Gas-Liquid Two-Phase Flow Studies using Three-Field Two-Fluid Model and Two-Group Interfacial Area Transport Equation

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Motivation and Objective

- Motivation: Computational Fluid Dynamics (CFD) simulation with a good two-phase model can give detailed three dimensional aspects for better design and performance of Two-phase systems (Nuclear Reactor, steam generator, heat exchanger, bubble column chemical reactor, electronic cooling system)
- Objective:
  - ✓ To develop a CFD framework within two-fluid model Formulation (Three-Field Two-fluid model with two group IATE) which can be used to predict the bubbly to churn-turbulent flow.
  - ✓ Select the right set of closure relations based on physics of adiabatic two-phase flow
    - – IATE models (coalescence and breakup models) – Sun et al. (2001) selected –Needs to be tested in 3D form against Non-Uniform Conditions in CFD code
    - – Existing Hydrodynamics model
    - – Develop appropriate model based on correct physics observed in experiments

Presentation Outline

- Introduction-Theoretical background
- Experimental setup and Non-Uniform data
- CFD Benchmark preparation
- Bubbly flow simulation Results
- Two-group Simulations (Cap-bubbly, Cap-Turbulent)
- Summary and Future work

Theoretical Background

- 3-D two-fluid model (Ishii, 1975; Ishii & Hibiki, 2010)

$$\frac{\partial \alpha_k \rho_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k \vec{v}_k) = \Gamma_k$$

$$\frac{\partial \alpha_k \rho_k \vec{v}_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k \vec{v}_k \vec{v}_k) = -\alpha_k \nabla p_k + \nabla \cdot \alpha_k (\overline{\overline{\tau}} + \tau_k^t) + \alpha_k \rho_k \vec{g} + \vec{v}_{ki} \Gamma_k + \overrightarrow{M}_{ik} - \nabla \alpha_k \cdot \overline{\overline{\tau}}_{ki}$$

$$\frac{\partial \alpha_k \rho_k i_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k i_k \vec{v}_k) = -\nabla \cdot \alpha_k (\overline{\overline{q}}_k + q_k^t) + \alpha_k \frac{D_k p_k}{Dt} + i_{ki} \Gamma_k + a_i q_{ki}'' + \phi_k$$

- Interfacial transfer terms due to time average

$$\left(\text{Interfacial transfer term}\right) = a_i \times (\text{Driving flux})$$

- Conventional approach
  - Flow regime dependent correlations- **Static** approach**, C**auses Numerical Bifurcation during transition
- Advanced Approach
  - Interfacial Area Transport Equation (IATE)- Dynamic approach, Multidimensional (1-D, 3-D)

$$\frac{\partial a_i}{\partial t} + \nabla \cdot (a_i \vec{v}_i) = \frac{2}{3}\left(\frac{a_i}{\alpha_g}\right)\left[\frac{\partial \alpha_g}{\partial t} + \nabla \cdot (\alpha_g \vec{v}_g) - \eta_{ph}\right] + \frac{1}{3\Psi}\left(\frac{a_i}{\alpha_g}\right)^2 \sum_j R_j + \pi D_{bc}^2 R_{ph}$$

One Group IATE

- Particle transport equation

$$\frac{\partial f}{\partial t}+\nabla\cdot(f\boldsymbol{v})+\frac{\partial}{\partial V}\left(f\frac{dV}{dt}\right)=\sum_j S_j+S_{ph}$$

- One Group number density transport Equation: $n(\boldsymbol{x},t)=\int_{V_{min}}^{V_{max}} f(V,\boldsymbol{x},t)dV$

$$\frac{\partial n}{\partial t}+\nabla\cdot\left(n\,\boldsymbol{v}_{pm}\right)=\sum_j R_j+R_{ph}$$

- One Group Void transport: $\alpha_g(\boldsymbol{x},t)=\int_{V_{min}}^{V_{max}} f(V,\boldsymbol{x},t)V dV$

$$\frac{\partial \alpha_g}{\partial t}+\nabla\cdot\left(\alpha_g\boldsymbol{v}_g\right)+\boxed{\int_{V_{min}}^{V_{max}}\left\{V\frac{\partial}{\partial V}\left(f\frac{dV}{dt}\right)\right\}dV}=\int_{V_{min}}^{V_{max}}\left(\sum_j S_j+S_{ph}\right)VdV$$

$$\int_{V_{min}}^{V_{max}}\left\{V\frac{\partial}{\partial V}\left(f\frac{dV}{dt}\right)\right\}dV=\boxed{\left(\frac{\dot{V}}{V}\right)}\left\{-\alpha_g\right\}$$

$$\frac{dm_b}{dt}=\left(\Gamma_g-\eta_{ph}\rho_g\right).\frac{V_b}{\alpha_g}$$

$$\frac{dm_b}{dt}=\frac{d\left(\rho_g V_b\right)}{dt}=V_b\frac{d\rho_g}{dt}+\rho_g\frac{dV_b}{dt}$$

$$\frac{1}{V}\frac{dV}{dt}=\frac{1}{\rho_g}\left(\frac{\Gamma_g-\eta_{ph}\rho_g}{\alpha_g}-\frac{d\rho_g}{dt}\right)$$

$$\eta_{ph}\equiv\int_{V_{min}}^{V_{max}}S_{ph}VdV$$

$$\frac{\partial \alpha_g\rho_g}{\partial t}+\nabla\cdot\alpha_g\rho_g\boldsymbol{v}_g=\Gamma_g$$

- One Group IATE transport: $a_i(\boldsymbol{x},t)=\int_{V_{min}}^{V_{max}} f(V,\boldsymbol{x},t)A_i(V)\,dV$

$$\frac{\partial a_i}{\partial t}+\nabla\cdot\left(a_i\,\boldsymbol{v}_i\right)+\boxed{\int_{V_{\min}}^{V_{\max}}\left\{A_i\frac{\partial}{\partial V}\left(f\frac{dV}{dt}\right)\right\}dV}=\int_{V_{\min}}^{V_{\max}}\left(\sum_j S_j+S_{ph}\right)A_i\,dV$$

$$\downarrow$$

$$\int_{V_{\min}}^{V_{\max}}\left\{A_i\frac{\partial}{\partial V}\left(f\frac{dV}{dt}\right)\right\}dV=\boxed{\left(\frac{\dot V}{V}\right)}\left(-\frac{2}{3}a_i\right)$$

$$\frac{\partial a_i}{\partial t}+\nabla\cdot\left(a_i\,\boldsymbol{v}_i\right)-\frac{2}{3}\frac{a_i}{\alpha_g}\left\{\frac{\partial\alpha_g}{\partial t}+\nabla\cdot\left(\alpha_g\boldsymbol{v}_g\right)-\eta_{ph}\right\}=\int_{V_{\min}}^{V_{\max}}\left(\sum_j S_j+S_{ph}\right)A_i\,dV$$

$$\frac{\partial a_i}{\partial t}+\nabla\cdot(a_i\vec{v}_i)=\left(\frac{2}{3}\right)\left(\frac{a_{i1}}{\alpha_{g1}}\right)\left(\frac{\partial\alpha_g}{\partial t}+\nabla\cdot\alpha_g\vec{v}_g-\eta_{ph}\right)+\sum_j\phi_j$$

$R_j\equiv\int_V S_j\,dV$: particle number density source/sink

$\phi_j\equiv\int_V S_jA_j\,dV$: interfacial area concentration source/sink

$\eta_j\equiv\int_V S_jV\,dV$: void fraction source/sink

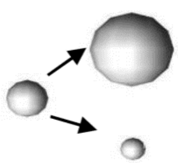
$$\frac{\partial a_i}{\partial t}+\nabla\cdot(a_i\vec{v}_i)=\frac{2}{3}\left(\frac{a_i}{\alpha_g}\right)\left[\frac{\partial\alpha_g}{\partial t}+\nabla\cdot(\alpha_g\vec{v}_g)-\eta_{ph}\right]+\frac{1}{3\Psi}\left(\frac{a_i}{\alpha_g}\right)^2\sum_j R_j+\pi D_{bc}^2R_{ph}$$

Interfacial Area Transport Equation

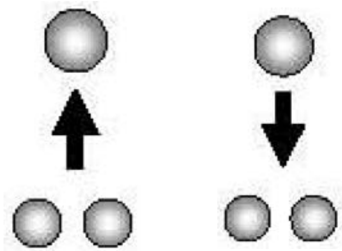
- Interfacial Area Transport Equation (Kojasoy and Ishii, 1995)

$$\frac{\partial a_i}{\partial t}+\nabla\cdot(a_i\vec{v}_i)=\frac{2}{3}\left(\frac{a_i}{\alpha_g}\right)\left[\frac{\partial\alpha_g}{\partial t}+\nabla\cdot(\alpha_g\vec{v}_g)-\eta_{ph}\right]+\frac{1}{3\Psi}\left(\frac{a_i}{\alpha_g}\right)^2\sum_j R_j+\pi D_{bc}^2 R_{ph}$$

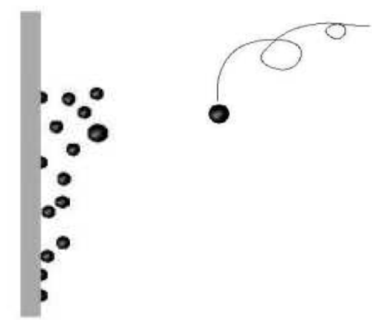
Contribution due to particle volume change



Contribution due to fluid particle interactions: coalescence & breakup



Contribution due to phase change: nucleation & condensation



Key Bubble Length Scale and Bubble Group Boundary

- Two-Group Approach
  - Group-1 bubbles: spherical and distorted bubbles
  - Group-2 bubbles: cap, slug and churn bubbles

