Oil & Gas Reservoir Fluid Injection & Flowback Management

Report by

Ritvik Agrawal

Project Mentors

Dr. Krishnaraj Sambath & Dr. Soumyadeep Ghosh

Academic Mentor

Dr. Ray Mentzer

Davidson School of Chemical Engineering, Purdue University

Summer 2021

ABSTRACT

The objective of the project is to develop a model to simulate the flow of a multiphase multicomponent fluid in an Oil & Gas reservoir. The model is based on the Buckley Leverett Theory, which is the most commonly used approach for estimating the water penetration in a reservoir as a function of time. The results of the model compare a realistic transient flow profile to an idealistic plug flow profile. The results show that the analysis in the Cylindrical system provides a more accurate description of the fluid flow, as compared to a Cartesian system. A Sensitivity Analysis of the model shows that water penetration is heavily influenced by the Pore Volume Injected (PVI). Conversely, the Corey Exponent for oil (n^o) tends to have a minimal effect on the distanced travelled by the fluid. The model can be developed further by introducing the Residual Oil Saturation to increase the accuracy of the results & Mobility Ratio for making the model dimensionless. Furthermore, the model can also be used to study other multiphase multicomponent fluid flows such as Zinc in drilling & completion fluids.

TABLE OF CONTENTS

TABLE OF FIGURES

ACKNOWLEDGEMENT

I would like to thank my project mentors Dr. Krishnaraj Sambath & Dr. Soumyadeep Ghosh for their valuable guidance throughout the duration of the project & ensuring that I thoroughly understand the requisite concepts. I would also like to thank my academic mentor, Dr. Ray Mentzer, for providing me timely access to all the resources that were essential for the execution of this project.

I would also like to thank Dr. Bill Clark for regularly meeting with me in order to ensure that my work progressed as per schedule. I would also like to express my appreciation for Dr. Catherine Fields for helping me hone my technical communication skills.

1. INTRODUCTION

The rate at which the global energy demand is increasing far exceeds the rate at which new reserves are being discovered. The efforts undertaken by the Oil & Gas industry to mitigate this problem can be split into 2 categories: Enhanced Oil Recovery to produce more incremental oil & production from reservoirs that were unviable earlier due to technological constraints. Each of these methods involve multiphase multicomponent fluid flows within the reservoir. The production of crude oil is also constrained by the price of oil, which has been consistently low for the past decade. Therefore, it is essential to understand & predict the behavior of such multiphase fluids in the reservoir, in order to produce the maximum amount of oil at minimum cost. The most prominent instances of multiphase fluid flows in the production process are Drilling & Completion Fluids, Enhanced Oil Recovery techniques, & Waterflooding.

1.1 Drilling & Completions Fluids

Drilling & completions fluids are high-density fluids that are used to contain the reservoir pressure before the well is completed & production can commence [1]. In order to meet the rapidly growing global energy demand, the Oil & Gas industry has been venturing into ultra-deep oil rich reservoirs. These reserves pose a new challenge to the industry, as they require the use of infrastructure that can withstand high pressures & high temperatures. A cost effective solution to meet this challenge is to include the combination of a halide solution of Zinc & Calcium in the completions fluid.

The use of Zinc brings along a set of new challenges. Zinc is recognized as a priority pollutant by the Environmental Protection Agency (EPA) & thus it is essential to design & implement efficient Zinc management & disposal strategies. Additionally, a certain amount of completions fluid seeps through the adjacent rock formation and can potentially pollute the aquifers. Therefore, understanding the flow behavior of multiphase fluids containing Zinc is critical to anticipating the seepage and taking corrective action in a timely manner to avoid a well blowout as well as prevent aquifers from being polluted.

1.2 Enhanced Oil Recovery

Enhanced Oil Recovery (EOR) is another example of the use of multiphase multicomponent fluid flow. EOR is a concept that encompasses several distinct techniques that are used to increase the amount of oil recovered from a reservoir. The techniques can be divided into 3 categories: thermal, chemical, & solvent methods [2]. Depending on the producing life of the reservoir, oil recovery can be categorized into 3 phases: Primary, Secondary & Tertiary. In the primary phase, the natural pressure of the reservoir drives the reservoir fluids towards the production well [3]. There is no need for external fluids to be injected [3]. In the secondary phase, external fluids such as water & gas are injected for pressure maintenance & volumetric sweep efficiency [3]. The secondary phase is followed by the tertiary recovery phase. In this phase, special fluids such as chemicals & miscible gases are injected to increase the total recovery [3]. On average only 30% of the crude oil is recovered from a reservoir, after which the well is plugged and abandoned due to unfavorable economics [3].

In the past, the producing life of a reservoir was relatively shorter than it is today. The constant discovery of new reserves made it less economical to invest in producing incremental oil from an existing field. However, the rate of discovery of new fields has decrease sharply [2]. Yet, the global energy demand is growing. Thus, it is gradually becoming economically favorable to employ develop & employ sophisticated EOR techniques. Therefore, understanding multiphase fluid behavior is essential to manipulating the properties to the injected chemicals to recover the maximum amount of crude oil possible.

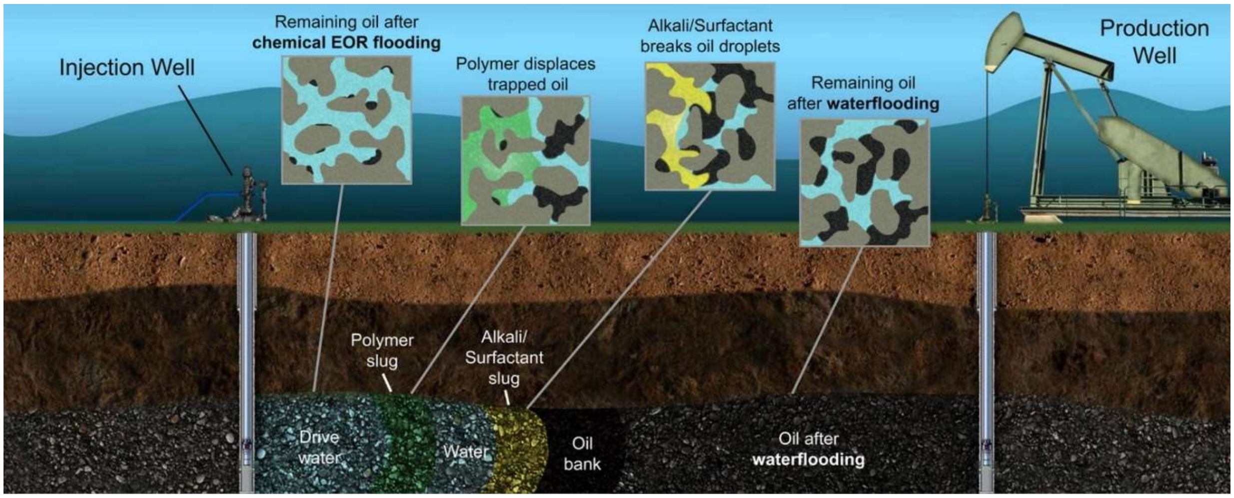

Figure 1. Tertiary Oil Recovery [4].

Figure 1 shows the tertiary phase of oil recovery after waterflooding. After waterflooding, the pores still contain a certain amount of recoverable oil that is immovable after waterflooding. Thus, special chemicals such as polymers, alkali & surfactants are injected to dislodge the trapped oil. Alkali & surfactant injections break the interfacial tension between the oil & displacing fluid [3]. Polymers can influence the mobility ratio by increasing the viscosity of the displacing fluid [3]. A favorable mobility ratio improves the sweep efficiency [3].

1.3 Waterflooding

Water injection into a reservoir is a type of multiphase fluid flow that is crucial to the production of crude oil. Initially, the pressure in the reservoir is enough to drive the oil towards the production well. However, the reservoir pressure gradually decreases as oil is produced from the well. Eventually, the reduced reservoir pressure is not enough to drive the flow of oil. In order to maintain production, a separate water injection well is drilled at a certain distance away from the production well. As water is injected into the reservoir, the reservoir pressure increases. Additionally, water pushes the oil in place towards the production well. Water injection is a cost intensive process. Therefore, it is essential to understand the flow behavior of water in a reservoir to estimate the incremental increase in crude oil production. Figure 2, illustrates the different types of flow profiles for water injection in a reservoir. In a real world scenario, the displacement profile of water is between that of a plug flow & a fully mixed flow.

