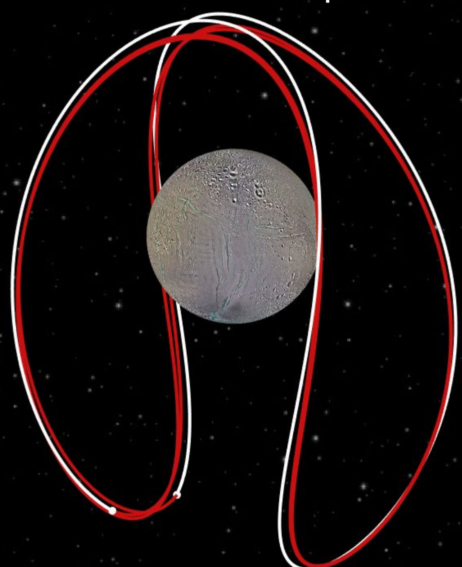
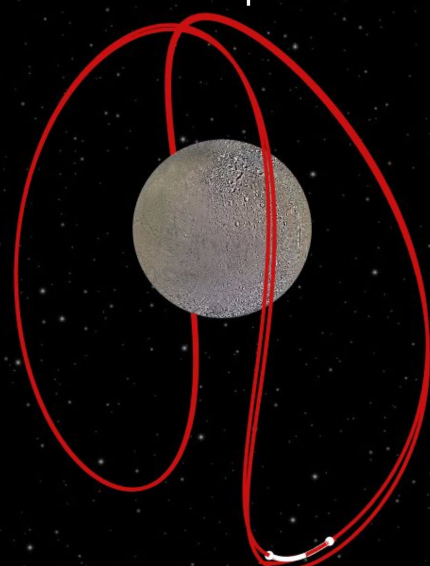
CR3BP Stationkeeping and Orbit Transfers

Find station-keeping maneuvers that will keep the spacecraft in an unstable butterfly orbit around Enceladus. The orbit would be periodic except for the inclusion of the J2 perturbation from Enceladus. Use indirect method two point boundary value problem for a low-thrust trajectory as well as a differential corrector to accomplish the same with an impulsive maneuver.

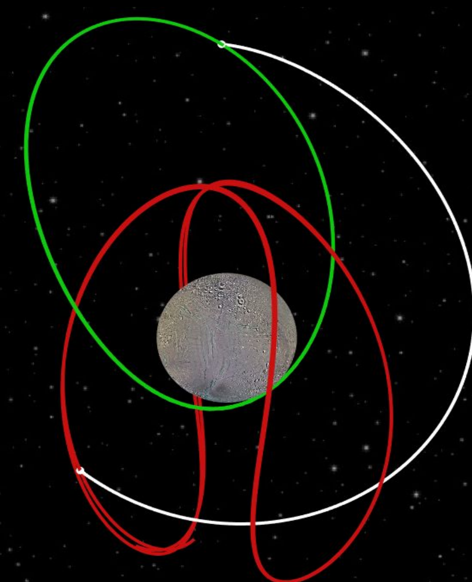


Impulsive Maneuver



Low-Thrust: Min-Energy

The same approach can be used to transfer between three-body orbits by setting different initial and final states as the boundary conditions.



Halo-Butterfly Transfer

A minimum fuel trajectory would be preferable but does not reliably converge.