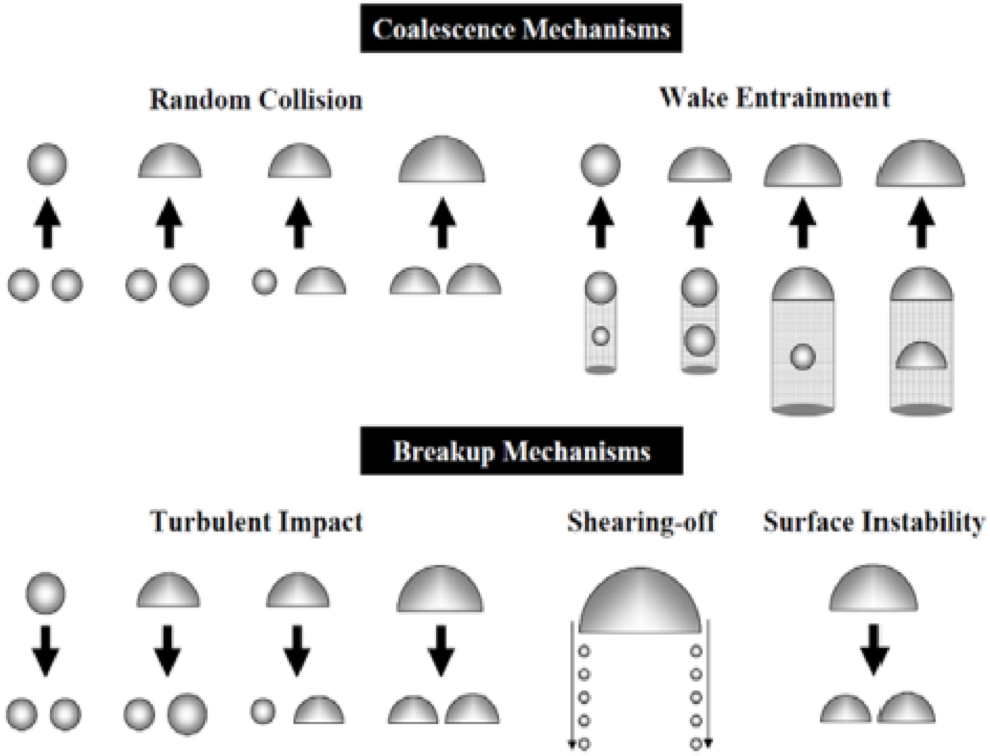
Description	Length scale
Spherical bubble limit	$D_{ds} = 4\sqrt{\frac{2\sigma}{g\Delta\rho}} N_{\mu f}^{1/3}$
Maximum distorted bubble limit	$D_{d,\max} = 4\sqrt{\frac{\sigma}{g\Delta\rho}}$
Maximum cap bubble limit	$D_{c,\max} = 40\sqrt{\frac{\sigma}{g\Delta\rho}}$
Critical bubble size at the group boundary in narrow channel	$D_c = 1.7 G^{1/3}\left(\frac{\sigma}{g\Delta\rho}\right)^{1/3}$

Two-Group IATE transport:

$$\frac{\partial a_{i1}}{\partial t} + \nabla\cdot(a_{i1}\vec{v}_{i1}) = \left(\frac{2}{3} - \chi D_{c1}^{*\,2}\right)\left(\frac{a_{i1}}{\alpha_{g1}}\right)\left(\frac{\partial \alpha_{g1}}{\partial t} + \nabla\cdot\alpha_{g1}\vec{v}_{g1}\right) + \sum_j \phi_{j,1}$$

$$\frac{\partial a_{i2}}{\partial t} + \nabla\cdot(a_{i2}\vec{v}_{i2}) = \frac{2}{3}\left(\frac{a_{i2}}{\alpha_{g2}}\right)\left(\frac{\partial \alpha_{g2}}{\partial t} + \nabla\cdot\alpha_{g2}\vec{v}_{g2}\right) + \chi D_{c1}^{*\,2}\left(\frac{a_{i1}}{\alpha_{g1}}\right)\left(\frac{\partial \alpha_{g1}}{\partial t} + \nabla\cdot\alpha_{g1}\vec{v}_{g1}\right) + \sum_j \phi_{j,2}$$

Schematics of Two-Group Bubble Interaction (Ishii et al., 2002)



Two-Group IATE with Three-Field Two-Fluid Model: Two Gas Momentum Equations

- Two-group gas continuity equation

$$\frac{\partial(\alpha_{g1}\rho_g)}{\partial t}+\nabla\cdot(\alpha_{g1}\rho_g\vec{v}_{g1})=-\Delta\dot{m}_{12}$$

$$\frac{\partial(\alpha_{g2}\rho_g)}{\partial t}+\nabla\cdot(\alpha_{g2}\rho_g\vec{v}_{g2})=\Delta\dot{m}_{12}$$

- Two-group gas momentum equation

$$\frac{\partial\alpha_{g1}\rho_g\vec{v}_{g1}}{\partial t}+\nabla\cdot\alpha_{g1}\rho_g\vec{v}_{g1}\vec{v}_{g1}=-\alpha_{g1}\nabla p_{g1}+\nabla\cdot\alpha_{g1}\left(\overline{\overline{\tau}}_{g1}+\tau^{t}_{g1}\right)+\alpha_{g1}\rho_g\vec{g}-\Delta\dot{m}_{12}\vec{v}_{g1i}+\overline{M}_{ig1}-\nabla\alpha_{g1}\cdot\overline{\overline{\tau}}_{g1i}$$

$$\frac{\partial\alpha_{g2}\rho_g\vec{v}_{g2}}{\partial t}+\nabla\cdot\alpha_{g2}\rho_g\vec{v}_{g2}\vec{v}_{g2}=-\alpha_{g2}\nabla p_{g2}+\nabla\cdot\alpha_{g2}\left(\overline{\overline{\tau}}_{g2}+\tau^{t}_{g2}\right)+\alpha_{g2}\rho_g\vec{g}+\Delta\dot{m}_{12}\vec{v}_{g2i}+\overline{M}_{ig2}-\nabla\alpha_{g2}\cdot\overline{\overline{\tau}}_{g2i}$$

$a_i \times$ (Driving flux)

- Two-group IATE

$$\frac{\partial a_{i1}}{\partial t}+\nabla\cdot(a_{i1}\vec{v}_{i1})=\left(\frac{2}{3}-\chi D^{*\,2}_{c1}\right)\left(\frac{a_{i1}}{\alpha_{g1}}\right)\left(\frac{\partial\alpha_{g1}}{\partial t}+\nabla\cdot\alpha_{g1}\vec{v}_{g1}\right)+\sum_j\phi_{j,1}$$

Coalescence and Breakup mechanism

$$\frac{\partial a_{i2}}{\partial t}+\nabla\cdot(a_{i2}\vec{v}_{i2})=\frac{2}{3}\left(\frac{a_{i2}}{\alpha_{g2}}\right)\left(\frac{\partial\alpha_{g2}}{\partial t}+\nabla\cdot\alpha_{g2}\vec{v}_{g2}\right)+\chi D^{*\,2}_{c1}\left(\frac{a_{i1}}{\alpha_{g1}}\right)\left(\frac{\partial\alpha_{g1}}{\partial t}+\nabla\cdot\alpha_{g1}\vec{v}_{g1}\right)+\sum_j\phi_{j,2}$$

Momentum closures

$$M_{id} = \frac{\alpha_d}{B_d}\left(F_d^D + F_d^L + F_d^W + F_d^{TD} + F_d^{VM} + F_d^B\right)$$

$$= M_d^D + M_d^L + M_d^W + M_d^{TD} + M_d^{VM} + M_d^B$$

Interfacial Forces	Phases	Nature	Coefficient
Drag Force	Gas and Liquid	Interfacial force	Ishii and Zuber (1979)
Wall Lubrication Force	Gas and Liquid	Interfacial force	Antal et al. (1991) $C_{W1} = -0.01, C_{W2} = 0.05$
Lift force	Gas and Liquid	Interfacial force	Hibiki and Ishii (2007) $C_{L,G1} = 0.01, C_{L,G2} = -0.5$
Turbulent dispersion force	Gas and Liquid	Interfacial force	Bertodano (1992) $C_{TD,1} = 0.25$
Momentum Transfer by Bubble interaction mechanism (Random Collision)	Gas and Liquid	Interfacial force	Sharma et al. (2017, 2019)
Bubble-induced turbulence	Liquid	Turbulence Induced by relative motions of Bubbles in liquid	Sato et al. (1981) $C_{Sato,G1} = 0.6$ $C_{Sato,G2} = 2.1$ Lee (2010); Lopez de Bertodano et al.(2006); Prabhudharwadkar et al. (2012)

Liquid phase Turbulence: k-ε model

$$\mu_{t,f} = \mu_{SI} + \mu_{BI}$$

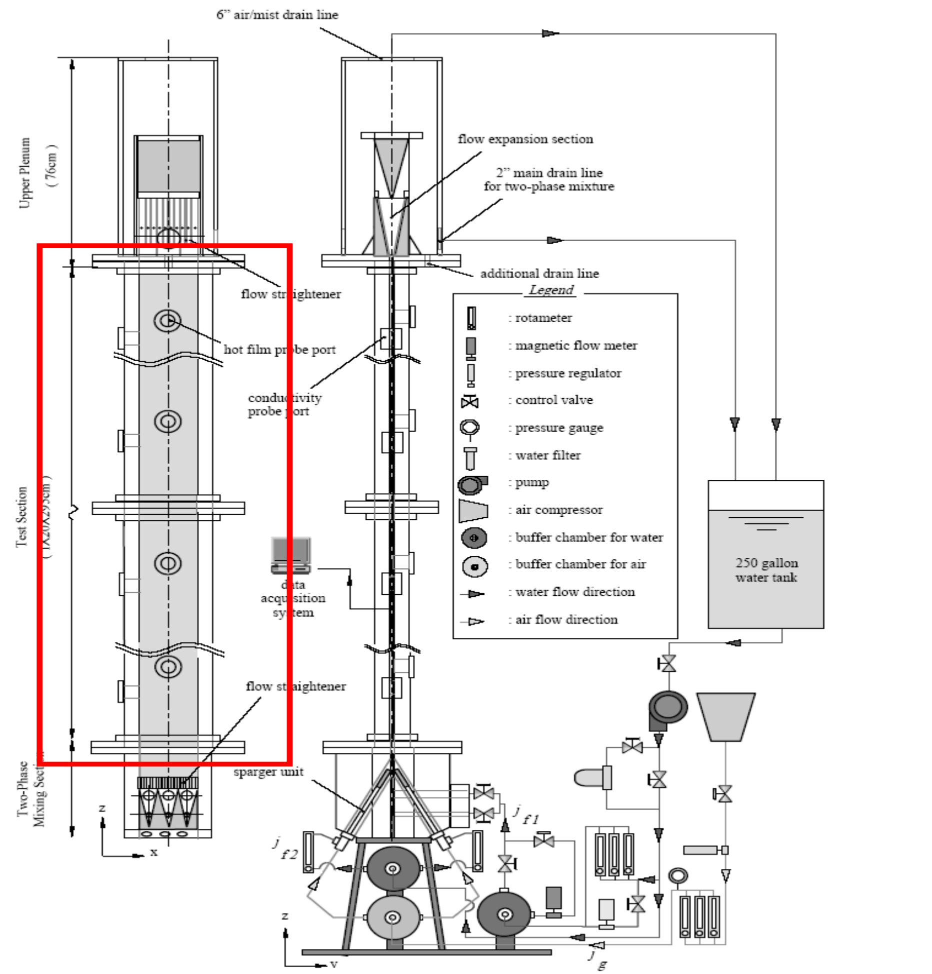
$$\mu_{SI} = C_\mu \rho_f \frac{k_f^2}{\varepsilon_f}; \mu_{BI} = C_{BI}\rho_f \alpha D_b \left|\upsilon_g - \upsilon_f\right|$$

