Gas-Liquid Two-Phase Flow Studies using Three-Field Two-Fluid Model and Two-Group Interfacial Area Transport Equation

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Motivation and Objective

- **Motivation**: Computational Fluid Dynamics (CFD) simulation with a good two-phase model can give detailed three dimensional aspects for better design and performance of Two-phase systems (Nuclear Reactor, steam generator, heat exchanger, bubble column chemical reactor, electronic cooling system).

- **Objective**:
  - To develop a CFD framework within two-fluid model Formulation (Three-Field Two-fluid model with two group IATE) which can be used to predict the bubbly to churn-turbulent flow.
  - Select the right set of closure relations based on physics of adiabatic two-phase flow
    - IATE models (coalescence and breakup models) – Sun et al. (2001) selected – Needs to be tested in 3D form against Non-Uniform Conditions in CFD code
    - Existing Hydrodynamics model
    - Develop appropriate model based on correct physics observed in experiments
Presentation Outline

• Introduction-Theoretical background
• Experimental setup and Non-Uniform data
• CFD Benchmark preparation
• Bubbly flow simulation Results
• Two-group Simulations (Cap-bubbly, Cap-Turbulent)
• Summary and Future work
Introduction
Theoretical Background

◆ 3-D two-fluid model (Ishii, 1975; Ishii & Hibiki, 2010)

\[
\frac{\partial \alpha_k \rho_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k \vec{v}_k) = \Gamma_k
\]

\[
\frac{\partial \alpha_k \rho_k \vec{v}_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k \vec{v}_k \vec{v}_k) = -\alpha_k \nabla p_k + \nabla \cdot \alpha_k (\vec{\tau} + \vec{\tau}'_k) + \alpha_k \rho_k \vec{g} + \vec{v}_{ki} \Gamma_k + M_{ik} - \nabla \alpha_k \cdot \vec{\tau}_{ki}
\]

\[
\frac{\partial \alpha_k \rho_k i_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k i_k \vec{v}_k) = -\nabla \cdot \alpha_k (q_k + q'_k) + \alpha_k \frac{D_k p_k}{Dt} + i_{ki} \Gamma_k + a_i q''_{ki} + \phi_k
\]

◆ Interfacial transfer terms due to time average

(Interfacial transfer term) = \( a_i \times \) (Driving flux)

◆ Conventional approach

➢ Flow regime dependent correlations- Static approach, Causes Numerical Bifurcation during transition

◆ Advanced Approach

➢ Interfacial Area Transport Equation (IATE)- Dynamic approach, Multidimensional (1-D, 3-D)

\[
\frac{\partial a_i}{\partial t} + \nabla \cdot (a_i \vec{v}_i) = \frac{2}{3} \left( \frac{a_i}{\alpha_g} \right) \left[ \frac{\partial \alpha_g}{\partial t} + \nabla \cdot (\alpha_g \vec{v}_g) - \eta_{ph} \right] + \frac{1}{3\Psi} \left( \frac{a_i}{\alpha_g} \right)^2 \sum_j R_j + \pi D_{bc}^2 R_{ph}
\]
One Group IATE

• Particle transport equation
\[
\frac{\partial f}{\partial t} + \nabla \cdot (f \mathbf{v}) + \frac{\partial}{\partial V} \left( f \frac{dV}{dt} \right) = \sum_j S_j + S_{ph}
\]

• One Group number density transport Equation:
\[
\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{v}_m) = \sum_j R_j + R_{ph}
\]

• One Group Void transport:
\[
\alpha_g(x,t) = \int_{V_{min}}^{V_{max}} f(V,x,t) V dV
\]
\[
\frac{\partial \alpha_g}{\partial t} + \nabla \cdot (\alpha_g \mathbf{v}_g) + \int_{V_{min}}^{V_{max}} \left\{ V \frac{\partial}{\partial V} \left( f \frac{dV}{dt} \right) \right\} dV = \int_{V_{min}}^{V_{max}} \left\{ \sum_j S_j + S_{ph} \right\} V dV
\]
\[
\int_{V_{min}}^{V_{max}} \left\{ V \frac{\partial}{\partial V} \left( f \frac{dV}{dt} \right) \right\} dV = \frac{dV}{dt} = -\alpha_g
\]