Optimal Control for Variable Thrust Bounds Due to Propulsion Constraints

Affan Nazeer

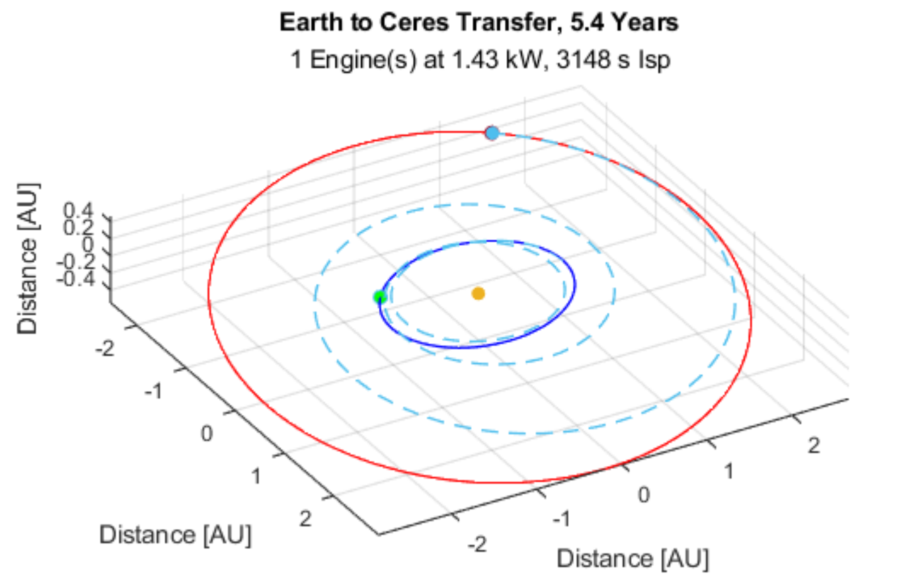
Objective:
Optimize spacecraft maneuver dependent on external thrust bounds and Engine Efficiency

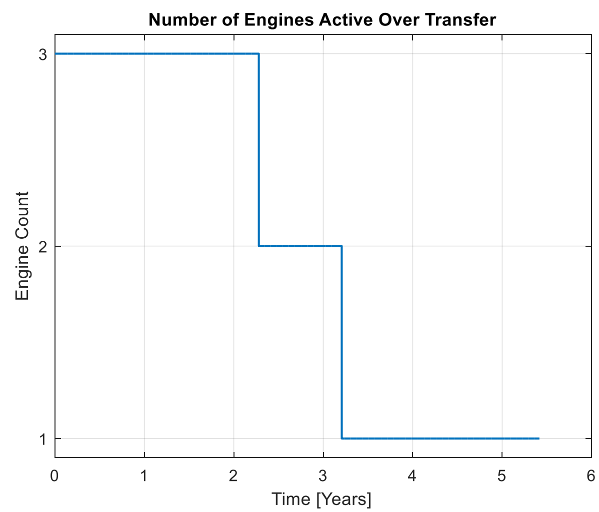
- Approach

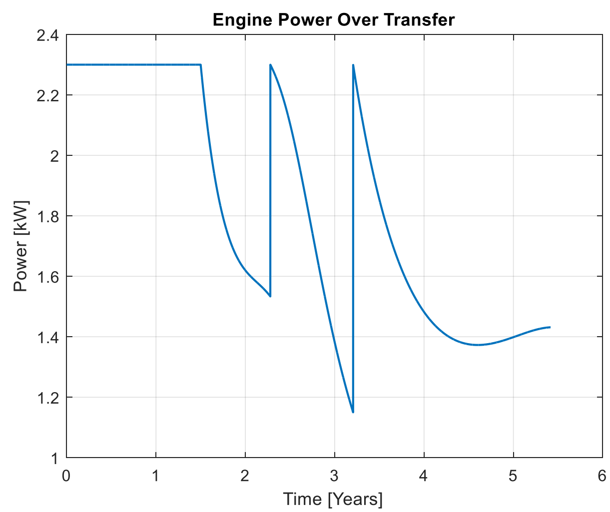
Use Pontryagin's Minimum Principle to optimize controller for minimum time of flight. Spacecraft utilizes Solar Electric Propulsion with three engines. Control maximum is constrained by power input, inversely proportional to the squared orbital radius. Utilize experimental data to find optimal engine count as a function of total power.