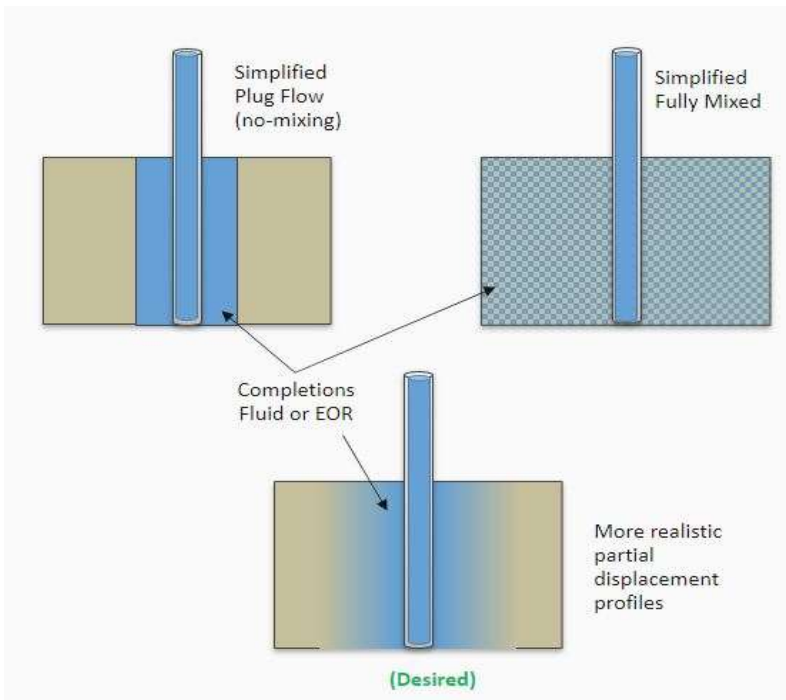

Figure 2. Types of Fluid Displacement Profiles [5].

The Buckley Leverett Theory is the most widely used approach to model the permeation of water in a reservoir. However, this approach is limited to the permeation of water in a reservoir containing oil. The purpose of the project is to expand the Buckley Leverett Equation to the Cylindrical co-ordinate system. Furthermore, the model can be used to simulate the flow of contaminants such as Zinc in the reservoir, as a function of time.

2. Model Development

The model is based on the Buckley Leverett Theory for fluid flow through a porous media. It is the simplest & most widely used approach for estimating the advance of a fluid displacement front in an immiscible displacement [6]. This method was first proposed in 1942. It is used to estimate the rate at which an injected water bank moves in a porous medium [2]. The Buckley-Leverett approach is based on the fractional flow theory and makes the following assumptions [6]:

- Water is injected into a reservoir containing only oil.
- Oil & water are immiscible.
- The viscosities of oil & water remain constant.
- The effects of capillary pressure & gravity are neglected.
- The flow is linear & horizontal.

The derivation for the Buckley Leverett equation begins by defining the volumetric flowrates of oil & water & obtaining the fractional flow equation. The total volumetric flowrate of the multiphase fluid, q is defined as the sum of the volumetric flowrate of water & oil, as shown in eqn. (1). The volumetric flowrates of water, q_w, & oil, q_o, can be defined using the Darcy's Equation for Flow through a Porous Media, as shown in eqn. (2) & (3) respectively [7]. In the following equations, $\frac{dp}{dx}$ represents the pressure gradient.

$$q = q_w + q_o \quad (1)$$

$$q_w = \frac{k_{rw} kA}{\mu_w} \frac{dp}{dx} \quad (2)$$

$$q_o = \frac{k_{ro} kA}{\mu_o} \frac{dp}{dx} \quad (3)$$

The fractional flow of water, f_w, is defined as the fraction of water in the total volumetric flowrate of the fluid. The fraction flow of water is a function of water saturation. Eqn. (4) defines the fractional flow of water in terms of the fraction of water flowing in the fluid [6]. The definition can be expanded by substituting the relationships for the volumetric flow of water & oil as defined in eqn. (2) & (3) respectively. Eqn. (5) shows a simple definition for the fraction flow of water [7].

$$f_w = \frac{q_w}{q_w + q_o} = \frac{\frac{k_{rw} kA}{\mu_w}\frac{dp}{dx}}{\frac{k_{rw} kA}{\mu_w}\frac{dp}{dx} + \frac{k_{ro} kA}{\mu_o}\frac{dp}{dx}} \quad (4)$$

$$f_w = \frac{\frac{k_{rw}}{\mu_w}}{\frac{k_{rw}}{\mu_w} + \frac{k_{ro}}{\mu_o}} = \frac{1}{1 + \frac{k_{ro}\mu_w}{k_{rw}\mu_o}} \quad (5)$$

Eqn. (5) shows that the fractional flow of water depends on the viscosities (μ_w & μ_o) & relative permeability (k_{rw} & k_{ro}) of water & oil. The relative permeability can be calculated using the Corey Model, as shown in eqn. (6) [8]. The Endpoint Relative Permeability, k^0_{rj}, is the permeability of one phase at the residual saturation of the other phase [2]. The saturation of the oil or water in the fluid is represented by S_j. The Corey Exponent, n_j, is a constant that is experimentally determined for each type of system.

$$k_{rj} = k^o{}_{rj} S_j^{n_j}, \qquad j = water\ or\ oil \quad (6)$$

The permeation of injected water in a reservoir can be analyzed in the Cartesian as well as the Cylindrical system. In the real world application, the water is injected through a cylindrical well and it permeates radially from the well bore. However, the system can also be visualized as vertical slices being cut into the sides of the well. The water permeating through each one of the slices can be considered as a separate Cartesian system. The model is designed to analyze the flow of water in both the systems.

2.1 Cartesian Co-ordinates

The Buckley Leverett equation can be derived using the mass balance for the flow of water across a control volume. For this iteration of the model, it assumed that the well bore has 3 slits. The arrangement of the slits are shown in Figure 3 [9].

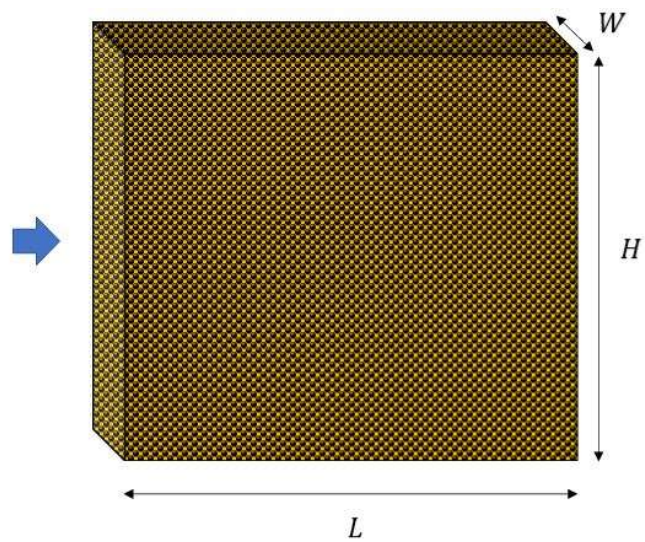

Figure 3. Flow Through a Slit in a Wellbore [9].

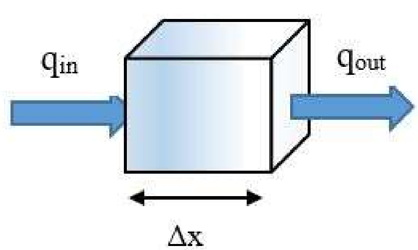

Figure 4. Water Flow Through a Control Volume in Cartesian System.