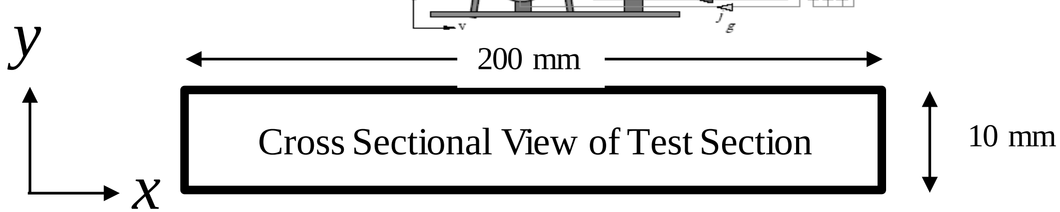
Gas phase turbulence: zero order

$$\upsilon_{t,g} = \frac{\upsilon_{t,f}}{\sigma} \Rightarrow \mu_{t,g} = \frac{\rho_g}{\rho_f}\frac{\mu_{t,f}}{\sigma}; \sigma = 1$$

Experimental Facility

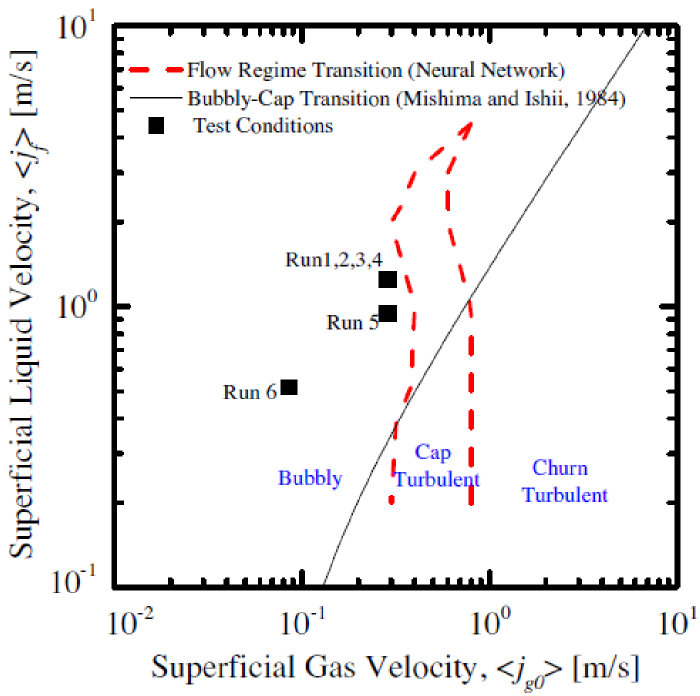
- Rectangular channel
  - 200 mm x 10 mm x 2950 mm
- Total test section length
  - z/D_h~150
- Local instrumentation ports
  - z/D_h=34.8 (Port 2), 88.2 (Port 4), 142 (Port 6)
- Flow regime of interest
  - Bubbly
  - Cap-turbulent
  - Churn-turbulent
- Measured flow parameters
  - $\alpha_g, a_i, v_g, D_{Sm}$
- Experimental database
  - Uniform inlet injection
  - Non-uniform inlet injection





Experimental Database Used for Benchmarking with Non-Uniform Inlet Boundary Conditions

Non-Uniform inlet injection



Uncertainty and Possible Error:

Kim et al (2000):

Void Fraction: 5%

Interfacial Area conc.:10%

Le Corre et al (2003):

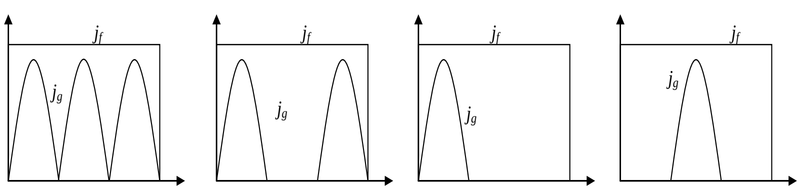
Void Fraction:7%

Gas velocity:12%

Interfacial Area conc.:25%

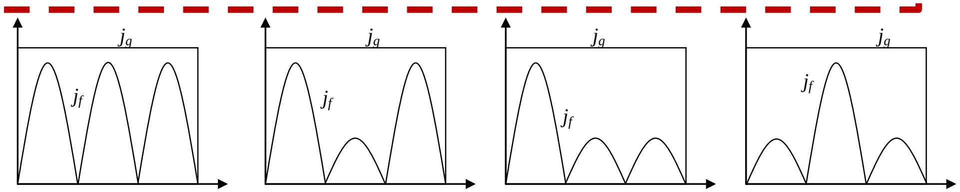
Factors affecting:

- finite size probe effect (missing bubbles)
- data acquisition (sampling frequency, total sampling time)
- signal processing (filtering, threshold)



Uniform j_g Profile | Double Side j_g Profile | Single Side j_g Profile | Center j_g Profile

Uniform Liquid Injection



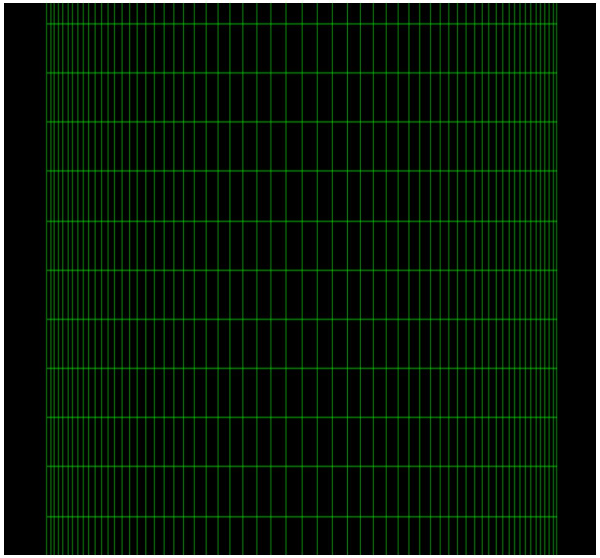
Uniform j_f Profile | Double Side j_f Profile | Single Side j_f Profile | Center j_f Profile

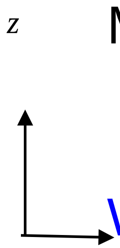
Uniform Gas Injection

CFD simulation strategy

- **B.C**
  - **Inlet :**
    - ✓ **Local measured void fraction, gas velocity , IAC at Port 2**
    - ✓ **Liquid velocity Profile** variation in x- direction obtained from gas velocity by subtracting slip velocity, in y- direction 1/7th power law variation used
  - Outlet atmospheric pressure

Mesh



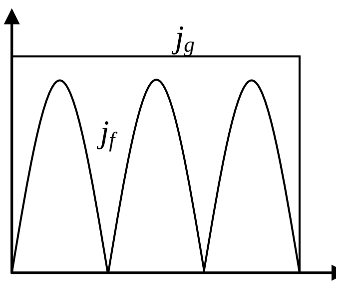


Mesh Configuration: 60x10x162

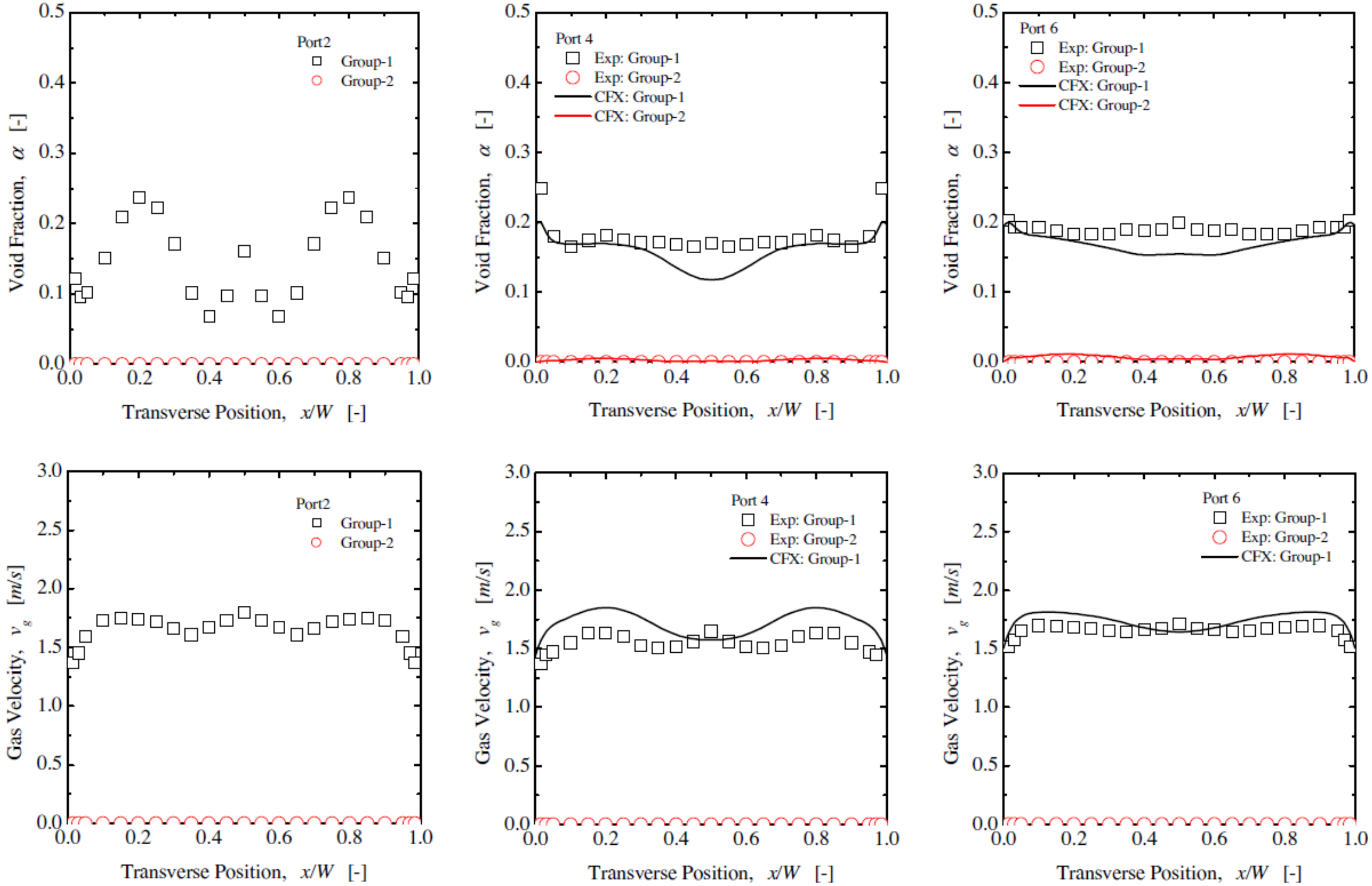
Wall B.C. No slip for liquid phase and Free slip for Gas phase