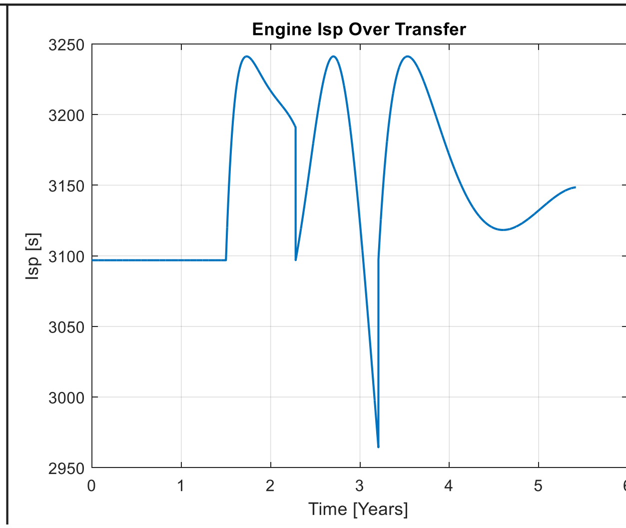
- Results

Utilizing a shooting method of indirect control optimization, a solution was found for a Dawn-like mission to Ceres. Optimal controller determined engine count and corresponding engine power to input. Introducing variable maximum thrust allows for low thrust Solar Electric Propulsion orbit transfer missions to be optimized. Further introducing extra granularity in engine operation such as discrete engine count optimization allows for slight gains in ΔV.









Lyapunov Control for Autonomous Asteroid SLAM

AAE 590ACA Project - Aditya Arjun Anibha

Objective

- Develop Lyapunov Controller to Improve Observability of Asteroid Optical Navigation Method.

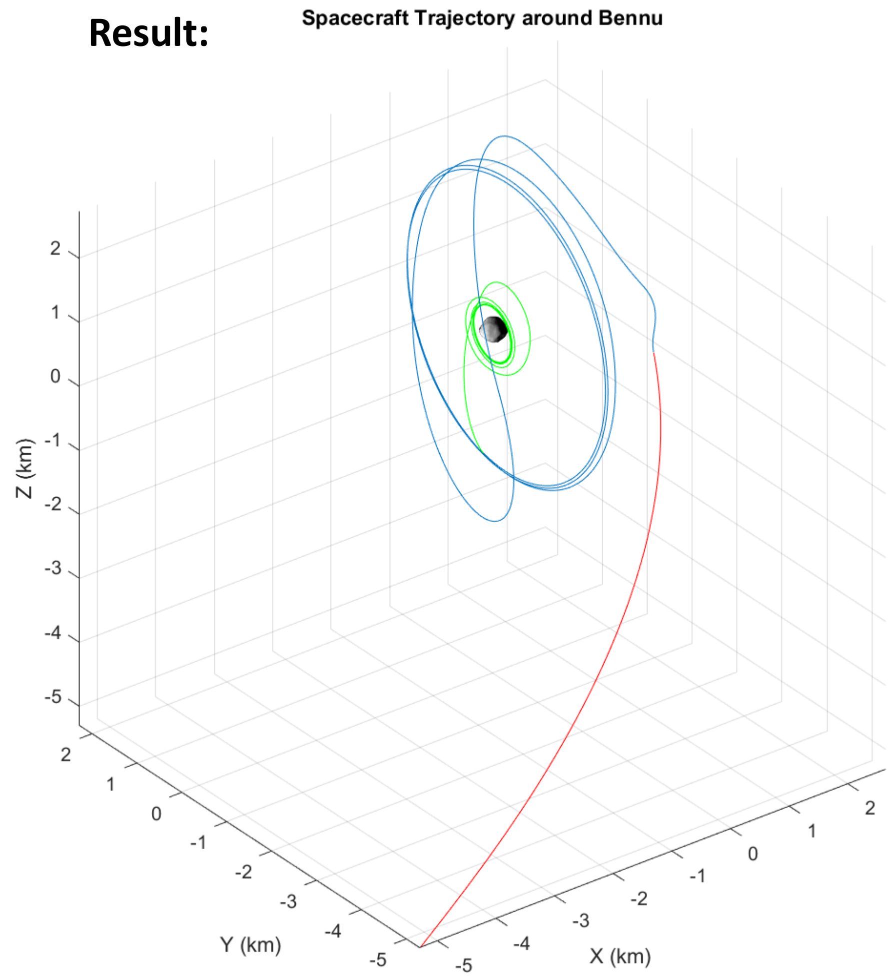
Approach

- Multi-stage Lyapunov Controller that approaches asteroid, transfers to terminator plane and to desired final orbit. Uses penalty functions for maximum distance and cone to keep spacecraft in regions where the OpNav algorithm is effective and accurate.

Discussion

- Successful with low control input, near-critical damping, and avoiding penalty zones. Further potential with improved tuning of gains and penalties, trying extreme initial conditions, along with testing OpNav algorithm on the produced trajectories.