Figure 4, shows the flow of water through a control volume of length Δx. The mass balance of the water flow across the control volume is shown in eqn. (7) [6]. The equation shows that the water accumulated in the control volume (R.H.S) is equal to the mass flowrate of water entering – the mass flowrate of water leaving the element (L.H.S) [6]. The density of water is represented by ρ_w, & the porosity of the reservoir is shown as $\emptyset$.

$$[(q_w\rho_w)_x - (q_w\rho_w)_{x+\Delta x}]\Delta t = A\emptyset\Delta x[(S_w\rho_w)_{t+\Delta t} - (S_w\rho_w)_t] \quad (7)$$

$$As\ \Delta t \to 0\ \&\ \Delta x \to 0,$$

$$-\frac{\partial(q_w\rho_w)}{\partial x} = A\emptyset\frac{\partial S_w}{\partial t} \quad (8)$$

$$Assuming\ incompressiblity, \rho_w = constant\ \&\ q_w = f_w q$$

$$-\frac{\partial f_w}{\partial x} = \frac{A\emptyset}{q}\frac{\partial S_w}{\partial t} \quad (9)$$

$$u\frac{\partial f_w}{\partial x} + \emptyset\frac{\partial S_w}{\partial t} = 0\,, where\ u = \frac{q}{A} \quad (10)$$

The Buckley Leverett equation is shown in eqn. (10)[2]. Here u is the Darcy velocity of the saturation plane. This equation is incorporated into the model in its dimensionless form, as shown below [10].

$$Taking\ the\ Dimensionless\ form\ of\ the\ equation,$$

$$x_D = \frac{x}{L} \quad (11)$$

$$\frac{\partial f_w}{\partial x_D}\frac{\partial x_D}{\partial x} \to \frac{\partial f_w}{\partial x_D}\frac{1}{L} \quad (12)$$

$$t_D = \frac{qt}{A\emptyset L} \quad (13)$$

$$\frac{\partial S_w}{\partial t_D}\frac{\partial t_D}{\partial t} \rightarrow \frac{\partial S_w}{\partial t_D}\frac{u}{\emptyset L} \quad (14)$$

$$Substituting\ eqn. (12) \&\ eqn. (14) in\ eqn. (10),$$

$$Subsituting\ t\ for\ t_D \&\ x\ for\ x_D,$$

$$\frac{\partial S_w}{\partial t} + \frac{\partial f_w}{\partial x} = 0 \quad (15)$$

Eqn. (15) shows the dimensionless form of the Buckley Leverett equation [10]. The numerical solution of the equation is shown in Appendix A.1.

2.2 Cylindrical System

Similar to the Cartesian system, the mass balance for the flow of water in a cylindrical system is shown below. Figure 5 illustrates the flow of water through a cylindrical control volume. In the figure, r_e is the radius of the wellbore. The mass balance is shown in eqn. (16) [11].

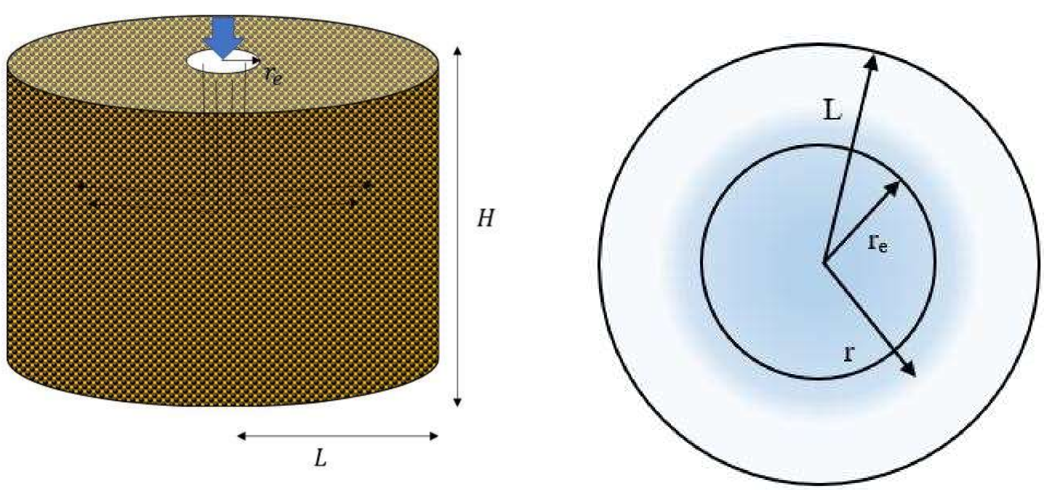

Figure 5. Water Flow Through a Control Volume in Cylindrical System [9].

$$[(q_w\rho_w)_r - (q_w\rho_w)_{r+\Delta r}]\Delta t = 2\pi rh\emptyset[(S_w\rho_w)_{t+\Delta t} - (S_w\rho_w)_t]\Delta r \quad (16)$$

$$As\ \Delta t \rightarrow 0\ \&\ \Delta r \rightarrow 0,$$

$$-\frac{\partial q_w \rho_w}{\partial r} = 2\pi r h \emptyset \frac{\partial S_w \rho_w}{\partial t} \quad (17)$$

$$Assuming\ incompressiblity, \rho_w = constant\ \&\ q_w = f_w q$$

$$-\frac{\partial f_w q}{\partial r} = 2\pi r h \emptyset \frac{\partial S_w}{\partial t} \quad (18)$$

$$-\frac{\partial f_w}{\partial r} = \frac{2\pi r h \emptyset}{q} \frac{\partial S_w}{\partial t} \quad (19)$$

$$u \frac{\partial f_w}{\partial r} \frac{1}{r} + \emptyset \frac{\partial S_w}{\partial t} = 0\ , where\ u = \frac{q}{2\pi h} \quad (20)$$

The Buckley Leverett equation for the cylindrical system is shown in eqn. (20) [11]. The equation can be reduced to its dimensionless form as follows.

$$r_D = \frac{r}{L} \quad (21)$$

$$\frac{\partial f_w}{\partial r_D} \frac{\partial r_D}{\partial r} \rightarrow \frac{\partial f_w}{\partial r_D} \frac{1}{L} \quad (22)$$

$$t_D = \frac{qt}{2\pi h \emptyset L} \quad (23)$$

$$\frac{\partial S_w}{\partial t_D} \frac{\partial t_D}{\partial t} \rightarrow \frac{\partial S_w}{\partial t_D} \frac{u}{\emptyset L} \quad (24)$$

$$Substituting\ eqn. (22)\&\ eqn. (24) in\ eqn. (20),$$

$$Subsituting\ t\ for\ t_D \&\ r\ for\ r_D,$$

$$\frac{\partial S_w}{\partial t} + \frac{\partial f_w}{\partial r} = 0 \quad (25)$$

Eqn. (25) shows the dimensionless form of the Buckley Leverett equation.

3. RESULTS

The model was used to simulate the flow profile of a multiphase fluid in a reservoir containing oil. In this iteration of the model, 1,000 barrels of water was injected into the reservoir. The results obtained from the model show the penetration of the injected water in the reservoir from the time of injection, up to 2 hours & 24 minutes later. The period for observation was chosen as such since in case of a plug flow, it would take 1,000 barrels of injected water 144 minutes or 2 hours & 24 minutes to completely displace an equal volume of oil in the reservoir. The resultant plots compare an idealistic plug flow with a more realistic transient flow. The input values for the relevant reservoir & fluid properties are shown in Table 1.

Parameter	Symbol	Value
End point relative permeability of water	Kr^0_w	0.6
End point relative permeability of oil	Kr^0_o	0.4
Corey exponent of water	n_w	2
Corey exponent of oil	n_o	2
Viscosity of Water	μ_w	1 cP
Viscosity of Oil	μ_o	5 cP
Volume of water injected	V	1,000 bbl.
Flowrate of water	q	10,000 bbl. /d
Height of the reservoir	h	1 m
Wellbore radius	re	0.05 m
Porosity	Ø	0.33

Table 1. Parameters used in the model.

Figure 6, shows the Relative Permeability curves for an oil & water system as a function of the water saturation in the reservoir, S_w. The relative permebilities are used to calculate the fractional flow of water, F_w as a function of S_w. Figure 7, shows that the fractional flow curve of water in an oil-water system. The fraction of water in the flowing multiphase fluid exponentially increases as the saturation of water in the reservoir increases beyond ~ 7%. However, the slope of the curve gradually decreases as S_w increases beyond 40%.