Bubbly Flow - Run1

$\langle j_{g,0} \rangle = 0.289$ m/s
$\langle j_f \rangle = 1.25$ m/s

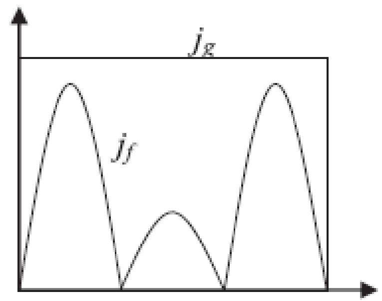


Uniform j_f Profile

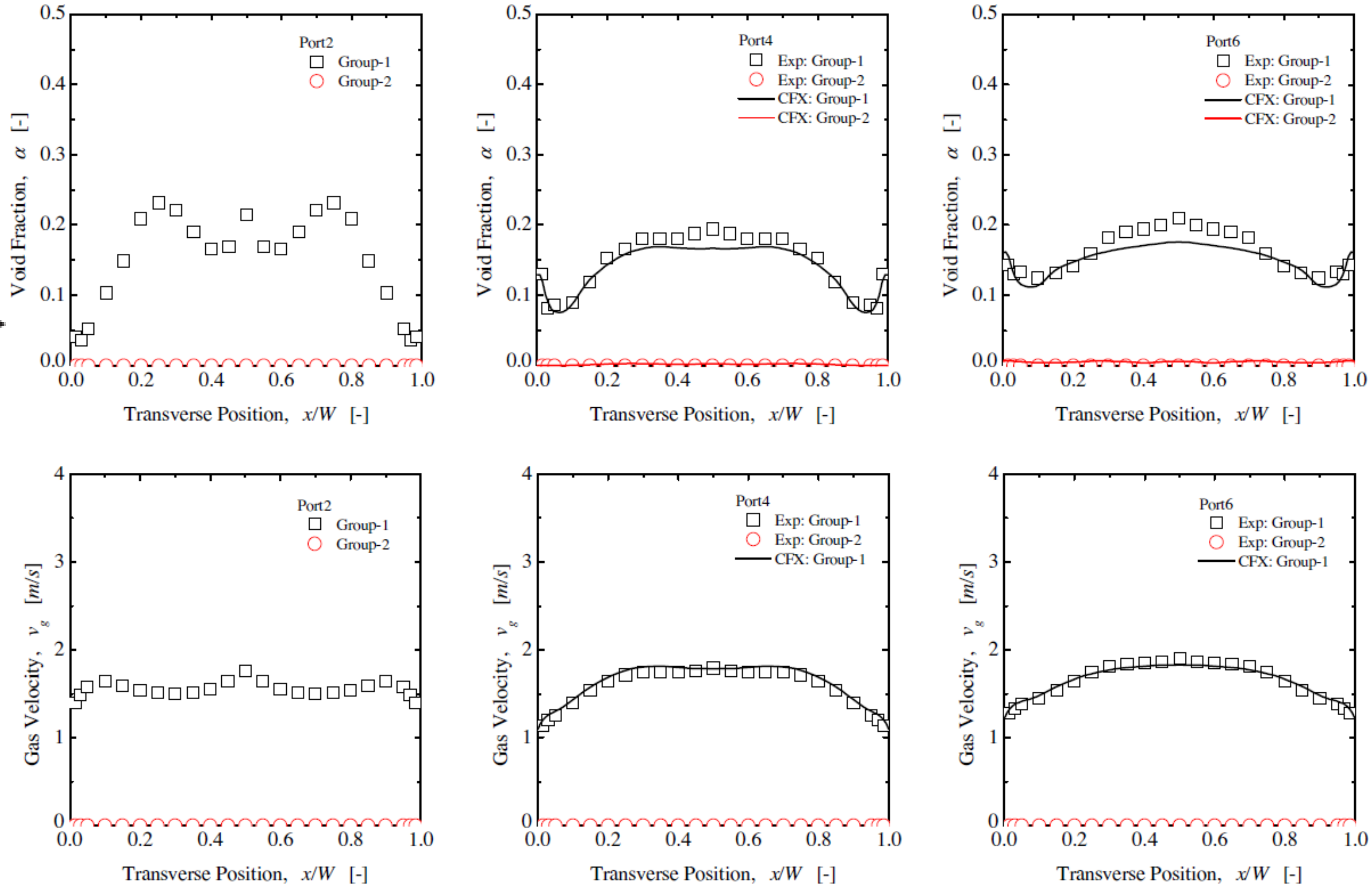


Bubbly Flow - Run 2

$\langle j_{g,0} \rangle = 0.289$ m/s
$\langle j_f \rangle = 1.25$ m/s

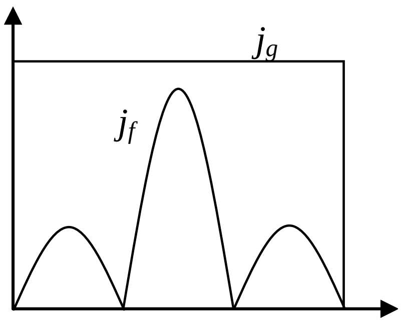


Double Side j_f Profile

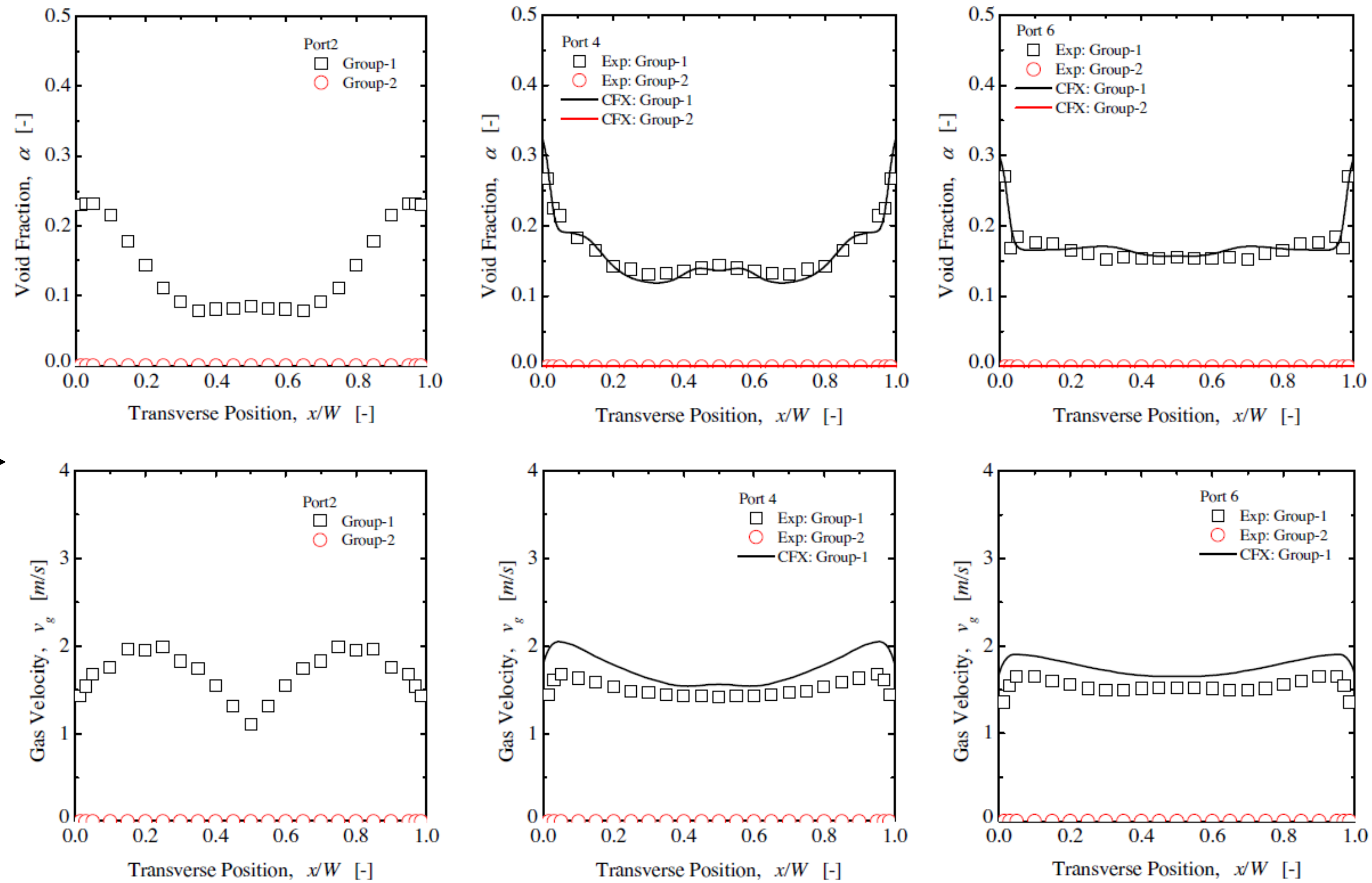


Bubbly Flow - Run 3

$\langle j_{g,0} \rangle$=0.289 m/s
$\langle j_f \rangle$=1.25 m/s

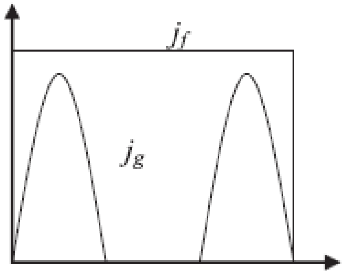


Center j_f Profile

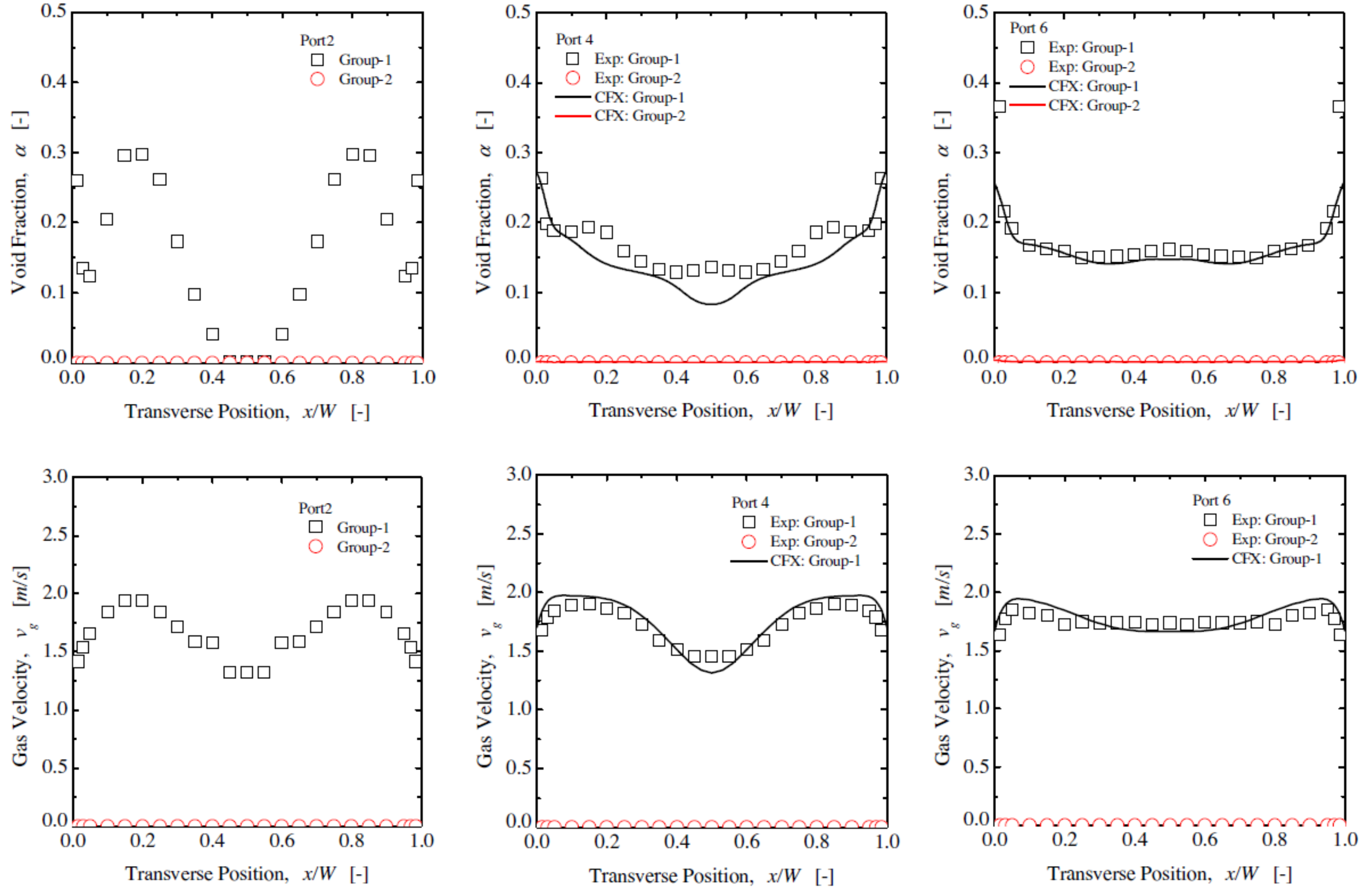


Bubbly Flow - Run 4

$\langle j_{g,0} \rangle = 0.295$ m/s
