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Answer 1:

All the points $[\ 0, 0, k\ ]^T$ (where k is a non-zero constant) in the $\mathbf{R^3}$ representational space are the homogeneous coordinates of the origin in the physical space $\mathbf{R^2}$.
Since any non-zero scalar multiplied with a point in $\mathbf{R^3}$ will also represent the same point $\mathbf{R^2}$, hence it is equivalent to say that the homogeneous coordinate for the origin in $\mathbf{R^2}$ is $[\ 0, 0, 1\ ]^T$.

Answer 2:

All points at infinity in $\mathbf{R^2}$ lie on the same line in $\mathbf{R^2}$, but they are not the same.
These points are represented in $\mathbf{R^3}$ as ideal points in the form $[\ u, v, 0\ ]^T$.
So if we have two of these ideal points $[\ u1, v1, 0\ ]^T$ and $[\ u2, v2, 0\ ]^T$, then on the $\mathbf{R^2}$ plane, both represents points at infinity (as the corresponding x and y value of both these points will be infinity when divided by the 0, which is the 3rd component of their homogeneous representation).
But the corresponding u1, v1 values may not be equal to the u2, v2 values. So, these points reaches infinity in different directions (as governed by their corresponding u, v values).

Answer 3:

The degenerate conic $C = LM^T + ML^T$. Now, both terms are outer product of two vectors M and L.
Since the outer product of vectors results in a matrix whose columns are all linearly dependent, so each of these outer product terms can only result in a matrix of rank 1 (at the most), i.e. has only one independent column vector.
Now, if the column vector of each of these two outer product terms are different, then the final sum of these terms will have two independent column vectors (at the most). Hence the degenerate conic can only have a rank of 2 at the most.

Answer 4:

$L1 = [\ 0, 0, 1\ ]^T \times [\ 2, 6, 1\ ]^T = [\ -6, 2, 0\ ]^T$
$L2 = [\ -6, 8, 1\ ]^T \times [\ -3, 2, 1\ ]^T = [\ 6, 3, 12\ ]^T$
$X12 = [\ -6, 2, 0\ ]^T \times [\ 6, 3, 12\ ]^T = [\ 24, 72, -30\ ]^T = [\ -0.8, -2.4, 1\ ]^T$ = point of intersection = (-0.8, -2.4) in $\mathbf{R^2}$.

If $L2 = [\ -10, -3, 1\ ]^T \times [\ 10, 3, 1\ ]^T = [\ -6, 20, 0\ ]^T$
Then it implies that both L1 and L2 pass through the origin $[\ 0, 0, 1\ ]^T$. Thus the point of intersection will be the origin (0, 0) in $\mathbf{R^2}$. So it will take 2 steps then, finding the values of L1 and L2.

Answer 5:

$L1 = [\ 0, 0, 1\ ]^T \times [\ 2, -2, 1\ ]^T = [\ 2, 2, 0\ ]^T$
$L2 = [\ -3, 0, 1\ ]^T \times [\ 0, -3, 1\ ]^T = [\ 3, 3, 9\ ]^T$

$X12 = [\ 2, 2, 0\ ]^T \times [\ 3, 3, 9\ ]^T = [\ 18, -18, 0\ ]^T$ = an ideal point in $\mathbf{R^3}$. Which implies that the lines L1 and L2 are parallel.

Answer 6:

Equation of the circle is $(x - 5)^2 + (y - 5)^2 = 1$ or $x^2 + y^2 - 10x - 10y + 49 = 0$

$$C = \begin{vmatrix} 1, & 0, & -5 \\ 0, & 1, & -5 \\ -5, & -5, & 49 \end{vmatrix}$$

$X = [\ 0, 0, 1\ ]^T$

$L = CX = [\ -5, -5, 49\ ]^T$ = polar line = $[\ a, b, c\ ]^T$ which is $-5x - 5y + 49 = 0$ in $\mathbf{R^2}$.

So the line intersects the x axis in $\mathbf{R^2}$ at $x = 9.8$ and intersects the y axis in $\mathbf{R^2}$ at $y = 9.8$.

Answer 7:

$x= 1$ or $x - 1 = 0$. So $[\ a, b, c\ ]^T = [\ 1, 0, -1\ ]^T$

$X = [\ 1, 0, -1\ ]^T$

$y = 1$ or $y - 1 = 0$. So $[\ a, b, c\ ]^T = [\ 0, 0, -1\ ]^T$

$Y = [\ 0, 1, -1\ ]^T$

$P12 = [\ 1, 0, -1\ ]^T \times [\ 0, 1, -1\ ]^T = [\ 1, 1, 1\ ]^T$ = point of intersection = (1, 1) in $\mathbf{R^2}$.

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