\[
\frac{dm_b}{dt} = \left( \Gamma_g - \eta_{ph} \rho_g \right) \frac{V_b}{\alpha_g}
\]
\[
\frac{dm_b}{dt} = \frac{d(\rho_g V_b)}{dt} = V_b \frac{d\rho_g}{dt} + \rho_g \frac{dV_b}{dt}
\]
\[
\frac{1}{V} \frac{dV}{dt} = \frac{1}{\rho_g} \left[ \Gamma_g - \eta_{ph} \rho_g - \frac{d\rho_g}{dt} \right]
\]
\[
\frac{\partial \alpha_g \rho_g}{\partial t} + \nabla \cdot \alpha_g \rho_g \mathbf{v}_g = \Gamma_g
\]

\[\eta_{ph} \equiv \int_{V_{min}}^{V_{max}} S_{ph} V dV\]
• One Group IATE transport:

\[
\frac{\partial a_i}{\partial t} + \nabla \cdot (a_i \mathbf{v}_i) = \int_{V_{\text{min}}}^{V_{\text{max}}} \left[ A_i \frac{\partial}{\partial V} \left( f \frac{dV}{dt} \right) \right] dV = \int_{V_{\text{min}}}^{V_{\text{max}}} \left[ \sum_j S_j + S_{\text{ph}} \right] A_i dV
\]

\[
\int_{V_{\text{min}}}^{V_{\text{max}}} \left[ A_i \frac{\partial}{\partial V} \left( f \frac{dV}{dt} \right) \right] dV = \left( \frac{\dot{V}}{V} \right) \left( -\frac{2}{3} a_i \right)
\]

\[
\frac{\partial a_i}{\partial t} + \nabla \cdot (a_i \mathbf{v}_i) - \frac{2}{3} \frac{a_i}{\alpha_g} \left[ \frac{\partial \alpha_g}{\partial t} + \nabla \cdot (\alpha_g \mathbf{v}_g) - \eta_{\text{ph}} \right] = \int_{V_{\text{min}}}^{V_{\text{max}}} \left[ \sum_j S_j + S_{\text{ph}} \right] A_i dV
\]

\[
\frac{\partial a_i}{\partial t} + \nabla \cdot (a_i \mathbf{v}_i) = \left( \frac{2}{3} \right) \left( \frac{a_i}{\alpha_g} \right) \left[ \frac{\partial \alpha_g}{\partial t} + \nabla \cdot (\alpha_g \mathbf{v}_g) - \eta_{\text{ph}} \right] + \sum_j \phi_j
\]

\[
R_j \equiv \int_V S_j dV : \text{particle number density source/sink}
\]

\[
\phi_j \equiv \int_V S_j A_j dV : \text{interfacial area concentration source/sink}
\]

\[
\eta_j \equiv \int_V S_j \eta dV : \text{void fraction source/sink}
\]

\[
\frac{\partial a_i}{\partial t} + \nabla \cdot (a_i \mathbf{v}_i) = \left( \frac{2}{3} \right) \left( \frac{a_i}{\alpha_g} \right) \left[ \frac{\partial \alpha_g}{\partial t} + \nabla \cdot (\alpha_g \mathbf{v}_g) - \eta_{\text{ph}} \right] + \frac{1}{3 \Psi} \left( \frac{a_i}{\alpha_g} \right)^2 \sum R_j + \pi D_{bc}^2 R_{\text{ph}}
\]
Interfacial Area Transport Equation

Interfacial Area Transport Equation (Kojasoy and Ishii, 1995)

\[ \frac{\partial a_i}{\partial t} + \nabla \cdot (a_i \vec{v}_i) = \frac{2}{3} \left( \frac{a_i}{\alpha_g} \right) \left[ \frac{\partial \alpha_g}{\partial t} + \nabla \cdot (\alpha_g \vec{v}_g) - \eta_{ph} \right] + \frac{1}{3 \Psi} \left( \frac{a_i}{\alpha_g} \right)^2 \sum_j R_j + \pi D_{bc}^2 R_{ph} \]

Contribution due to particle volume change

Contribution due to fluid particle interactions: coalescence & breakup

Contribution due to phase change: nucleation & condensation
Key Bubble Length Scale and Bubble Group Boundary

◆ Two-Group Approach
  - Group-1 bubbles: spherical and distorted bubbles
  - Group-2 bubbles: cap, slug and churn bubbles