$\langle j_f \rangle = 1.25$ m/s

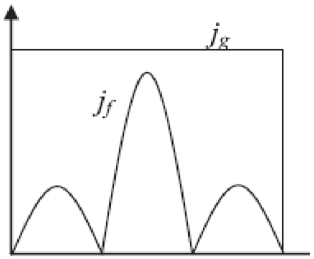


Double Side j_g Profile

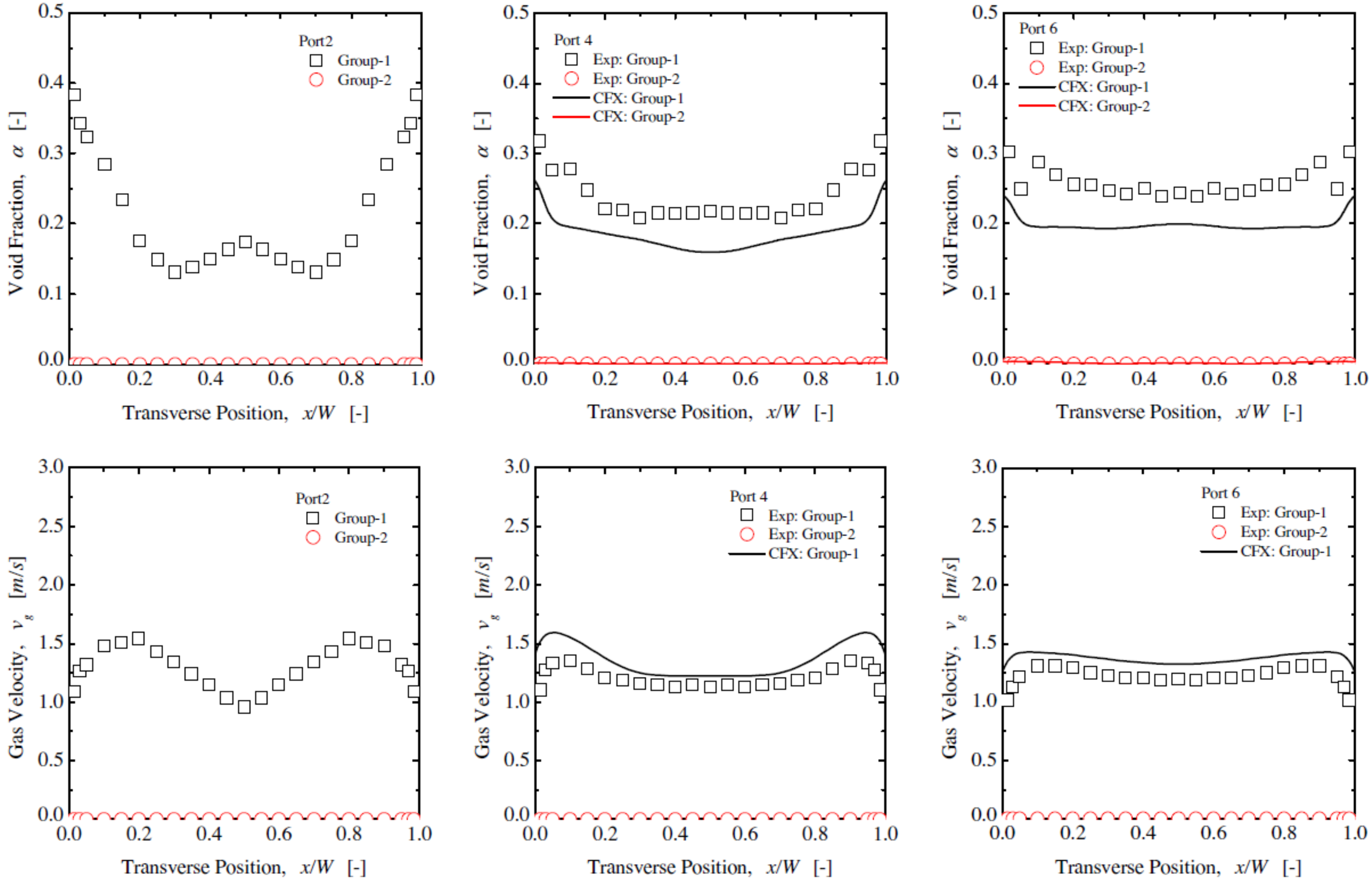


Bubbly Flow - Run 5

$\langle j_{g,0} \rangle$ = 0.289 m/s
$\langle j_f \rangle$ = 0.943 m/s

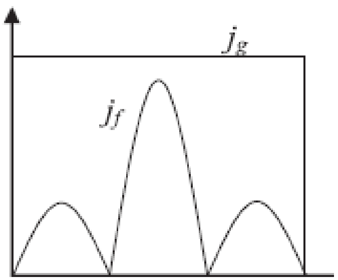


Center j_f Profile

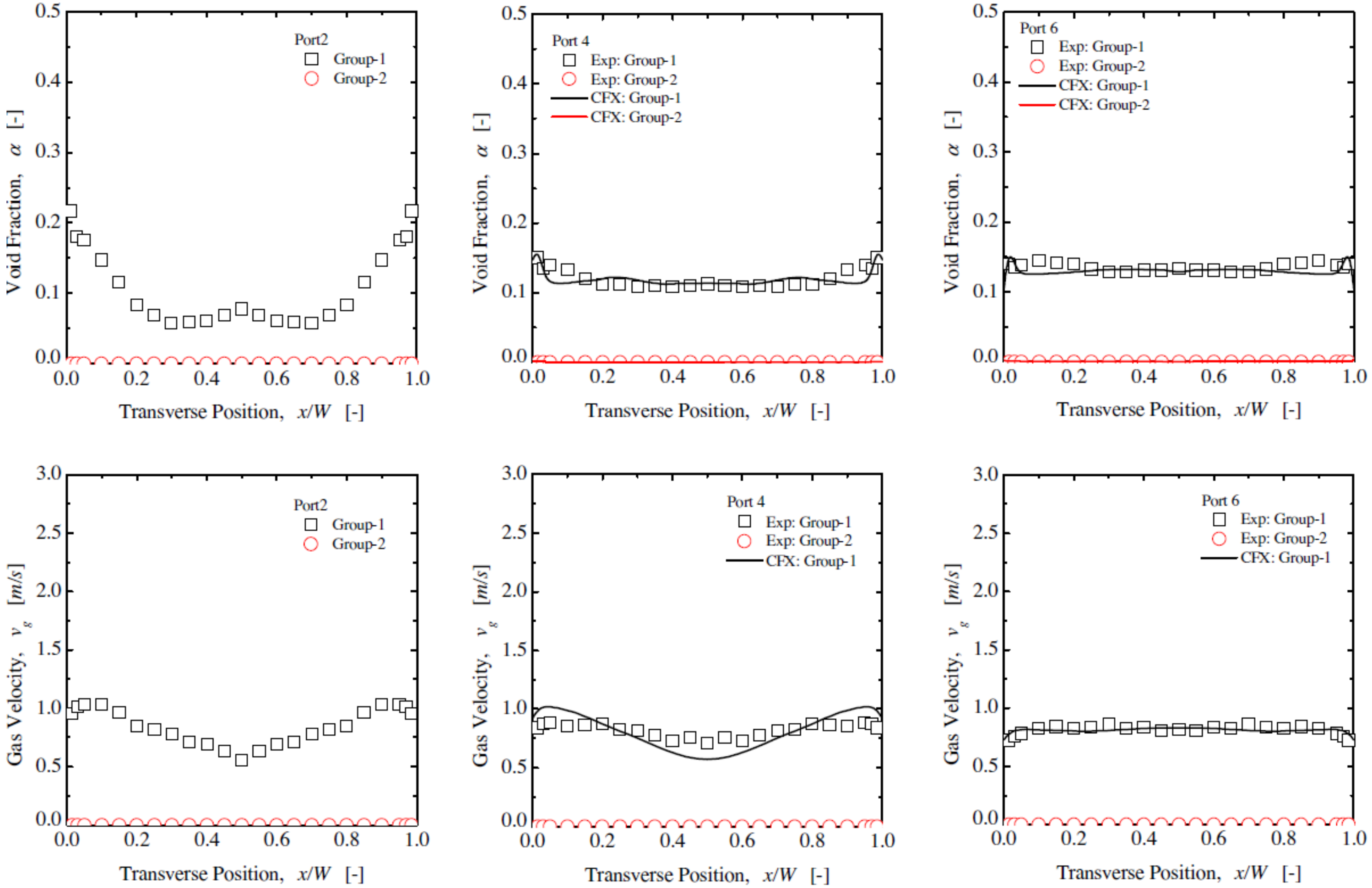


Bubbly Flow - Run 6

$\langle j_{g,0} \rangle = 0.086$ m/s
$\langle j_f \rangle = 0.515$ m/s



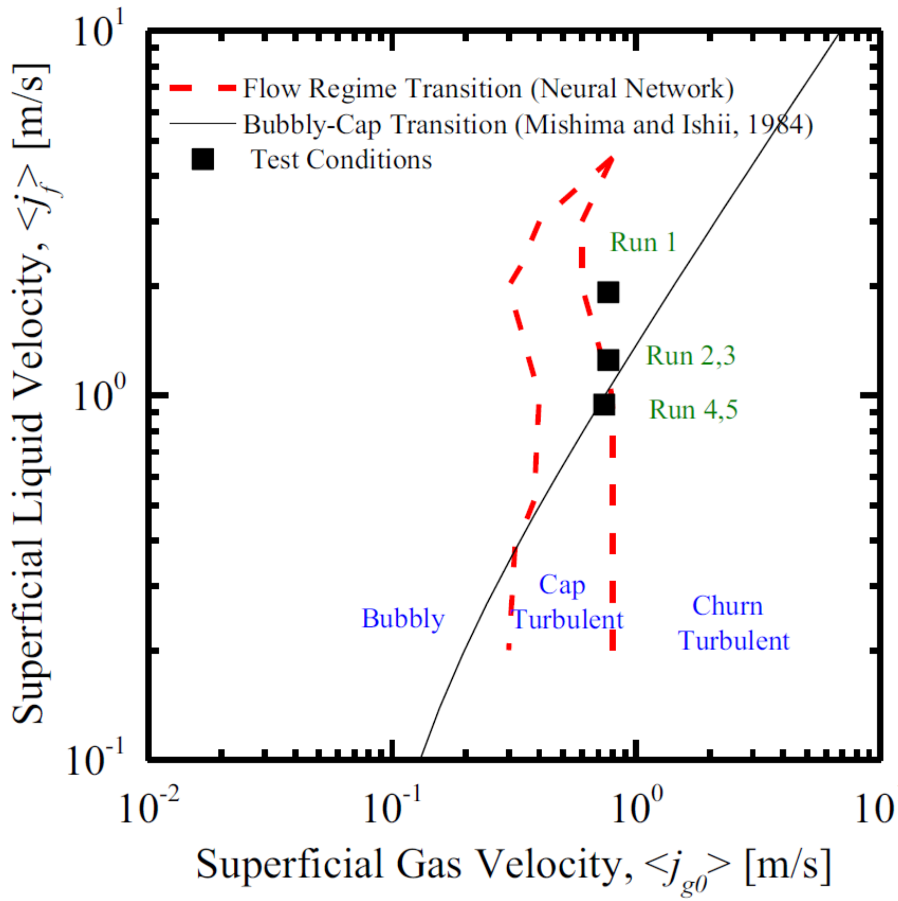
Center j_f Profile



-Hydraodynamics model predicting the trend in measured void fraction
-Good prediction of development of velocity profile by models

