AAE 553 – Elasticity in Aerospace Engineering

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Course Description
AAE 553 is a fundamental course in the theory of elasticity with emphasis on understanding the governing principles and solution techniques used in the stress analysis of elastic solids and structures. Cartesian tensors are introduced for formulations of general deformations and states of stress. Constitutive relations and field equations are derived for large deformations and then reduced to small deformation. Two dimensional problems are solved by using the Airy stress function method. Energy methods and approximate solutions using variational principles are included.

Prerequisites
Elementary courses in mechanics of materials (AAE 204 and AAE 352 or equivalent), linear algebra, and differential equations. Elementary proficiency with a mathematical computer tool such as Matlab, Mathematica, or Maple is also needed for some homework assignments.

Course Goals and Learning Objectives
The objective of this course is to give the student an in-depth background in mechanics of solids including tensor analysis, large deformations, fundamentals of stress and strain, constitutive relations, and the ability to perform stress analyses in elastic bodies.

Additional information on the course and learning outcomes can be found at https://engineering.purdue.edu/AAE/academics/courses_descriptions/AAE553

Required Texts
Theory of Elasticity – a first course on fundamental principles and methods of analysis, James F. Doyle and C-T. Sun (available for download on Blackboard).


Policies
Exams
There will be two in-class exams in addition to the final exam. Each exam will be worth 25% of your overall grade. Students are allowed a handwritten, self-made note sheet for exams. Reasonable accommodations will be made for students having exceptional circumstances preventing them from taking exams during the scheduled times. However, requests for such accommodation must be made as soon as possible. The instructor reserves the right to use his discretion regarding this, and conflicts will be mediated through the college/university. See also Grief Absence Policy for Students, Students with Disabilities, and Academic Dishonesty.
Remote students are responsible for arranging a proctor and testing site for their exams. There is a process by which proctors are vetted through Purdue Engineering Professional Education. More information can be found via the following link.

https://engineering.purdue.edu/ProEd/student-resources/taking-exams/exam-process

**Homework**

Six homework sets will be assigned and are collectively worth 25% of your overall grade. Late homework will not be accepted. Working with classmates is encouraged, but simply copying answers will be considered cheating and receive no credit. The grader may use his or her discretion in the aforementioned. See also Grief Absence Policy for Students, Students with Disabilities, and Academic Dishonesty.

On-campus students must submit a hard copy of their homework in class on the day that it is due. Please staple multiple pages together.

Remote students must send an electronic copy of their homework to the teaching assistant via email before the end of the day (11:59 PM EST) that it is due. Please submit your homework as a single .pdf file and use ‘firstname_lastname_AAE553_HW#' in the subject line of your email and as the file name of your submission.

**Grading**

Homework – 25%
Exam I – 25%
Exam II – 25%
Final Exam – 25%

A+ ≥ 96.67% (note that both an A+ and an A are worth 4.0)
96.67% > A ≥ 93.33%
93.33% > A- ≥ 90%

90% > B+ ≥ 86.67%
86.67% > B ≥ 83.33%
83.33% > B- ≥ 80%

80% > C+ ≥ 76.67%
76.67% > C ≥ 73.33%
73.33 > C- ≥ 70%

70% > D+ ≥ 67.67%
67.67% > D ≥ 63.33%
63.33% > D- ≥ 60%

F < 60%

The instructor reserves the right to curve or to not curve the class or individual exams.
**Academic Dishonesty**

Purdue prohibits “dishonesty in connection with any University activity. Cheating, plagiarism, or knowingly furnishing false information to the University are examples of dishonesty.” [Section B.2.a, Code of Student Conduct] Furthermore, the University Senate has stipulated that “the commitment of acts of cheating, lying, and deceit in any of their diverse forms (such as the use of substitutes for taking examinations, the use of illegal cribs, plagiarism, and copying during examinations) is dishonest and must not be tolerated. Moreover, knowingly to aid and abet, directly or indirectly, other parties in committing dishonest acts is in itself dishonest.” [University Senate Document 72-18, December 15, 1972]

The instructor reserves the right to reprimand cheating at a level commensurate with the offense. This includes up to reporting to the college/university and failing the course.

Additional information on Purdue academic integrity can be found at www.purdue.edu/odos/osrr/academic-integrity-brochure.

**Use of Copyrighted Materials**

Among the materials that may be protected by copyright law are the lectures, notes, and other material presented in class or as part of the course. Always assume the materials presented by an instructor are protected by copyright unless the instructor has stated otherwise. Students enrolled in, and authorized visitors to, Purdue University courses are permitted to take notes, which they may use for individual/group study or for other non-commercial purposes reasonably arising from enrollment in the course or the University generally.

Notes taken in class are, however, generally considered to be “derivative works” of the instructor’s presentations and materials, and they are thus subject to the instructor’s copyright in such presentations and materials. No individual is permitted to sell or otherwise barter notes, either to other students or to any commercial concern, for a course without the express written permission of the course instructor. To obtain permission to sell or barter notes, the individual wishing to sell or barter the notes must be registered in the course or must be an approved visitor to the class. Course instructors may choose to grant or not grant such permission at their own discretion, and may require a review of the notes prior to their being sold or bartered. If they do grant such permission, they may revoke it at any time, if they so choose.

**Grief Absence Policy for Students**

Purdue University recognizes that a time of bereavement is very difficult for a student. The University therefore provides the following rights to students facing the loss of a family member through the Grief Absence Policy for Students (GAPS). GAPS Policy: Students will be excused for funeral leave and given the opportunity to earn equivalent credit and to demonstrate evidence of meeting the learning outcomes for missed assignments or assessments in the event of the death of a member of the student’s family.