Result:



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Indirect Optimization of Solar Sailing trajectories

An AAE 590ACA Project by Kaushik Rajendran

Objective

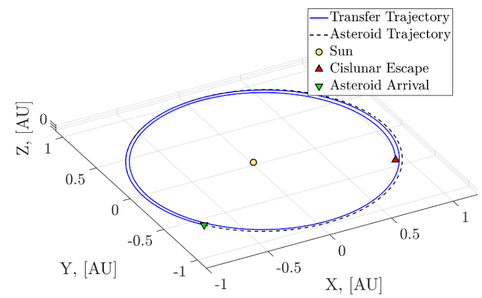
- To optimize trajectories of solar sailing craft implementing practical mission considerations

Approach

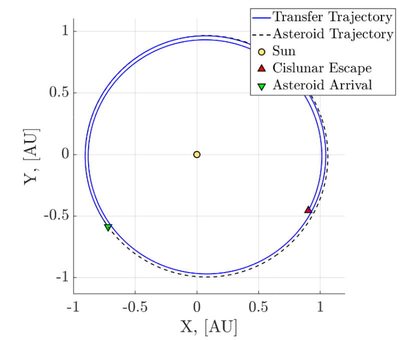
- Pontryagin's minimum principle based optimization utilizing solar sailing primer vector theory [1]
- Optimized for minimum-time and minimum-solar angle/maximum power generation objectives

Results

- Solar sailing trajectories optimizing for desired objective while considering practical mission requirements such as sail angle constraint and imperfect solar reflectivity

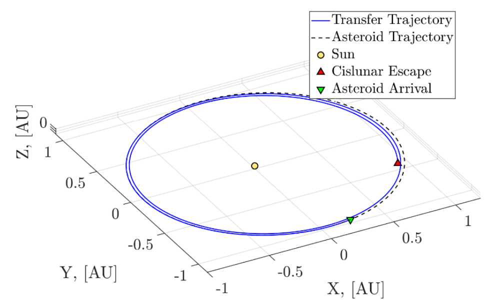


3D trajectory of asteroid transfer

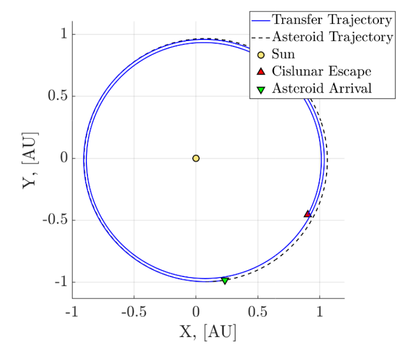


Asteroid transfer trajectory projected onto the x-y plane

Minimum time optimal transfer



3D trajectory of asteroid transfer



Asteroid transfer trajectory projected onto the x-y plane

Minimum solar angle optimal transfer

Key Takeaway

- Solar sailing primer vector is a robust framework allowing consideration of several mission constraints and requirements, while also facilitating the incorporation of technical developments in low-thrust trajectory optimization literature into the realm of solar sailing orbit transfer problems

[1] Oguri, K., Lantoine, G., and McMahon, J. W., "Solar Sailing Primer Vector Theory: Indirect Trajectory Optimization with Practical Mission Considerations," Journal of Guidance, Control, and Dynamics, American Institute of Aeronautics and Astronautics (AIAA), 45, 1, Jan2022, pp. 153–161. https://doi.org/10.2514/1.g006210

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Optimal Control and Momentum Exchange Tethers

PURDUE UNIVERSITY
School of Aeronautics and Astronautics

Andrew Binder, Abdulrahman Abdrabou

AAE 59000 ACA Final Project

Objectives

Momentum exchange tethers present a new and numerous challenges in the astrodynamics field. The use of these tethers can provide a promising solution for spacecraft propulsion and trajectory optimization. By developing advanced control algorithms and optimization techniques to guide payloads into catch opportunities with the tether, this research seeks to provide valuable insights and guidelines for the design and implementation of future cislunar missions and space infrastructure. Our objectives were classified into three main levels. These levels were defined in a way that makes it easy to follow and understand and makes a smooth transition in complexity levels using content from class. Our objectives list can be summarized below. These are goals we will work towards achieving:

1. Level 1 **(COMPLETE)**: Acquire a variety of catch opportunities using a min. energy formulation
2. Level 2 **(COMPLETE)**: Achieve a min. energy → min. fuel homotopy between transfer objectives
3. Level 3 **(INCOMPLETE)**: Control of the tether's rotation rate, reducing fuel costs of the payload

Methodologies Used

Minimum Energy

The tether sits in an L_1 Halo orbit. We can decompose the motion of the tip of the tether as the motion of the tether's center of mass in it's orbit, plus the rotation of the tether.

$$\boldsymbol{h}_{tip} = \boldsymbol{r}_{halo} + \bar{l}\begin{bmatrix}\cos[\tilde{\alpha}(\tau-\tau_0)] \\ \sin[\tilde{\alpha}(\tau-\tau_0)] \\ 0\end{bmatrix}$$

Payload state/costate dynamics under control evolve typically from a fixed initial condition:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}_0 + B\boldsymbol{u}$$

$$\dot{\boldsymbol{\lambda}} = -\left[\frac{\partial[\boldsymbol{f}_0]}{\partial \boldsymbol{x}}\right]^T \boldsymbol{\lambda}, \qquad \frac{\partial[\boldsymbol{f}_0]}{\partial \boldsymbol{x}} = \begin{bmatrix}\boldsymbol{0}_{3x3} & \boldsymbol{I}_{3x3} \\ U_{pp} & \Omega\end{bmatrix}$$

Payload optimal control follows primer vector:

$$\boldsymbol{u}^* = -\frac{1}{2}B^T\boldsymbol{\lambda} = \frac{1}{2}\boldsymbol{p}$$

Our goal is precise rendezvous with tether tip:

$$\boldsymbol{\psi} = [\boldsymbol{x}(\boldsymbol{\tau_f}) - \boldsymbol{h_{tip}}(\boldsymbol{\tau_f})], \qquad \boldsymbol{Z} = \begin{bmatrix}\boldsymbol{\lambda_0} \\ \boldsymbol{\tau_f}\end{bmatrix}$$

Elements of $\partial\boldsymbol{\psi}/\partial\boldsymbol{Z}$ can be integrated alongside the state/costate evolution, avoiding finite diff.:

$$\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{X_{aug}}} = \begin{bmatrix}\begin{bmatrix}\boldsymbol{0}_{3x3} & \boldsymbol{I}_{3x3} \\ U_{pp} & \Omega\end{bmatrix} & \begin{bmatrix}\boldsymbol{0}_{3x3} & \boldsymbol{0}_{3x3} \\ \boldsymbol{0}_{3x3} & -1/2\,\boldsymbol{I}_{3x3}\end{bmatrix} \\ \boldsymbol{M} & \begin{bmatrix}\boldsymbol{0}_{3x3} & -U_{pp} \\ -\boldsymbol{I}_{3x3} & -\Omega\end{bmatrix}\end{bmatrix}$$

Min. Energy → Min. Fuel Homotopy

A blended control Lagrangian permits a smooth homotopy between min. energy and min. fuel:

$$\mathcal{L} = (1-\beta)\|u\|_2^2 + \beta\|u\|_2$$

The tether tip's motion, as well as payload's state and costate evolve like min. energy. Payload optimal control is more complex here:

$$\mathrm{u}^* = \underset{\|u\|_2 \le u_{max}}{\arg\min}\left((1-\beta)\|u\|_2^2 + \beta\|u\|_2 + \boldsymbol{\lambda}^T B\boldsymbol{u}\right)$$

$$\mathrm{u}^* = \Gamma^*\hat{\boldsymbol{u}}^*, \qquad \hat{\boldsymbol{u}}^* = \hat{\boldsymbol{p}}$$

$$\Gamma^* = \begin{cases} u_{max} & (2u_{max}(1-\beta) < \|p\|_2 - \beta) \\ \frac{1}{2}\frac{\|p\|_2 - \beta}{1-\beta} & (0 \le \|p\|_2 - \beta \le 2u_{max}(1-\beta)) \\ 0 & (\|p\|_2 - \beta < 0)\end{cases}$$