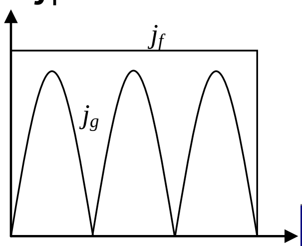
Two-group case

◆ Cap-bubbly, cap-turbulent, churn-Turbulent flows

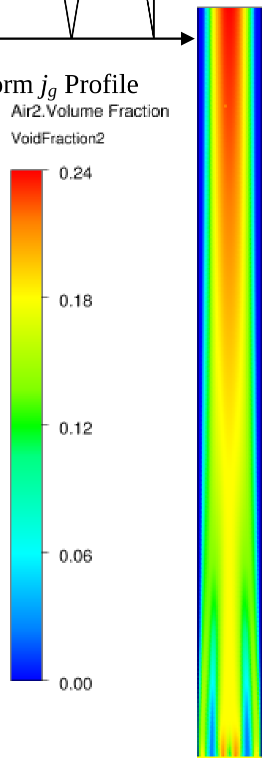


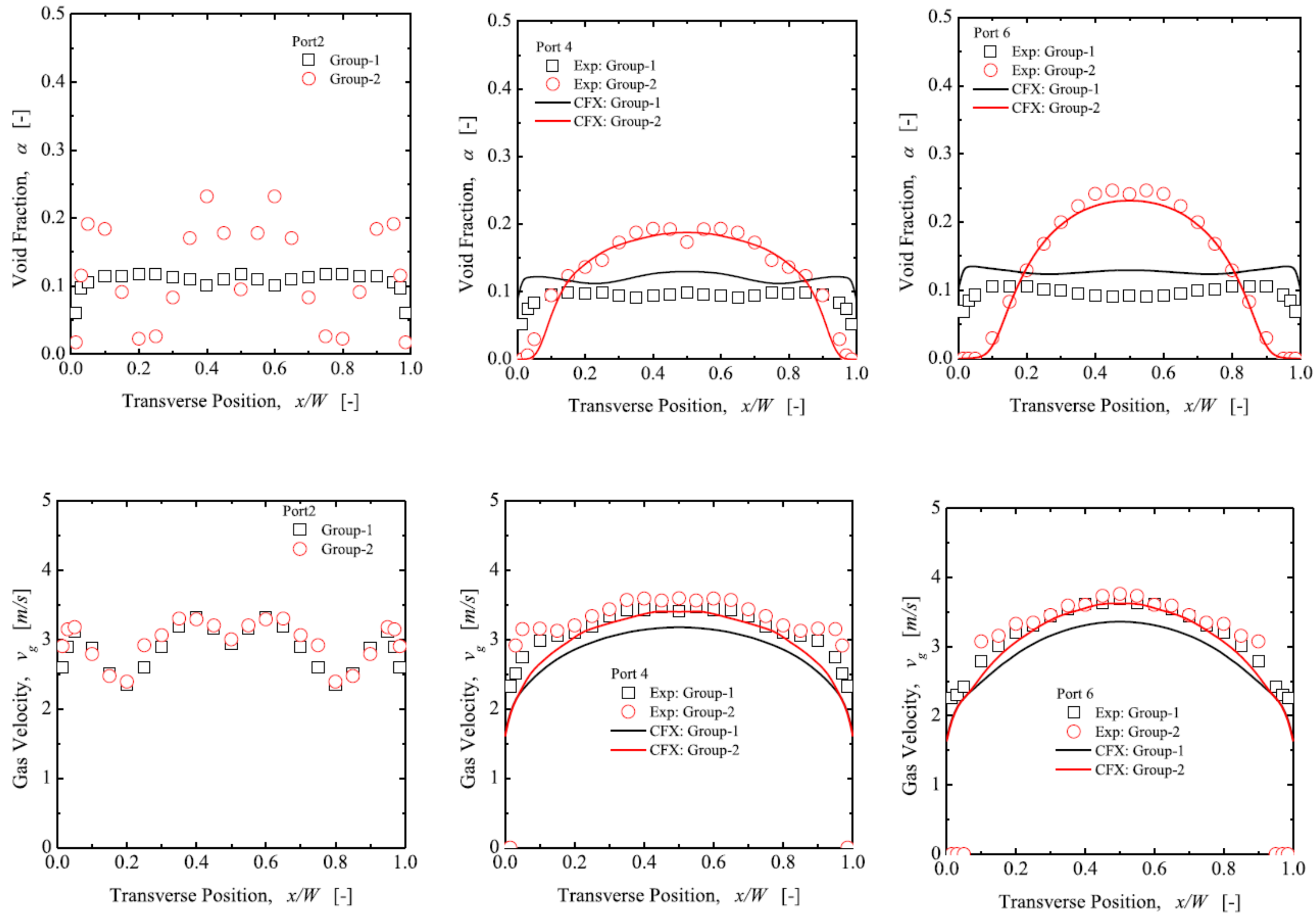
Two-Group cases: Run 1

$\langle j_{g,0} \rangle$=0.77 m/s
$\langle j_f \rangle$=1.92 m/s



Uniform j_g Profile

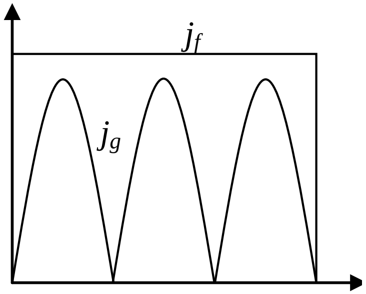




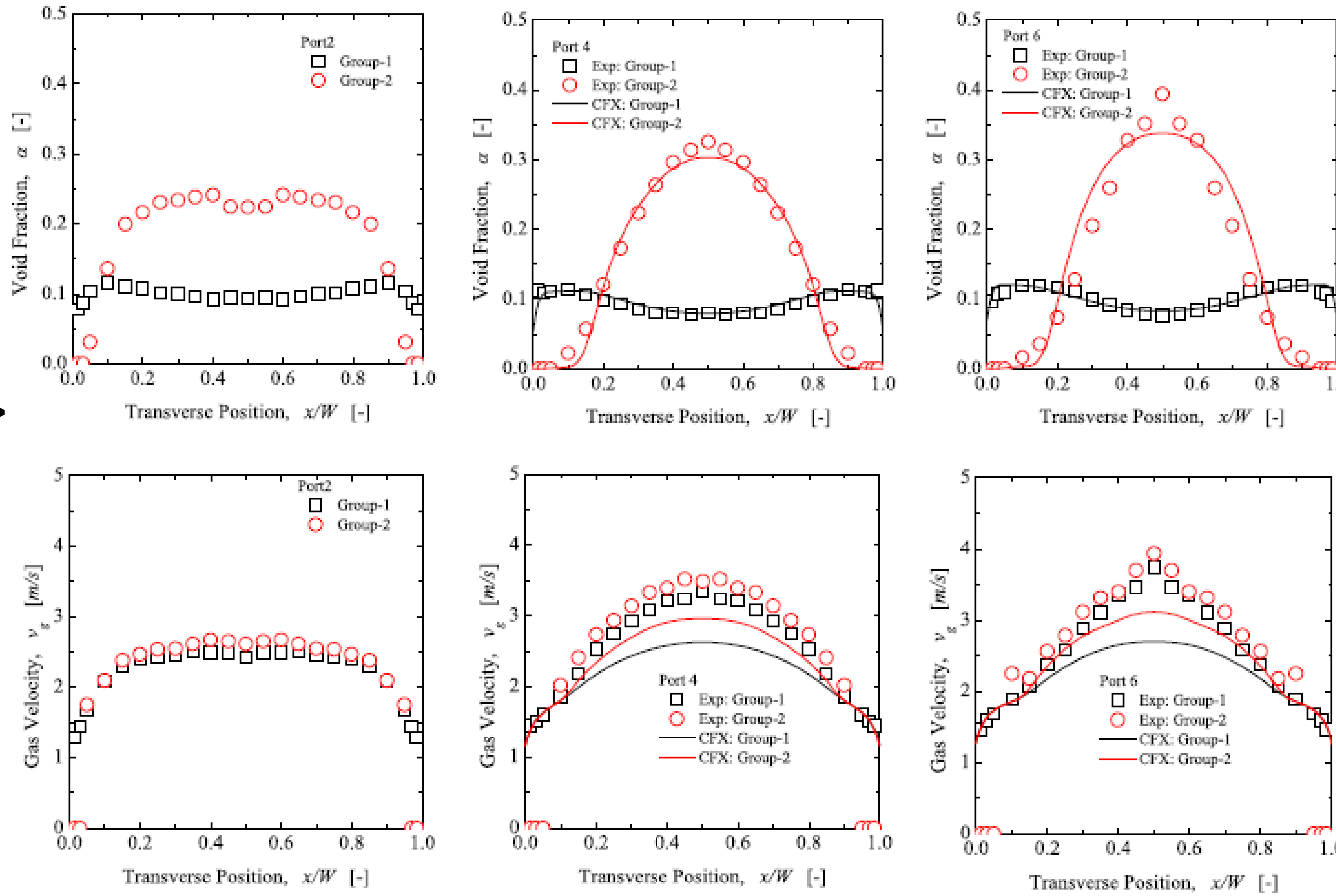
❑ Good prediction for overall trend and magnitude of each bubble group in transverse direction

Two-Group cases: Run 2

$\langle j_{g,0} \rangle = 0.74$ m/s
