$.

Case 2:

$$l_2 = \begin{pmatrix} -10 \\ -3 \\ 1 \end{pmatrix} \times \begin{pmatrix} 10 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 20 \\ 0 \end{pmatrix}$$

- ⇨ Both l_1 and l_2 pass through the origin.
- ⇨ The point of intersection is $(0, 0)$.
- ⇨ Two steps calculation.

Problem 5:

$$l_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$$

$$l_2 = \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 9 \end{pmatrix}$$

$$x = l_1 \times l_2 = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 3 \\ 3 \\ 9 \end{pmatrix} = \begin{pmatrix} 18 \\ -18 \\ 0 \end{pmatrix}$$

∴ The intersection point is at infinity.

- ⇨ l_1 and l_2 are parallel.

Problem 6:

The equation of the circle in R^2 space is given by

$$(x - 5)^2 + (y - 5)^2 = 1$$

- ⇨ $x^2 + y^2 - 10x - 10y + 49 = 0$

- ⇨ $C = \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -5 \\ -5 & -5 & 49 \end{bmatrix}$
- ⇨ $l = Cx = \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -5 \\ -5 & -5 & 49 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ -5 \\ 49 \end{bmatrix}$
- ⇨ $x_{intercept} = l \times l_x = \begin{bmatrix} -5 \\ -5 \\ 49 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -49 \\ 0 \\ -5 \end{bmatrix}$
- ⇨ $y_{intercept} = l \times l_y = \begin{bmatrix} -5 \\ -5 \\ 49 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 49 \\ 5 \end{bmatrix}$

∴ Intersection point of the polar line with x axis = $(49/5\,,0)$.

∴ Intersection point of the polar line with y axis = $(0{,}49/5)$.

Problem 7:

$l_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

$l_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

$x = l_1 \times l_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

∴ The intersection point is $(1, 1)$.