**Students with Disabilities**

Purdue University is required to respond to the needs of the students with disabilities as outlined in both the Rehabilitation Act of 1973 and the Americans with Disabilities Act of 1990 through
the provision of auxiliary aids and services that allow a student with a disability to fully access and participate in the programs, services, and activities at Purdue University. If you have a disability that requires special academic accommodation, please make an appointment to speak with the instructor within the first three weeks of the semester in order to discuss any adjustments. It is important that we talk about this at the beginning of the semester. It is the student's responsibility to notify the Disability Resource Center (http://www.purdue.edu/drc) of an impairment/condition that may require accommodations and/or classroom modifications.

Class Schedule
A tentative schedule is provided below with required readings from the text. Next to the date of each week is a short summary of the topics covered. Important dates are in red and boldfaced.

Week 1 (8/21 and 8/23) – introduction and mathematical preliminaries
Required readings: chapter 1 (Doyle and Sun) and chapter 1 (Sadd)
• introduction to elasticity
• introduction to tensors
• tensor manipulation
• special tensors
• tensor transformations

Week 2 (8/28 and 8/30) – mathematical preliminaries continued
Required readings: chapter 1 (Doyle and Sun) and chapter 1 (Sadd)
• isotropic tensors
• principal values of symmetric, second order tensors
• calculus of tensor fields

Week 3 (9/4 and 9/6) – deformation
Required readings: sections 2.1 and 2.2 (Doyle and Sun)
• introduction to deformation
• Lagrangian and Eulerian descriptions of displacement
• derivation of the deformation gradient tensor
• deformation of lines
• deformation of areas
• deformation of volumes

Week 4 (9/11 and 9/13) – strain
Required readings: sections 2.3 – 2.5 (Doyle and Sun) and chapter 2 (Sadd)
• introduction to strain
• derivation of the Lagrangian and Eulerian strain tensors
• physical interpretation of Lagrangian and Eulerian strain tensors
• strain tensors in terms of displacements
• simplification for small displacements

Week 5 (9/18 and 9/20) – special deformations
Required readings: section 2.6 (Doyle and Sun)
• special deformations
• review for exam I
• exam I (9/20) – mathematical preliminaries, deformation, and strain

Week 6 (9/25 and 9/27) – stress
Required readings: sections 3.1 and 3.2 (Doyle and Sun) and chapter 3 (Sadd)
• introduction to stress
• description of the traction vector
• relation between the traction vector and normal vectors
• derivation of equilibrium equations
• normal, shear, and principal stresses

Week 7 (10/2 and 10/4) – stress continued
Required readings: sections 3.3 and 3.4 (Doyle and Sun) and chapter 3 (Sadd)
• equilibrium in the reference configuration
• physical derivation of the Lagrangian stress tensor
• the Kirchhoff stress tensor
• physical interpretation of stress tensors
• stresses in special deformations

Week 8 (10/9 and 10/11) – constitutive relations
Required readings: sections 4.1 and 4.2 (Doyle and Sun) and chapter 4 (Sadd)
• no class 10/9, October break
• introduction to constitutive relations
• constitutive relations from strain energy density
• major and minor symmetry of the stiffness tensor
• Voigt notation
• reduction of the stiffness tensor through material symmetry

Week 9 (10/16 and 10/18) – constitutive relations continued and linear elasticity problems
Required readings: sections 4.3 and 5.1 (Doyle and Sun) and sections 5.1-5.4 (Sadd)
• derivation of elastic constants for HILE materials
• introduction to general elasticity problems
• reduction to linear elasticity
• displacement formulation
• stress formulation

Week 10 (10/23 and 10/25) – linear elasticity problems continued
Required readings: section 5.1 (Doyle and Sun) and sections 5.6, 5.7, 6.1, and 6.2 (Sadd)
• solution strategies for linear elasticity problems
• uniqueness of linear elasticity solutions
• review for exam II
• exam II (10/25) – stress and constitutive relations

Week 11 (10/30 and 11/1) – plane elasticity in Cartesian coordinates
Required readings: sections 5.2 and 5.3 (Doyle and Sun) sections 7.1, 7.2, and 8.1 (Sadd)
• introduction to plane problems
• plane strain formulation
• plane stress formulation
• Airy stress function
• Cartesian coordinate solutions using polynomials

**Week 12 (11/6 and 11/8) – plane elasticity in cylindrical coordinates**
Required readings: section 5.4 (Doyle and Sun) and section 7.6 (Sadd)
• field equations in cylindrical coordinates
• plane strain and plane stress in cylindrical coordinates
• Airy stress function in cylindrical coordinates
• general Michell solution
• axisymmetric solution
• guidelines for selecting stress functions

**Week 13 (11/13 and 11/15) – examples of plane elasticity problems**
Required readings: review plane examples in chapter 5 (Doyle and Sun)
• bending of a beam by uniform transverse loading
• notch/crack problem
• hole in a tensile field
• the Flamant problem

**Week 14 (11/20 and 11/22) – variational methods**
Required readings: sections 6.2 and 7.1 (Doyle and Sun) and sections 6.5 and 6.6 (Sadd)
• introduction to calculus of variations
• principle of virtual work
• **no class 11/22, Thanksgiving break**

**Week 15 (11/27 and 11/29) – variational methods continued**
Required readings: section 7.2 (Doyle and Sun) and section 6.7 (Sadd)
• principle of minimum potential energy
• derivation of governing equations and boundary conditions from variational methods
• Rayleigh-Ritz method
• Galerkin method

**Week 16 (12/4 and 12/6) – introduction to the finite element method**
Required readings: section 6.1 (Doyle and Sun) and section 16.1 (Sadd)
• introduction to the finite element method for 3D linear elastic media
• catch up/closure
• special topics (time permitting)
• final exam review

**Week 17**
• **final exam (date TBD)**