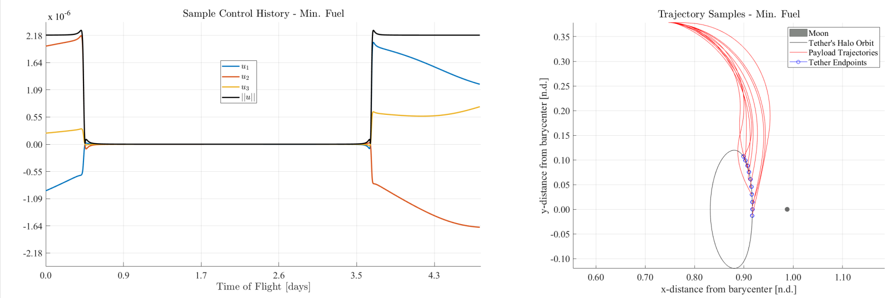
Piecewise functions are written with the Heaviside step, which has an approximation:

$$H(x) \approx \frac{1}{2}\left[\frac{x}{\sqrt{x^2+\varepsilon^2}} + 1\right]$$

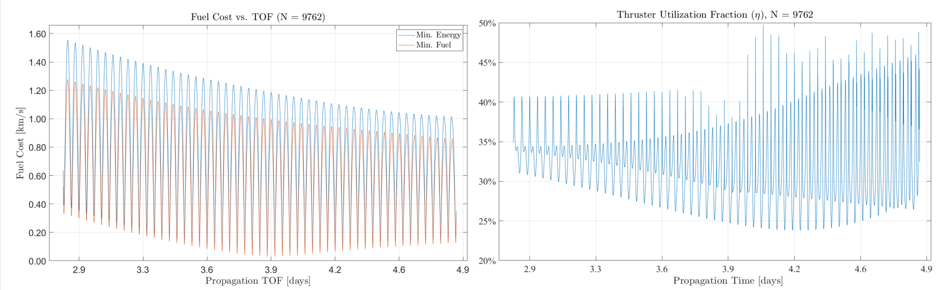
Where $\varepsilon \ll 1$ is a sharpness parameter. $\beta = 0$ is the minimum energy trajectory, $\beta = 1$ is the minimum fuel trajectory, and by sweeping between the two values $\beta \in (0,1)$ and solving the TPBVP at each β (while feeding previous results into future convergence steps) smoothly deforms minimum energy into minimum fuel.

Visualization of Results (N = 9762, $TOF \in [0.65, 1.12]\ [nd]$)

Typical Trajectories:



Aggregate Results:



Discussion and Conclusions

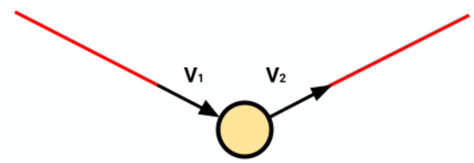
In conclusion, optimal control can be used to guide payloads into catch opportunities with the rotating tip of a momentum exchange tether. For a 48-hour window, optimal trajectories are solved for in a minimum energy formulation. These trajectories are all transitioned into a minimum-fuel formulation using a homotopy process between objective functions. Sometimes, finer steps in β are required to smoothly deform samples. The minimum fuel's resultant bang-bang control profile is shown to provide a net-savings over the minimum energy result in all cases, proving the validity of the results. Future work could find a heuristic floor for thruster strength on the payload could be derived within the min. fuel framework. Finally, by increasing maximum thrust permitted, burns performed begin approximating impulsive maneuvers. Using this technique, the minimum fuel investment could be found via approximate impulsive maneuvers. Additionally, control techniques beyond *Pontryagin's Minimum Principle* could solve similar or related problems.