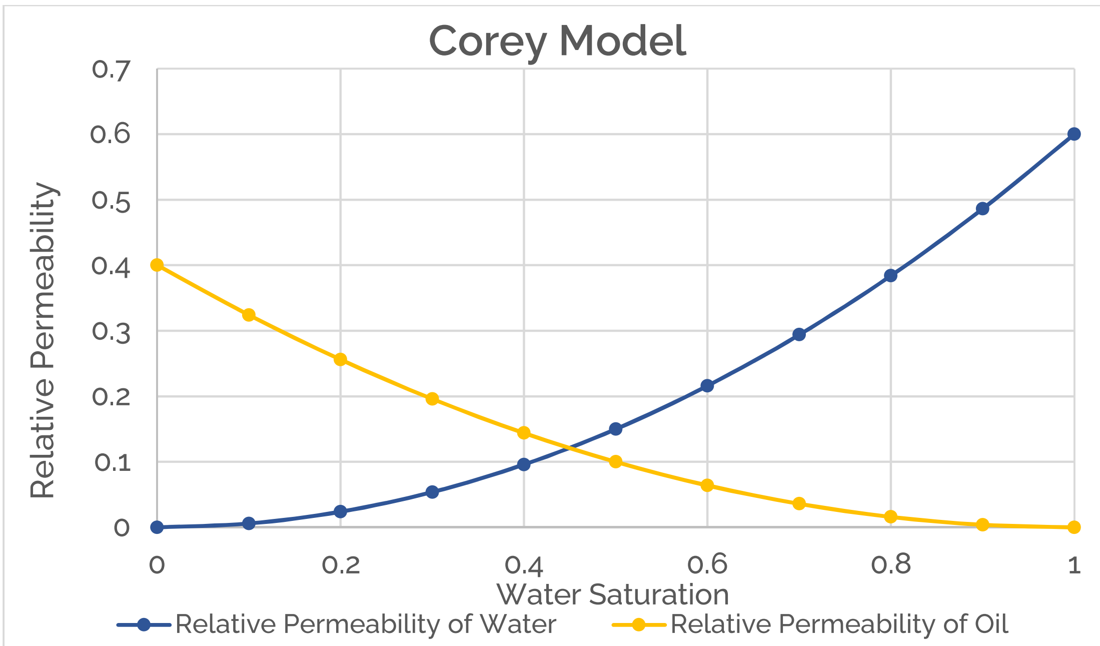

Figure 6. Relative Permeability of water & oil w.r.t S_w.

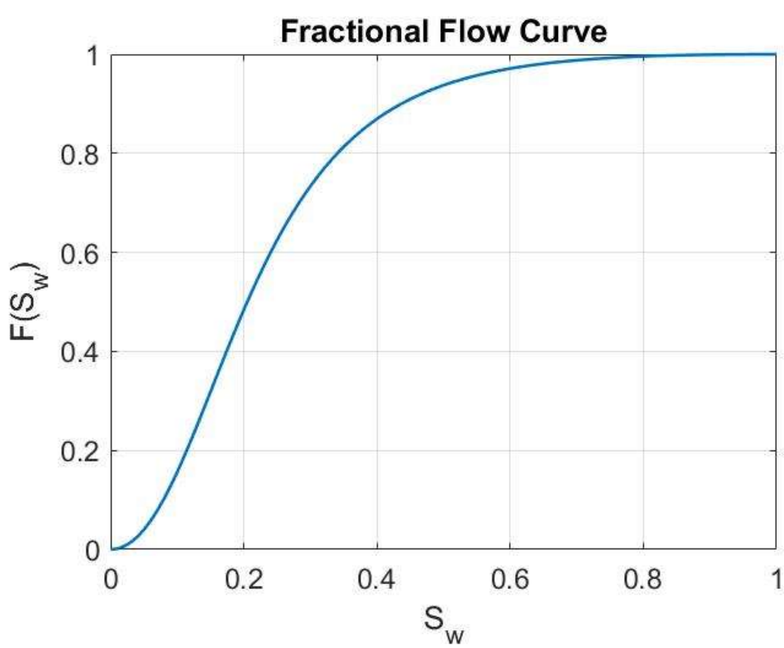

Figure 7. Fractional Flow Curve for Oil-Water system.

Figures 8 & 9 show the Flow Profile Plots for the Cartesian & Cylindrical co-ordinate systems. The vertical axis shows the Volume Fraction or Water Saturation, S_w. The horizontal axis shows the length of the reservoir penetrated by the injected water. The black lines show the distribution of water saturation & the water penetration in the reservoir, at different time intervals over a period of 2 hours & 24 minutes. The red line shows the extent of the reservoir penetrated by the water at the final time interval. The plots also show a comparison of the transient flow to a plug flow. The green line shows the distance travelled by the plug flow line once the injected water displaces an equal volume of oil.

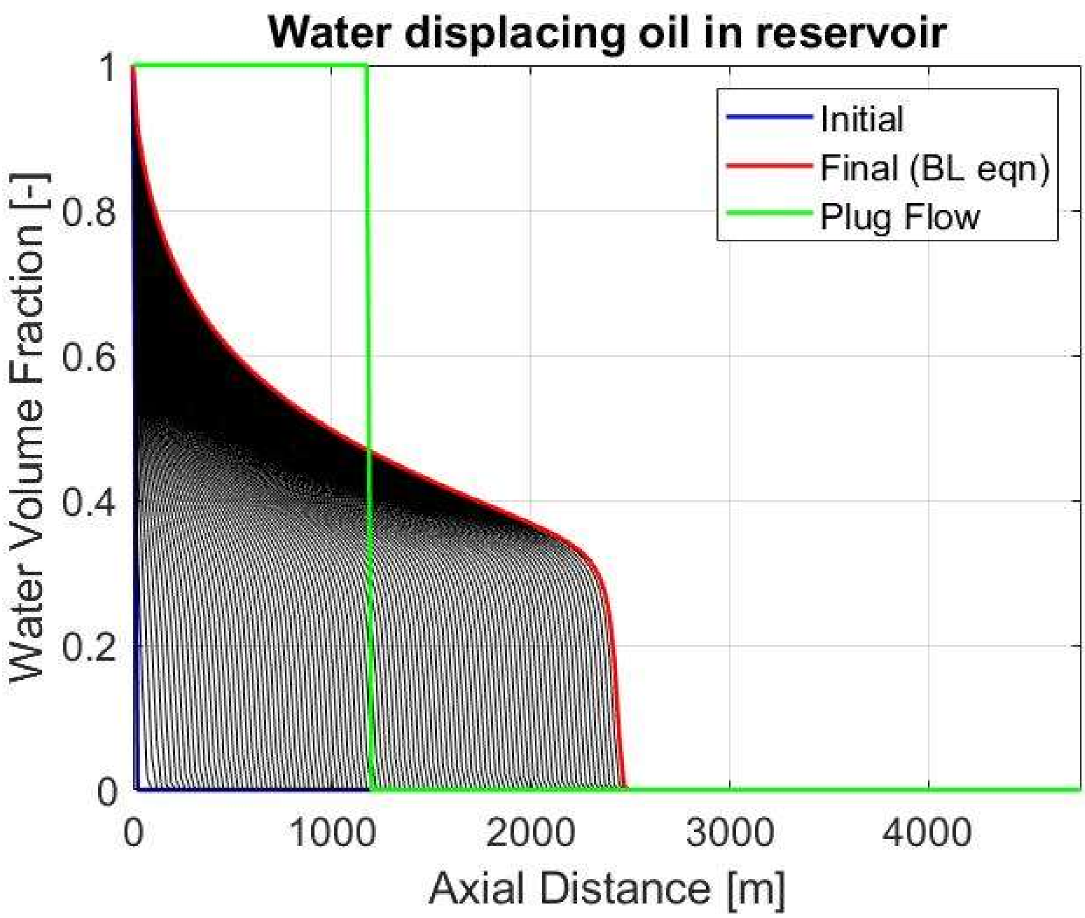

Figure 8. Flow Profile for Cartesian system.