$\langle j_f \rangle = 1.25$ m/s



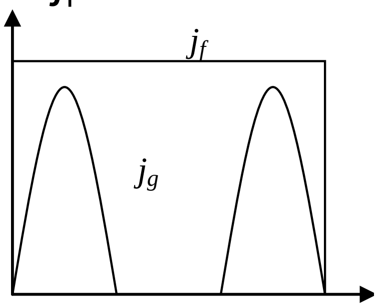
Uniform j_g Profile



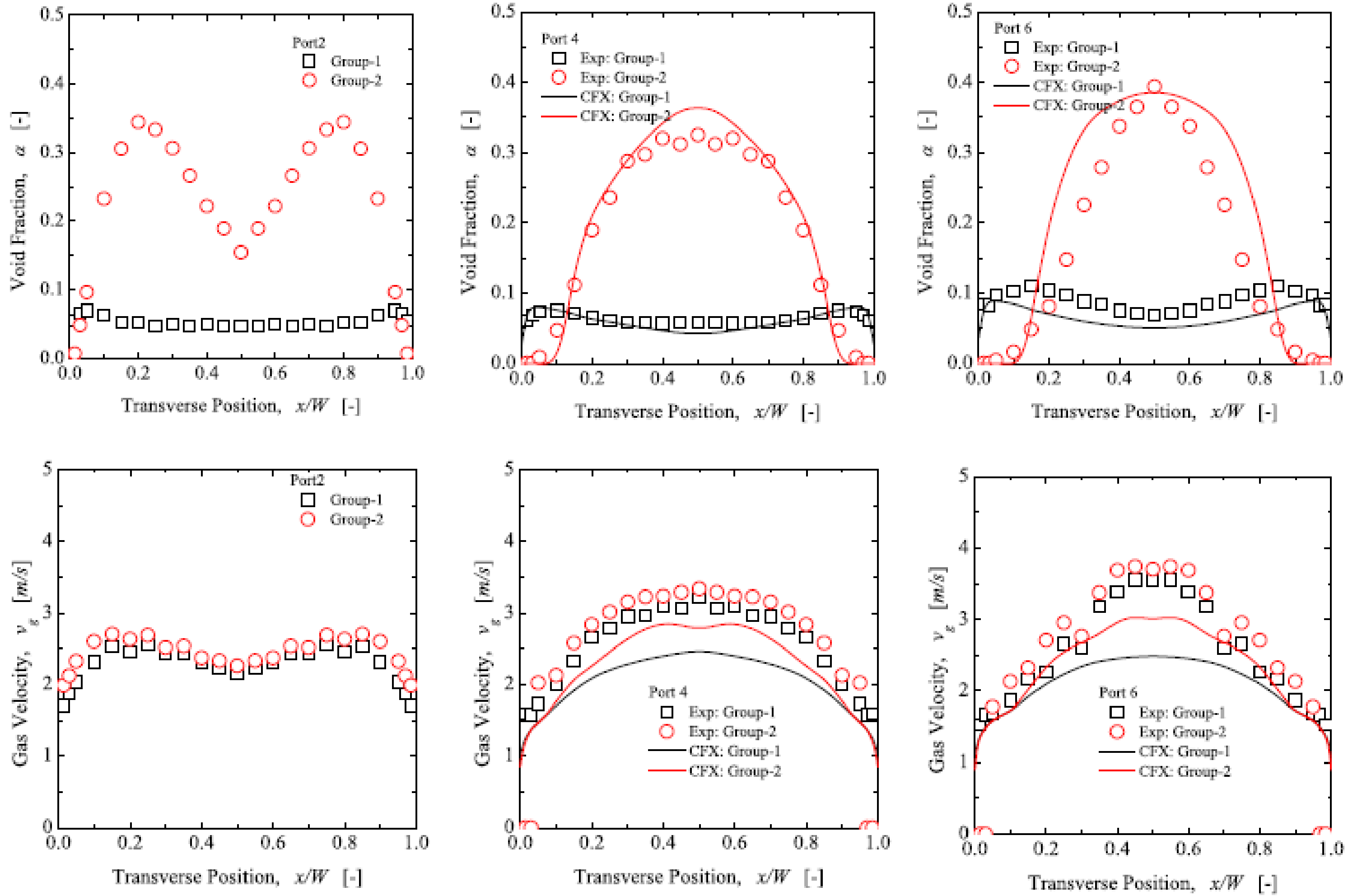
- Good prediction for overall trend of phase distribution in transverse direction.
-

Two-Group cases: Run 3

$\langle j_{g,0} \rangle = 0.77$ m/s
$\langle j_f \rangle = 1.25$ m/s



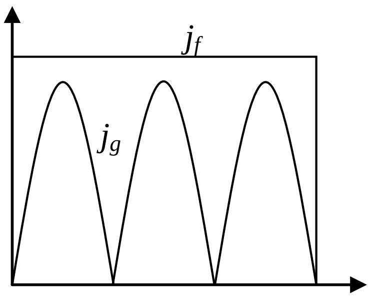
Double Side j_g Profile



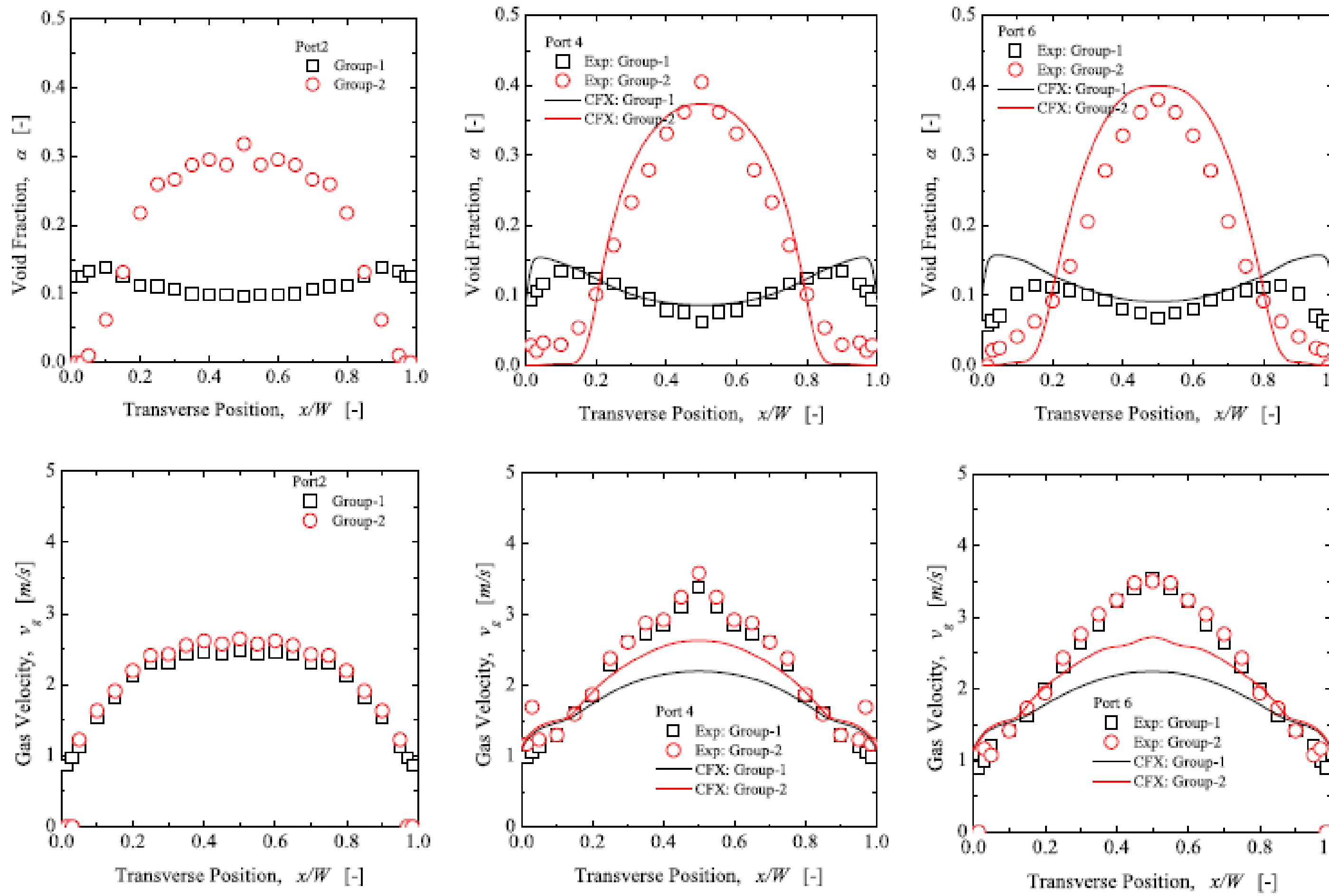
- Very Good prediction for overall trend of phase distribution in transverse direction.
- Underprediction of volume transfer from G-2 to G-1 bubbles