<table>
<thead>
<tr>
<th>Description</th>
<th>Length scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical bubble limit</td>
<td>( D_{ds} = 4 \sqrt{\frac{2\sigma}{g \Delta \rho}} N^{1/3} )</td>
</tr>
<tr>
<td>Maximum distorted bubble limit</td>
<td>( D_{d,max} = 4 \sqrt{\frac{\sigma}{g \Delta \rho}} )</td>
</tr>
<tr>
<td>Maximum cap bubble limit</td>
<td>( D_{c,max} = 40 \sqrt{\frac{\sigma}{g \Delta \rho}} )</td>
</tr>
<tr>
<td>Critical bubble size at the group boundary in narrow channel</td>
<td>( D_{c} = 1.7G^{1/3} \left( \frac{\sigma}{g \Delta \rho} \right)^{1/3} )</td>
</tr>
</tbody>
</table>

Two-Group IATE transport:

\[
\frac{\partial a_{i1}}{\partial t} + \nabla \cdot (a_{i1} \bar{v}_{i1}) = \left( \frac{2}{3} - \chi D_{c1}^* \right) \left( \frac{a_{i1}}{\alpha_{g1}} \right) \left( \frac{\partial \alpha_{g1}}{\partial t} + \nabla \cdot \alpha_{g1} \bar{v}_{g1} \right) + \sum_{j} \phi_{j,1}
\]

\[
\frac{\partial a_{i2}}{\partial t} + \nabla \cdot (a_{i2} \bar{v}_{i2}) = \frac{2}{3} \left( \frac{a_{i2}}{\alpha_{g2}} \right) \left( \frac{\partial \alpha_{g2}}{\partial t} + \nabla \cdot \alpha_{g2} \bar{v}_{g2} \right) + \chi D_{c1}^* \left( \frac{a_{i1}}{\alpha_{g1}} \right) \left( \frac{\partial \alpha_{g1}}{\partial t} + \nabla \cdot \alpha_{g1} \bar{v}_{g1} \right) + \sum_{j} \phi_{j,2}
\]
Schematics of Two-Group Bubble Interaction (Ishii et al., 2002)
Two-Group IATE with Three-Field Two-Fluid Model: Two Gas Momentum Equations

- Two-group gas continuity equation

\[
\frac{\partial (\alpha_{g1}\rho_{g1})}{\partial t} + \nabla \cdot (\alpha_{g1}\rho_{g1} v_{g1}) = -\Delta m_{12}
\]

\[
\frac{\partial (\alpha_{g2}\rho_{g2})}{\partial t} + \nabla \cdot (\alpha_{g2}\rho_{g2} v_{g2}) = \Delta m_{12}
\]

- Two-group gas momentum equation

\[
\frac{\partial \alpha_{g1}\rho_{g1} v_{g1}}{\partial t} + \nabla \cdot (\alpha_{g1}\rho_{g1} v_{g1} v_{g1}) = -\alpha_{g1} \nabla p_{g1} + \nabla \cdot \alpha_{g1} \left( -\tau_{g1} + \tau_{g1}' \right) + \alpha_{g1} \rho_{g1} g - \Delta m_{12} v_{g11} + M_{ig1} - \nabla \alpha_{g1} \cdot \tau_{g11}
\]

\[
\frac{\partial \alpha_{g2}\rho_{g2} v_{g2}}{\partial t} + \nabla \cdot (\alpha_{g2}\rho_{g2} v_{g2} v_{g2}) = -\alpha_{g2} \nabla p_{g2} + \nabla \cdot \alpha_{g2} \left( -\tau_{g2} + \tau_{g2}' \right) + \alpha_{g2} \rho_{g2} g + \Delta m_{12} v_{g22} + M_{ig2} - \nabla \alpha_{g2} \cdot \tau_{g22}
\]

- Two-group IATE

\[
\frac{\partial a_{i1}}{\partial t} + \nabla \cdot (a_{i1} v_{i1}) = \left( \frac{2}{3} - \chi D_{c1}^{*} \right) \left( \frac{a_{i1}}{\alpha_{g1}} \right) \left( \frac{\partial \alpha_{g1}}{\partial t} + \nabla \cdot \alpha_{g1} v_{g1} \right) + \sum_{j} \phi_{j,1}
\]

\[
\frac{\partial a_{i2}}{\partial t} + \nabla \cdot (a_{i2} v_{i2}) = \left( \frac{2}{3} - \chi D_{c1}^{*} \right) \left( \frac{a_{i2}}{\alpha_{g2}} \right) \left( \frac{\partial \alpha_{g2}}{\partial t} + \nabla \cdot \alpha_{g2} v_{g2} \right) + \chi D_{c1}^{*} \left( \frac{a_{i1}}{\alpha_{g1}} \right) \left( \frac{\partial \alpha_{g1}}{\partial t} + \nabla \cdot \alpha_{g1} v_{g1} \right) + \sum_{j} \phi_{j,2}
\]

\[a_{i} \times \text{(Driving flux)}\]