Figure 8 shows the flow profile for the injected water in a Cartesian system. After 2 hours & 24 minutes, in case of a plug flow the water penetrates a length of ~ 1,200 meters. In the case of a transient flow, the water penetrates a distance of ~ 2,470 meters. Therefore, the water travels slightly more than double the distance covered by the plug flow. The lines showing the change in flow profile as a function of time are evenly spaced, indicating that the speed of the flow remains constant.

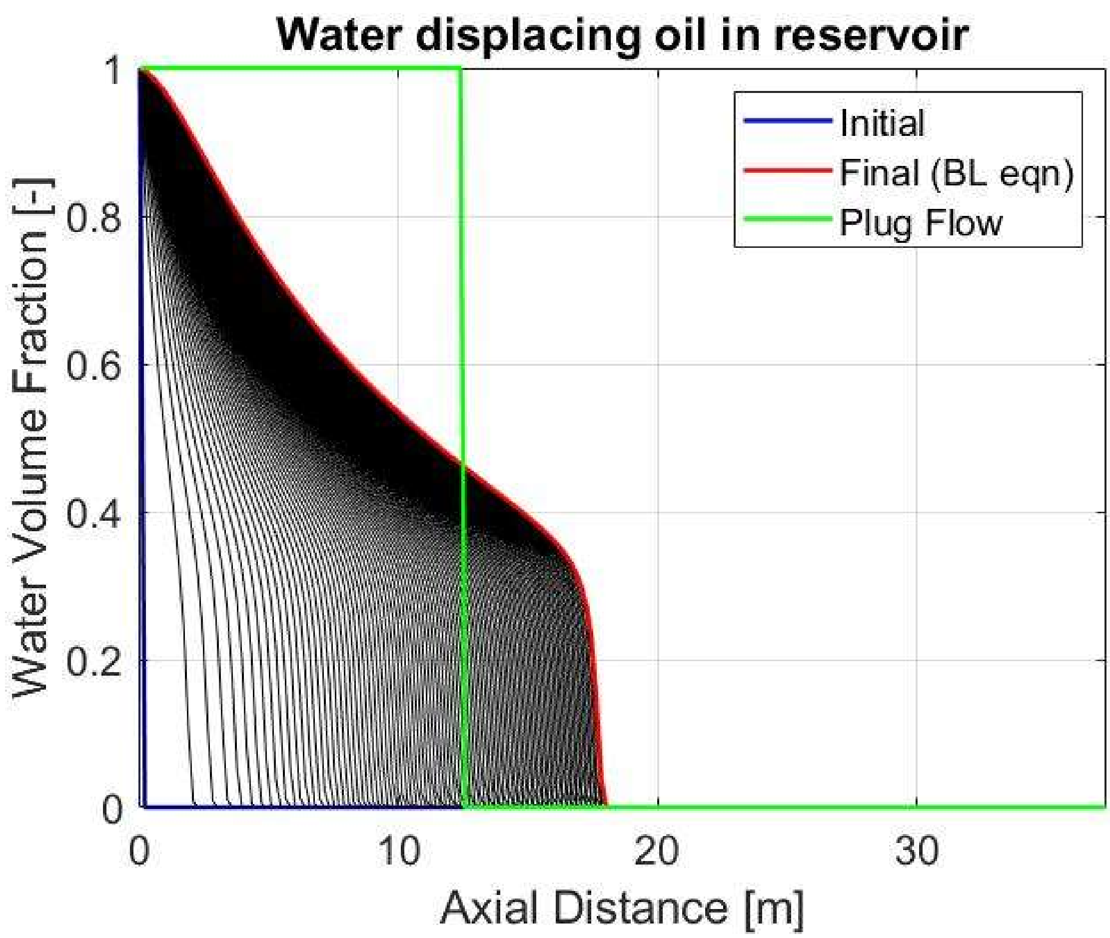

Figure 9. Flow Profile for Cylindrical System.

The flow profile for the injected water in the cylindrical system is shown in Figure 9. From the figure, it can be seen that in a plug flow, the water penetrates a distance of 12.6 meters. In the transient flow profile, after 2 hours & 24 minutes the water reaches a distance of 18.02 meters. Thus, the water penetration in the transient profile exceeds the plug flow by approximately 30%. This difference is significantly less than the Cartesian system, where it exceeds the plug flow by 106%. In the case of a radial flow, the lines showing the change in flow profile as a function of time are unevenly spaced. The space between those lines decreases as the water penetrates deeper into the reservoir. This indicates that the speed of flow for water decreases as time increases.

A sensitivity analysis was performed to test the robustness of the model. It was also performed to study the effect of changes in the reservoir & fluid properties. A range of input values for key reservoir & fluid properties were used to observe the change in water penetration. The results are shown in the form of a Tornado plot to rank the effect of the properties. The properties that were used in the analysis & their corresponding values are shown in Table 2 below.

Parameter	**Symbol**	**Low**	**Mid**	**High**
Water Corey Exponent	n_w	1.5	2	4
Oil Corey Exponent	n_o	1.5	2	4
Viscosity Ratio	μ_w / μ_o	0.1	0.2	1
Endpoint Relative Permeability of Water	kr0w	0.2	0.6	1
Endpoint Relative Permeability of Oil	kr0o	0.2	0.4	1
Pore Volume Injected (Cartesian)	PVI	0.37	0.75	1.50
Pore Volume Injected (Cylindrical)	PVI	64.58	129.16	258.32

Table 2. Parameters for Sensitivity Analysis

The term Pore Volume Injected (PVI) is introduced in order to make the analysis dimensionless. Pore Volume Injected is defined as the ratio of the cumulative volume of water injected (V) to the Pore Volume (V_P) [12]. The Pore Volume Injected accounts for any changes in the length, width, height, & porosity of the reservoir. Additionally it also accounts for any changes in the volume of water injected, as in this case. The volume of water injected was varied between 500 bbl. /day to 2000 bbl. /day for this study. The viscosity ratio is another dimensionless term used in the analysis. Since the study is concerned with the production of light oil, the viscosities of oil are varied from 1 cP to 10 cP.