Two-Group cases: Run 4

$\langle j_{g,0} \rangle = 0.74$ m/s
$\langle j_f \rangle = 0.943$ m/s



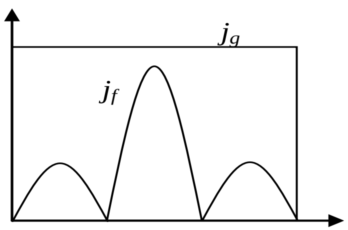
Uniform j_g Profile



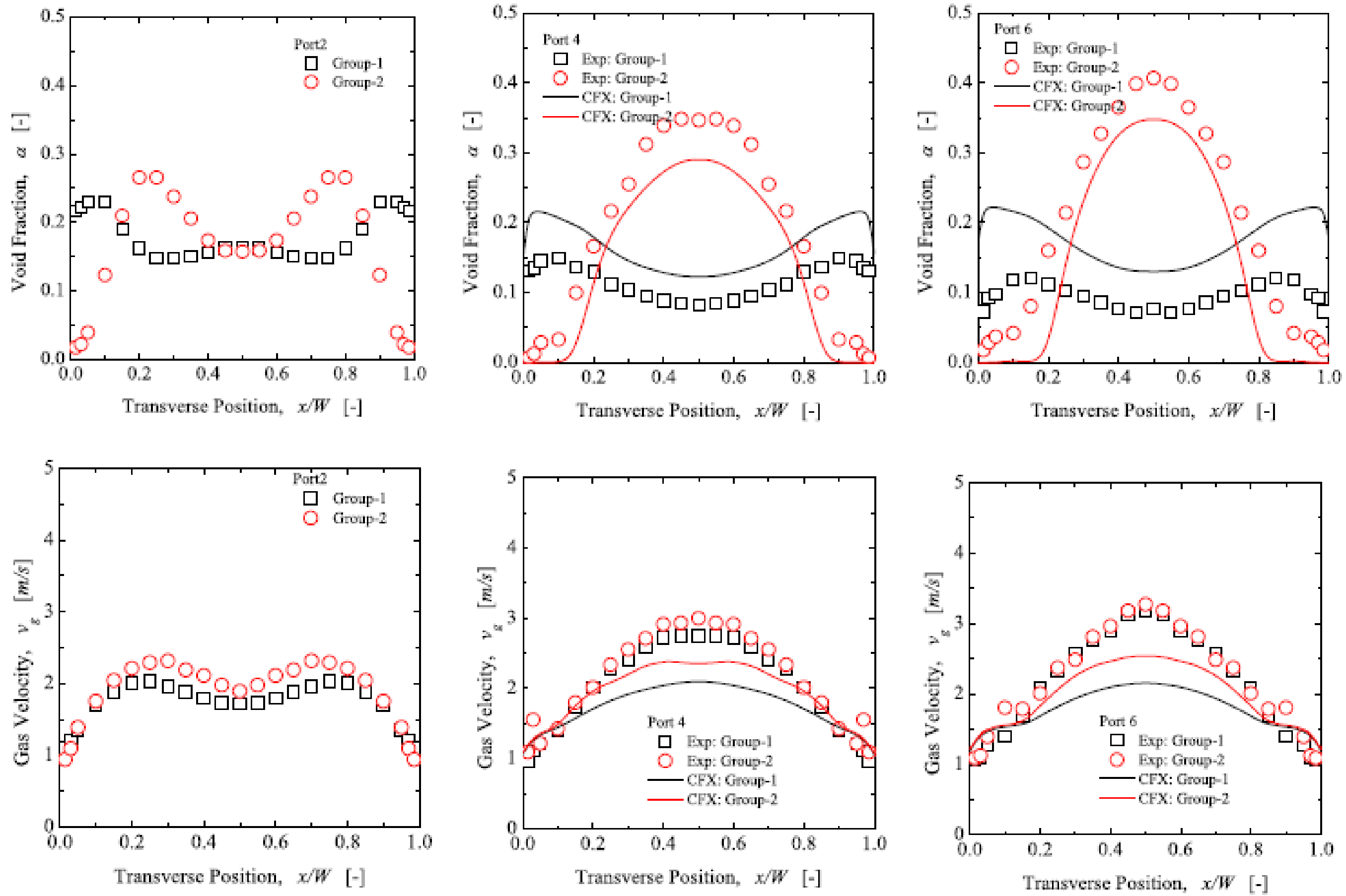
- Very Good prediction for overall trend of phase distribution in transverse direction.
- Underprediction of volume transfer from G-2 to G-1 bubbles

Two-Group cases: Run 5

$<j_{g,0}>$=0.74 m/s
$<j_f>$=0.942m/s

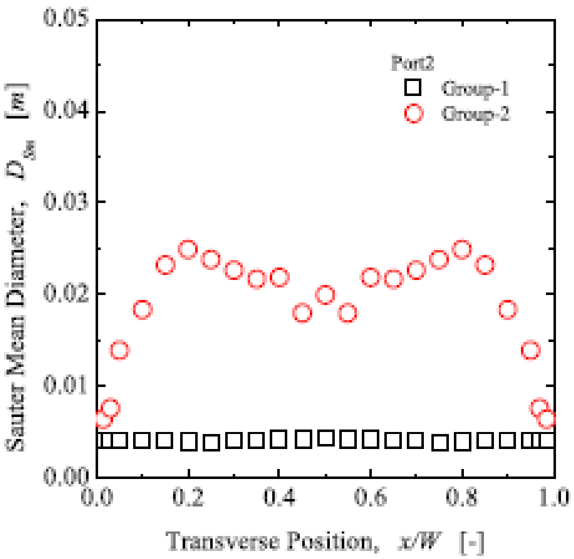


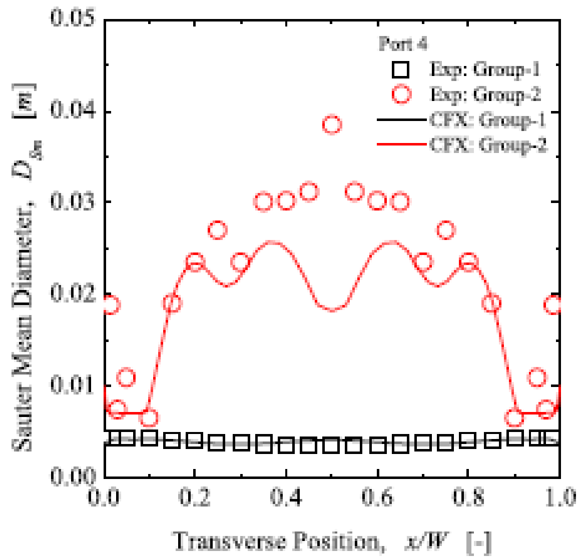
Center j_f Profile

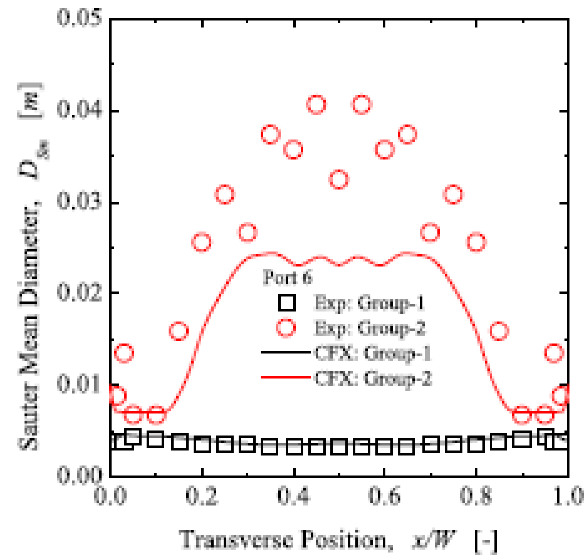


•Very Good prediction for overall trend of phase distribution in transverse direction.
•Underprediction of volume transfer from G-1 to G-2 bubbles by coalescence
•Underprediction of the increase in G-2 bubble sauter mean diameter.

Two-Group cases: Run 5







Summary :

-Hydrodynamics model gives satisfactory prediction in the trend of phase distribution

-Discrepancies were found in IATE model prediction in some cases , specially in intergroup transfer model

- Need to establish a good set of flow transition experimental database for IATE benchmarking
- Need to improve coalescence model

Conclusion

- CFD Framework with three-field two-fluid model and two-group IATE was prepared
- Benchmarking simulations was carried out against uniform and non-uniform inlet test conditions.
- In general, Hydrodynamic models along with IATE showing satisfactory prediction of phase distribution
  - Bubble coalescence model will be further investigated

Thank you!!

Publications

- **Sharma, S. L.**, Ishii, M., Hibiki, T., Schlegel, J., Liu, Y., Buchanan, Jr., J.R., 2019, "Beyond Bubbly Two-Phase Flow Investigation using a CFD Three-Field Two-Fluid Model", International Journal of Multiphase Flow, 113, p.1-15.
- **Sharma, S. L.**, Hibiki, T., Ishii, M., Brooks, C. S., Schlegel, J., Liu, Y., Buchanan, Jr., J.R., 2017, "Turbulence-induced Bubble Collision Force Modeling and Validation in Adiabatic Two-phase Flow using CFD", Nuclear Engineering and Design Journal, 312, p.399-409.