Coalescence and Breakup mechanism
Momentum closures

\[ M_{id} = \frac{Q_d}{B_d} \left( F^D_d + F^L_d + F^W_d + F^{TD}_d + F^{VM}_d + F^B_d \right) \]

\[ = M^D_d + M^L_d + M^W_d + M^{TD}_d + M^{VM}_d + M^B_d \]

<table>
<thead>
<tr>
<th>Interfacial Forces</th>
<th>Phases</th>
<th>Nature</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drag Force</td>
<td>Gas and Liquid</td>
<td>Interfacial force</td>
<td>Ishii and Zuber (1979)</td>
</tr>
<tr>
<td>Wall Lubrication Force</td>
<td>Gas and Liquid</td>
<td>Interfacial force</td>
<td>Antal et al. (1991) ( C_{W1} = -0.01, C_{W2} = 0.05 )</td>
</tr>
<tr>
<td>Lift force</td>
<td>Gas and Liquid</td>
<td>Interfacial force</td>
<td>Hibiki and Ishii (2007) ( C_{L,G1} = 0.01, C_{L,G2} = -0.5 )</td>
</tr>
<tr>
<td>Turbulent dispersion force</td>
<td>Gas and Liquid</td>
<td>Interfacial force</td>
<td>Bertodano (1992) ( C_{TD,1} = 0.25 )</td>
</tr>
<tr>
<td>Momentum Transfer by Bubble interaction mechanism (Random Collision)</td>
<td>Gas and Liquid</td>
<td>Interfacial force</td>
<td>Sharma et al. (2017, 2019)</td>
</tr>
</tbody>
</table>
| Bubble-induced turbulence           | Liquid                  | Turbulence Induced by relative motions of Bubbles in liquid            | Sato et al. (1981) \( C_{Sato,G1} = 0.6 \) \( C_{Sato,G2} = 2.1 \) \( Lee (2010); Lopez de Bertodano et al. (2006); Prabhudharwadkar et al. (2012) \)

\[ \mu_{t,f} = \mu_{Sl} + \mu_{Bl} \]

\[ \mu_{Sl} = C_{\mu} \rho_f \frac{k^2}{\varepsilon_f}; \mu_{Bl} = C_{Bl} \rho_f \alpha D_b \left| v_g - v_f \right| \]

\[ \nu_{t,g} = \frac{\nu_{t,f}}{\sigma} \Rightarrow \mu_{t,g} = \frac{\rho_g \mu_{t,f}}{\rho_f \sigma}; \sigma = 1 \]

Liquid phase Turbulence: k-\( \varepsilon \) model

Gas phase turbulence: zero order

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Experimental Facility

- Rectangular channel
  - 200 mm x 10 mm x 2950 mm
- Total test section length
  - z/D_h~150
- Local instrumentation ports
  - z/D_h=34.8 (Port 2), 88.2 (Port 4), 142 (Port 6)
- Flow regime of interest
  - Bubbly
  - Cap-turbulent
  - Churn-turbulent
- Measured flow parameters
  - α_g, a_i, v_g, D_{Sm}
- Experimental database
  - Uniform inlet injection
  - Non-uniform inlet injection

Cross Sectional View of Test Section
Experimental Database Used for Benchmarking with Non-Uniform Inlet Boundary Conditions

Non-Uniform inlet injection

- Flow Regime Transition (Neural Network)
- Bubbly-Cap Transition (Mishima and Ishii, 1984)

Test Conditions:
- Run 1, 2, 3, 4
- Run 5
- Run 6

- Bubbly
- Cap Turbulent
- Churn Turbulent

Uncertainty and Possible Error:

Kim et al (2000):
- Void Fraction: 5%
- Interfacial Area conc.: 10%

Le Corre et al (2003):
- Void Fraction: 7%
- Gas velocity: 12%
- Interfacial Area conc.: 25%

Factors affecting:
- finite size probe effect (missing bubbles)
- data acquisition (sampling frequency, total sampling time)
- signal processing (filtering, threshold)

Uniform Liquid Injection

Uniform Gas Injection
CFD simulation strategy

◆ B.C
  ● Inlet:
    ✓ Local measured void fraction, gas velocity, IAC at Port 2
    ✓ Liquid velocity Profile variation in x-direction obtained from gas velocity by subtracting slip velocity, in y-direction 1/7th power law variation used
  ● Outlet atmospheric pressure