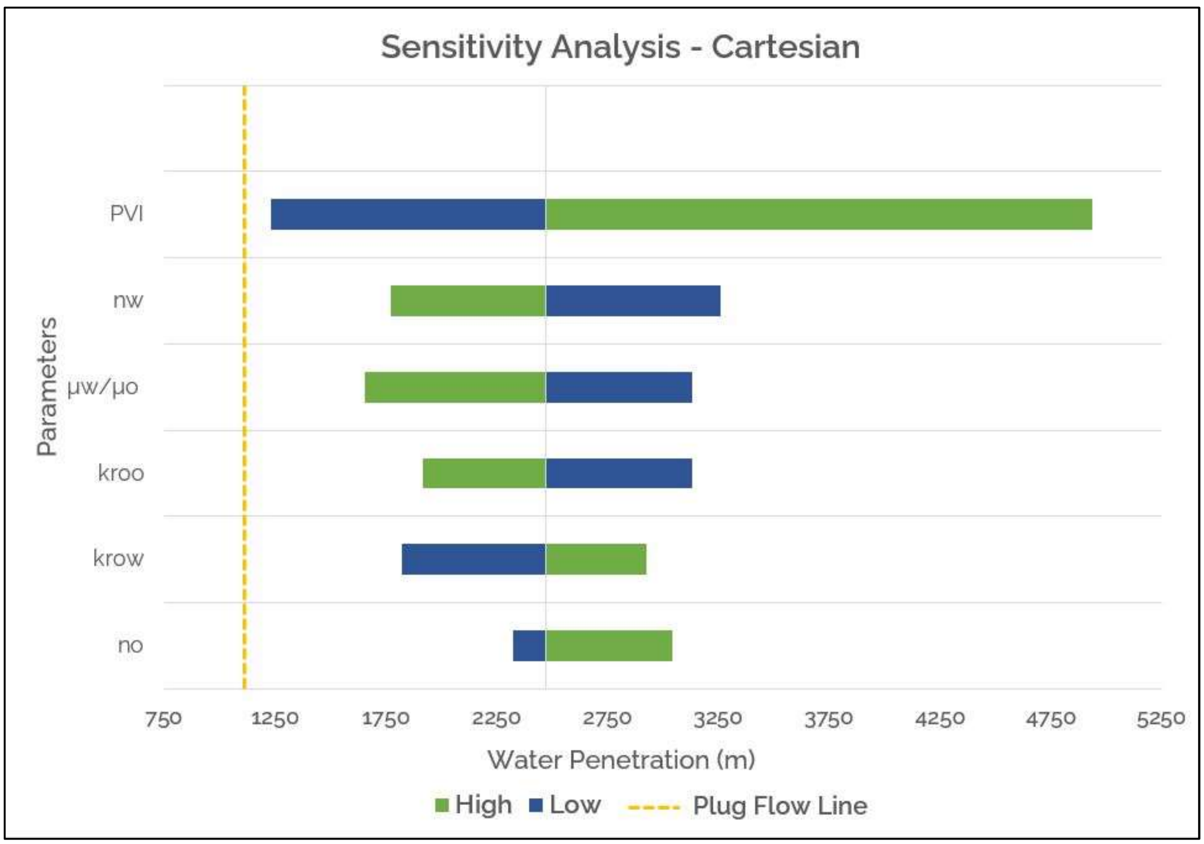

Figure 10. Sensitivity Analysis for Cartesian System.

Figure 10 shows the results of the sensitivity analysis study for the Cartesian system. The results show that the Pore Volume Injected has the most effect on the water penetration in the reservoir.

Conversely, the Corey exponent for oil has the least influence in a Cartesian system. The Pore Volume Injected, Endpoint Relative Permeability of water & Corey exponent for oil have a directly proportional relationship with the water penetration. However, the viscosity ratio, Corey exponent for water, & Endpoint Relative Permeability of oil have an inverse relationship with the water penetration.

The results of the sensitivity analysis study for the Cylindrical system can be seen in Figure 11. The results of the both the systems are similar as PVI has the most & n^o has the least influence on the water penetration. In addition, the results show a difference in the rank of the viscosity ratio & Corey exponent for water. However, the difference in their effect is marginal & can be attributed to inconsistencies in the model.

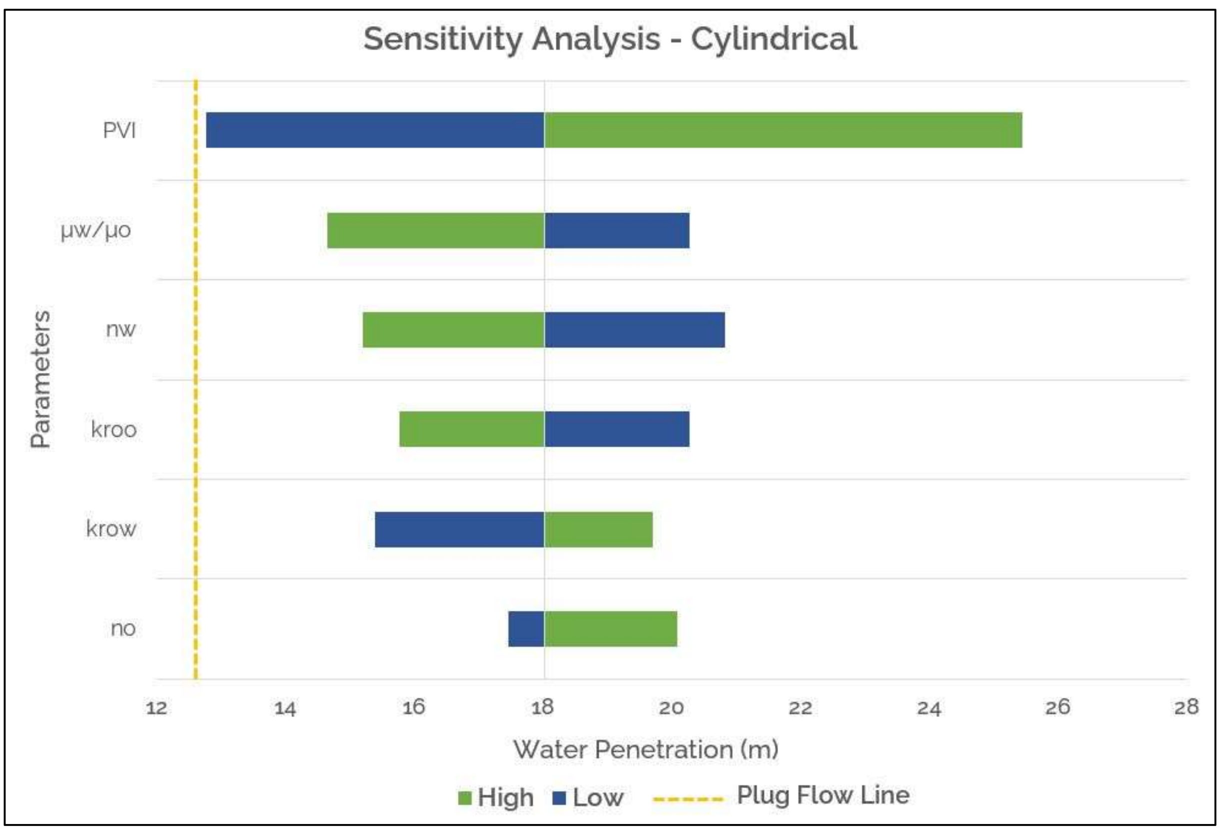

Figure 11. Sensitivity Analysis for Cylindrical System.

4. DISCUSSION & CONCLUSION

The results obtained from the analysis highlight significant attributes of the flow behavior of water in an oil-water system. From Figure 6, the change in relative permeability of oil & water as a function of water saturation can be observed. This change is dependent on the interfacial tension between the oil & water phases. As the water saturation increases, a greater volume of water is able to overcome the interfacial tension between the phases & flow through the pores. It can also be observed that the relative permeability of water remains at 0.6 even after the water saturation reaches 100%. These are called the Endpoint Relative Permeability. This is the constant relative permeability of one phase at the other phase's residual saturation [2]. It remains below 1 since some residual oil remains trapped within the rock crevices, as the saturation of water in the pore increases.

Figures 8 & 9 show significant disparities between the flow profiles in the Cartesian & Cylindrical systems. After 2 hours & 24 minutes, the water penetrates 137 times more in the Cartesian system. This is because in the Cartesian system, the entire volume of the water injected is split equally between 3 narrow slits, whereas in the radial system the water flows equally in all directions from the wellbore. Hence, the net volume of water flowing through a control volume in a particular dimension is much greater in the Cartesian system.

The penetration speed of water is uniform in the Cartesian system, while in the cylindrical system it gradually decreases. This can be attributed to the geometry of the control volume. As the water penetrates deeper into the reservoir, the volume of flowing water keeps decreasing as it displaces the oil in the pores. Due to the reduced volume of water flowing through a particular point in the radial system, the time taken for the water saturation to meet the fractional flow condition, as seen in Figure 7, increases.

The flow profile plots also display some similarities. For instance, the plots show that at the end of the observation period, the water displaces ~ 45% of the oil trapped in the pores, at the distance penetrated by the plug flow. The plug flow line shows an ideal flow of water, in which the water completely displaced the oil from the pores before penetrating further. However, the results of the Buckley Leverett line from the plots show that ~ 55% of the oil is left behind in the pores at that point. The amount of oil left behind in the pores increases as the distance from the injection well increases. The flow profiles also show that the curve drops off at 30% water saturation. This shows that until the water saturation increases above 30%, oil is displaced in a piston like displacement by water. This type of displacement is similar to a plug flow. As the saturation increases beyond 30%, the plug flow displacement is replaced by a transient flow profile.

The results of the sensitivity analysis in Figure 10 & 11, illustrate the relationship of the different reservoir & fluid properties with the water penetration. As the volume of water injected increases, PVI increases, which pushes the water flow further into the reservoir. The Endpoint Relative Permeability of water determines the ability of the water to push through the remaining oil trapped in the pores. As kr^0_w decreases, the ability of the water to penetrate through the oil decreases & the water ultimately penetrates a shorter distance. Similarly, as kr^0_o increases, the ability of oil to penetrate through the pores increases, while the permeability of water decreases.

The water penetration is also influenced by the viscosity ratio. The viscosity ratio decreases as the viscosity of oil increases. When the oil becomes more viscous, the water penetrates through the trapped oil in the pores and pushes the oil towards the rock crevices instead of out of the pores. Thus, the water penetration is directly related to the viscosity of oil. Lastly, it can be seen that the Corey Exponent for water has a greater effect on the water penetration than the Corey Exponent for oil. An increase in the Corey Exponent for water, n_w, reduces the relative permeability for water for lower saturations, thus increasing the slope of the curve. On the other hand, decreasing n_w, increases the relative permeability of water for lower saturations. Therefore, it is inversely related to the water penetration in the reservoir. The Corey Exponent for oil, n_o, has the opposite effect on the water penetration in the reservoir.

There results also point to certain limitations in the model. The model does not account for the Residual Saturation of oil & water in the reservoir. It is assumed that there is no oil trapped in the pores after the waterflooding. The Residual Saturation influences the fractional flow curve, which consequently affects the water penetration. Additionally, the reservoir rock structure is assumed to be homogeneous. In reality, reservoir rocks have a heterogeneous structure since they are made up of different rock layers with varying properties & geometries.

It can be concluded from the results that the analysis in the Cylindrical system provides a more accurate description of the fluid flow profile in the reservoir. There are significant differences between the flow profiles in the Cartesian & Cylindrical system. Therefore, the analysis should not be conducted in the Cartesian system. It can also be seen that oil is not completely displaced from the pores and the amount of oil left behind increases as the distance from the injection well increases. The water penetration in the reservoir can be changed by varying certain properties of the fluid or the reservoir rock. It is directly related to the Pore Volume Injected, Endpoint Relative Permeability of water and the Corey Exponent for oil. On the other hand, it is inversely related to the viscosity ratio of water to oil, Corey Exponent for water, and the Endpoint Relative Permeability of oil. Lastly, the accuracy of the model can be increased by incorporating the Residual Saturation of oil in the pores & accounting for the heterogeneity of the reservoir rock structure.

5. PROPOSED STEPS FORWARD

The model can be developed & refined in order to increase the accuracy of the results. One such development would be to incorporate the Residual Oil Saturation (S_{ro}) as an input parameter. The Residual Oil Saturation refers to the remaining saturation of oil in the pores after the movable oil has been completely displaced by the injected water [2]. Additionally, the model can be made further dimensionless by using Mobility Ratio (M^0). The Mobility Ratio is a product of the ratios of the viscosity of oil to water & Relative Permeability of water to oil [8]. The Mobility Ratio & Residual Oil Saturation can describe the effectiveness of waterflooding using a more commonly used metric, Oil Recovery Efficiency. Oil Recovery Efficiency is a function of the Mobility Ratio & Residual Oil Saturation [8]. It is defined as the fraction of oil in place that can be recovered economically [14].

Furthermore, the constituent equations can be expanded to simulate the flow of 3 or more components simultaneously. A reservoir usually contains some amount of natural gas as well. The natural gas dissolved in the crude oil drives the fluid towards the production well during the primary production stage. Thus, a 3 component multiphase fluid flow would simulate a more accurate water penetration profile in the reservoir.

The model can be used to simulate the flow of Zinc in the reservoir. A combination of Zinc & Calcium halides are used in high-density drilling & completions fluids in the case of high-pressure high-temperature reservoirs. Since Zinc is classified as a priority pollutant by the Environmental Protection Agency (EPA), it is essential to predict the extent to which the fluids can seep through the surrounding rock & pollute the aquifers.

6. REFERENCES

1. Completion fluid – Schlumberger Oilfield Glossary https://glossary.oilfield.slb.com/en/terms/c/completion_fluid (Accessed July 20, 2021)
2. Lake, Larry. W. *Enhanced Oil Recovery*; Prentice-Hall, Inc.: Englewood Cliffs, NJ, 1989.
3. Sheng, James. J. *Modern Chemical Enhanced Oil Recovery*; Gulf Professional Publishing: Burlington, MA, 2010.
4. Druetta, P; Raffa, P; Picchiomi, F. Chemical Enhanced Oil Recovery and The Role of Chemical Product Design. *Applied Energy* **2019**, Vol. 252, 113480.
5. Sambath, K. *Two Phase Flow in Porous Media: Kickoff Meeting*. **2021**.
6. Immiscible Displacement Buckley-Leverett Theory https://perminc.com/resources/fundamentals-of-fluid-flow-in-porous-media/chapter-4-immiscible-displacement/buckley-leverett-theory/ (Accessed June 15, 2021)
7. Amooie, M. A. Analysis of the Buckley-Leverett Solution and Comparison with Numerical Simulation. Technical Report, University of Wyoming, Laramie, WY, 2016.
8. Ding, L.; Wu, Q.; Zhang, L.; Guerillot, D. Application of Fractional Flow Theory for Analytical Modeling of Surfactant Flooding, Polymer Flooding, and Surfactant/Polymer Flooding for Chemical Enhanced Oil Recovery. *Water* **2020**, Vol. 12(8), p.2195.
9. Sambath, K. *Two Fluid Flow in Porous Media*. **2021**.
10. Rosland, F. A. Modelling of water-oil flow in reservoirs including effects from gravity, capillary pressure and water rock chemistry. Master's Thesis, Universitetet i Stavanger, Stavanger, Norway, 2013.
11. Ling, K. Fractional Flow in Radial Flow Systems: A Study for Peripheral Waterflood. *J. Petrol. Explor. Prod. Technol.* **2016**, Vol. 6(3), p.441-450.
12. Feng, G. Yu, M. Characterization of Pore Volume of Cumulative Water Injection Distribution. *Petroleum* **2015**, Vol. 1(2), p. 158-163.
13. Lyons, W. C. *Working Guide to Reservoir Engineering*; Gulf Professional Publishing: Burlington, MA, 2010.
14. Sambath, K.; Agrawal, R. *Cartesian Model Code*. **2021**.

APPENDIX

A.1 Numerical Solution of Buckley Leverett Equation

The discretization of the Buckley Leverett equation for the Cartesian model is as follows [10]:

$$\frac{\partial S_w}{\partial t} + \frac{\partial f_w}{\partial x} = 0 \quad 0 \leq x \leq L, t \geq 0 \quad (15)$$

$$Initial\ Condition: S(0, x) = 0$$

$$Boundary\ Condition: S(t, 0) = 1$$

$We\ use\ the\ Backward\ Difference\ method\ to\ discretize\ the\ equation\ \&\ Explicit\ Time\ Integration,$

$$\frac{\partial S_w}{\partial t} = \frac{S(t, x) - S(t-1, x)}{\Delta t}$$

$$\frac{\partial f_w}{\partial x} = \frac{F(t-1, x) - F(t-1, x-1)}{\Delta x}$$

$$Substituting\ \frac{\partial f_w}{\partial x}\ \&\ \frac{\partial S_w}{\partial t}\ in\ eqn.(15)\ , we\ get$$

$$\frac{S(t, x) - S(t-1, x)}{\Delta t} + \frac{F(t-1, x) - F(t-1, x-1)}{\Delta x} = 0$$