Mesh Configuration: 60x10x162

Wall B.C. No slip for liquid phase and Free slip for Gas phase
Bubbly Flow - Run1

\[ \langle j_{g,0} \rangle = 0.289 \text{ m/s} \]
\[ \langle j_t \rangle = 1.25 \text{ m/s} \]
Bubbly Flow - Run 2

$\langle j_{g,0} \rangle = 0.289 \text{ m/s}$

$\langle j_f \rangle = 1.25 \text{ m/s}$

Double Side $j_f$ Profile
\( <j_{g,0}> = 0.289 \text{ m/s} \)

\( <j_f> = 1.25 \text{ m/s} \)

Center \( j_f \) Profile
\langle j_{g,0} \rangle = 0.295 \text{ m/s} \\
\langle j_f \rangle = 1.25 \text{ m/s} \\

Double Side $j_g$ Profile
\( \langle j_{g,0} \rangle = 0.289 \text{ m/s} \)
\( \langle j_f \rangle = 0.943 \text{ m/s} \)

**Center \( j_f \) Profile**

![Graph showing \( j_f \) profile](image)

**Void Fraction, \( \alpha \)**

- Port 2
  - Group 1
  - Group 2

- Port 4
  - Exp: Group 1
  - Exp: Group 2
  - CFX: Group 1
  - CFX: Group 2

- Port 6
  - Exp: Group 1
  - Exp: Group 2
  - CFX: Group 1

**Gas Velocity, \( v \)**

- Port 2
  - Group 1
  - Group 2

- Port 4
  - Exp: Group 1
  - Exp: Group 2
  - CFX: Group 1

- Port 6
  - Exp: Group 1
  - Exp: Group 2

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$\langle \dot{V}_g \rangle = 0.086 \text{ m/s}$

$\langle \dot{V}_f \rangle = 0.515 \text{ m/s}$

- Hydraodynamics model predicting the trend in measured void fraction
- Good prediction of development of velocity profile by models
Two-group case

- Cap-bubbly, cap-turbulent, churn-turbulent flows
Two-Group cases: Run 1

$<j_g,0> = 0.77 \text{ m/s}$

$<j_f> = 1.92 \text{ m/s}$

- Uniform $j_g$ Profile
- Transverse Position, $x/W$ [-]

- Good prediction for overall trend and magnitude of each bubble group in transverse direction
Two-Group cases: Run 2

\[ \langle j_{g,0} \rangle = 0.74 \text{ m/s} \]
\[ \langle j_{f} \rangle = 1.25 \text{ m/s} \]

- Good prediction for overall trend of phase distribution in transverse direction.
Two-Group cases: Run 3

\[ \langle j_{g,0} \rangle = 0.77 \text{ m/s} \]
\[ \langle j_{f} \rangle = 1.25 \text{ m/s} \]

- Very Good prediction for overall trend of phase distribution in transverse direction.
- Underprediction of volume transfer from G-2 to G-1 bubbles
Two-Group cases: Run 4

\[<j_{g,0}> = 0.74 \text{ m/s} \]
\[<j_f> = 0.943 \text{ m/s} \]

- Very Good prediction for overall trend of phase distribution in transverse direction.
- Underprediction of volume transfer from G-2 to G-1 bubbles
Two-Group cases: Run 5

\[ \langle j_g,0 \rangle = 0.74 \text{ m/s} \]
\[ \langle j_f \rangle = 0.942 \text{ m/s} \]

- Very Good prediction for overall trend of phase distribution in transverse direction.
- Underprediction of volume transfer from G-1 to G-2 bubbles by coalescence.
- Underprediction of the increase in G-2 bubble sauter mean diameter.
Two-Group cases: Run 5

Summary:
- Hydrodynamics model gives satisfactory prediction in the trend of phase distribution
- Discrepancies were found in IATE model prediction in some cases, specially in intergroup transfer model
- Need to establish a good set of flow transition experimental database for IATE benchmarking
- Need to improve coalescence model
Conclusion

- CFD Framework with three-field two-fluid model and two-group IATE was prepared
- Benchmarking simulations was carried out against uniform and non-uniform inlet test conditions.
- In general, Hydrodynamic models along with IATE showing satisfactory prediction of phase distribution
  - Bubble coalescence model will be further investigated
Thank you!!