A.2 Cartesian Model Code [15]:

```
%% Modeling movement of lost completions fluid in reservoir
% Using Buckley - Leverett Equation & Corey Model
% 2021 - JUL - 2, KS & RA

clear
clc

global kr0w kr0o nw no muw muo

%% System Parameters

% Flushing parameters

n = 3; % no. of slices
```

```
V_bbl = 1000 / n; % [barrels of water flushed or injected]
q_bbld = 10000 / n; % [rate of flushing in barrels / day]

V = V_bbl / 6.2898; % [m3]
q = q_bbld / 543439.65; % [m3/s]

tf = V / q; % [s] Duration of flushing with Completions Fluid

% Reservoir parameters

H = 1; % [m] Height of reservoir (pay zone)
re = 0.1 / 2; % [m] Wellbore radius
W = 2 * sqrt(pi / n * tan(pi / n)) * re; % [m] Wellbore width per slice
phi = 0.33; % [m] Porosity of reservoir
L0 = V / (H * W * phi); % [m] Plow Length of Reservoir
L = 4 * L0; % [m] Length of reservoir monitored in the direction of flush

% Fw(Sw) curve parameters

kr0w = 0.6; % endpoint relative permeability of water
kr0o = 0.4; % endpoint relative permeability of oil
nw = 2; % Corey exponent for water
no = 2; % Corey exponent for oil
muw = 1; % Viscosity of water (Changed from 0.5 to 10)
muo = 5; % Viscosity of oil

S0 = linspace(0, 1, 100); % Used for plotting
F0 = fwfn(S0); % Used for plotting

%% Discretization of Space & Time domains

% Spatial discretization
nx = 200;
eta = linspace(0, 1, nx);
deta = eta(2) - eta(1);

tc = H * W * L * phi / q;

nt = 5000;
towf = tf / tc;
tow = linspace(0, towf, nt);
dtow = tow(2) - tow(1);

% Defining variables
S = zeros(nt, nx);
F = zeros(nt, nx);

% Initial Condition

S(1, :) = [1 zeros(1, nx - 1)];

F(1, :) = fwfn(S(1, :));

%% Integration

for it = 2:nt

    S(it, 1) = S(it - 1, 1) - 2 * dtow * (F(it - 1, 1) - 1) / deta;
```

```
    for ix = 2:nx

        S(it, ix) = S(it - 1, ix) - dtow * (F(it - 1, ix) - F(it - 1, ix -
1)) / deta ;

    end

    F(it, :) = fwfn(S(it, :));

end

%% Plotting Solutions

plot1 = 1; % Profile plots
plot2 = 0; % Transient plots
plot3 = 0; % F(S_w)
plot4 = 0; % Slices arrangement
plot5 = 0; %Plug Flow Analysis
anim1 = 0; % Create Animation

x = eta * L;
t_min = tow * tc / 60;

% Plug Flow Solution

SP = heaviside(x + deta) - heaviside(x - L0);

if plot1 % Profile plots

    h1 = plot(x, S(1, :), '-b', 'LineWidth', 1.5);
    grid on
    xlabel('Axial Distance [m]')
    ylabel('Water Volume Fraction [-]')
    ylim([0 1])
    xlim([0 L])
    set(gca, 'FontSize', 14)
    title('Water displacing oil in reservoir')
    hold on
    for it = 1:ceil(nt / 100):nt

        plot(x, S(it, :), '-k')
        pause(0.01)

    end
    hn = plot(x, S(nt, :), '-r', 'LineWidth', 1.5);
    hr = plot(x, SP, '-g', 'LineWidth', 1.5);
    hold off
    legend([h1 hn hr], 'Initial', 'Final (BL eqn)', 'Plug Flow')

end

if plot2 % Transient plots

    figure
    plot(t_min, S(:, 1), 'LineWidth', 1.5)
    hold on
    plot(t_min, S(:, floor(0.25 * nx)), 'LineWidth', 1.5)
    plot(t_min, S(:, floor(0.50 * nx)), 'LineWidth', 1.5)
```

```
    plot(t_min, S(:, floor(0.75 * nx)), 'LineWidth', 1.5)
    plot(t_min, S(:, floor(1.00 * nx)), 'LineWidth', 1.5)
    hold off
    xlabel('Time [min]')
    ylabel('Water Volume Fraction [-]')
    ylim([0 1])
    title('Water displacing oil in reservoir: Transients')
    legend('x = 0', 'x = 25% L', 'x = 50% L', 'x = 75% L', 'x = L',
'Location', 'Best')
    set(gca, 'FontSize', 14)
    grid on

end

if plot3 % F(S_w) - Phase Mobility vs. Saturation

    figure
    plot(S0, F0, 'LineWidth', 1.5)
    xlabel('S_w')
    ylabel('F(S_w)')
    title('Fractional Flow Curve')
    grid on
    set(gca, 'FontSize', 14)

end

if plot4 % F(S_w) - Phase Mobility vs. Saturation

    figure
    xp = [0 L L 0 0];
    yp = [0 0 W W 0];
    xp = xp + re * sqrt(pi / n / tan(pi / n));
    yp = yp - W / 2;
    plot(xp, yp, '-k', 'LineWidth', 1.5)
    axis equal
    axis off
    hold on

    for i = 1:n - 1

        th = 2 * pi / n;

        M = [cos(th) sin(th); -sin(th) cos(th)];

        tp = [xp; yp]' * M;

        xp = tp(:, 1)';
        yp = tp(:, 2)';


        plot(xp, yp, '-k', 'LineWidth', 1.5)

    end

    th = linspace(0, 2 * pi, 100);
    xr = re * cos(th);
    yr = re * sin(th);
    plot(xr, yr, '-r', 'LineWidth', 1.5)
```

```
    xlim([-1 1] * 5 * re)
    ylim([-1 1] * 5 * re)
    hold off

end

if anim1 % Profile plots

    % Preallocate movie structure.
    mov(1) = struct('cdata', [], 'colormap', []);

    h1 = plot(x, S(1, :), '-b', 'LineWidth', 1.5);
    grid on
    xlabel('Axial Distance [m]')
    ylabel('Water Volume Fraction [-]')
    ylim([0 1])
    set(gca, 'FontSize', 18)
    title('Water displacing oil in reservoir')
    hold on

    id = 1;

    mov(id) = getframe(gcf);

    for it = 10:10:nt

        plot(x, S(it, :), '-k')

        id = id + 1;

        mov(id) = getframe(gcf);

    end
    hn = plot(x, S(nt, :), '-r', 'LineWidth', 1.5);
    hr = plot(x, SP, '-g', 'LineWidth', 1.5);
    hold off
    legend([h1 hn hr], 'Initial', 'Final (BL eqn)', 'Plug Flow')

    for i = 1:15

        id = id + 1;
        mov(id) = getframe(gcf);

    end

    % Create AVI file

    v = VideoWriter('FlushingAnimation.avi');
    v.FrameRate = 15;
    open(v);
    writeVideo(v, mov);
    close(v);

end



%Plug Flow Analysis
```

```
%Distance covered by plug flow
plug_dist = 0;
for iter=1:nx
    if SP(1,iter)==0
        plug_dist = iter-1;
        break
    end
end

% Time taken by 20% Saturation plane to reach Plug Flow distance
for irow=1:nt
    if S(irow,plug_dist)>=0.2
        plug_trow = irow;
        break
    end
end

Splug = S(plug_trow:nt, plug_dist);
fplug = fwfn(Splug);
plug_flow_out = q*fplug;  %Flow of water from 20% saturation to final
saturation level

Total_volume_flow_out = 0;
Percent_Volume_in_Plug_temp = [];

rownum = length(plug_flow_out);
for flowiter = 1:rownum
    Total_volume_flow_out = Total_volume_flow_out +
plug_flow_out(flowiter,1);
    Percent_Volume_in_Plug_temp(flowiter,1) = (V - Total_volume_flow_out) *
100/V;
end

temp_volume = ones(plug_trow-1,1);
temp_volume = temp_volume*100;
Percent_Volume_in_Plug = [temp_volume;Percent_Volume_in_Plug_temp];


if plot5 % Water within Plug Flow line w.r.t time


    figure
    plot(t_min, Percent_Volume_in_Plug, 'LineWidth', 1.5)
    xlabel('Time (mins)')
    ylabel('Volume (%)')
    title('Volume of Water within Plug Flow')
    grid on
    grid minor
    set(gca, 'FontSize', 14)

end



%% Functions

function y = fwfn(S)
```

```
global kr0w kr0o nw no muw muo

Lrw = kr0w * S.^nw / muw;
Lro = kr0o * (1 - S).^no / muo;
f = Lrw ./(Lrw + Lro);

y = f;

end
```