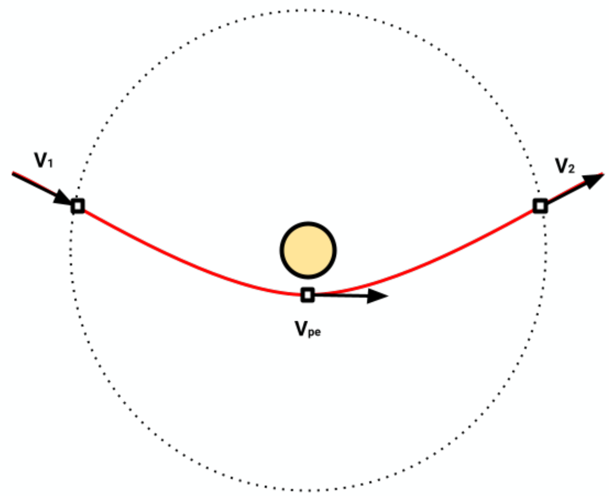
Bibliography

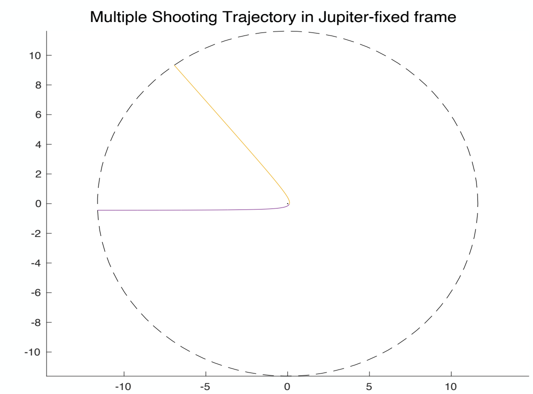
AAE 590 ACA Final Project – Converting VILM flybys using Pontryagin-based multiple shooting optimization

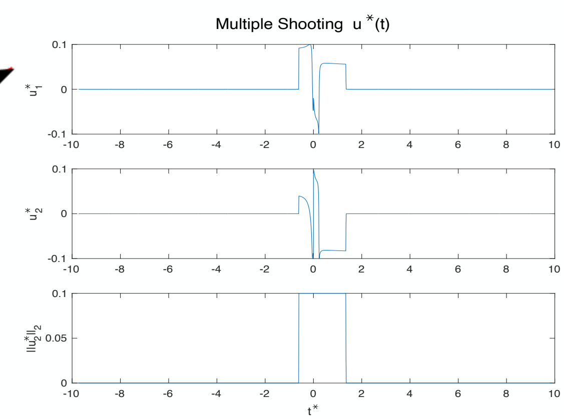
Daisy Mei Ehara

- Goal: Given $\vec{v}_1$ and $\vec{v}_2$, create a continuous trajectory with terminal velocities matching $\vec{v}_1$ and $\vec{v}_2$
- Approach: starting at periapsis, propagate 2 trajectories with the same initial state outwards using Pontryagin-based minimum fuel trajectories, and enforce boundary conditions using fsolve()
- Discussion: Less than ideal ΔV values, but approach demonstrated multiple shooting's robustness and decreased sensitivity to minor changes









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Optimal Trajectory Generation for Multiple Space-Debris Cleanups

AAE 590 Applied Controls in Aeronautics: Final Project Summary Slide • Spring 2024 • Emmett Hawkins, Amogha Sarang

PURDUE AERONAUTICS & ASTRONAUTICS

Introduction

Space debris objects in low-Earth orbit (LEO) pose a significant threat to present and future missions to space. One method of addressing this issue is by using specialized spacecraft to de-orbit space debris, cleaning up Earth's orbit. In advancement of this potential solution, this project seeks to develop an optimal trajectory generation method that will:

- Target space debris in LEO
- Avoid other objects in close proximity to target debris
- Maneuver to optimal position to provide de-orbiting impulse to debris object
- Accomplish task w/ minimum fuel usage
- Provide flexibility and extended lifespan

Project Objectives

Objectives divided into levels based on difficulty of task and importance of completion for project's success

Level 1:

- Implement minimum-fuel Pontryagin TPBVP to approach debris cloud
- Factor in LEO perturbations
  - Aerodynamic drag
  - J2 perturbation

Level 2:

- Apply convex optimization via CWH to reach debris location
- Impart impulse on debris for de-orbiting process

Level 3:

- Re-orient spacecraft after each de-orbiting impulse
- Enhance adaptability and efficiency of impulses
- Test control robustness using Monte-Carlo simulations

Approach: Pontryagin

State Dynamics: $\dot{x} = f_0(x) + Ba_d(x) + Bu$ $\quad {}^{\mathcal{N}}x = \begin{bmatrix} {}^{\mathcal{N}}r & {}^{\mathcal{N}}v \end{bmatrix}^T = \begin{bmatrix} x & y & z & \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^T \quad {}^{\mathcal{N}}u = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}$

$$f_0(x) = \begin{bmatrix} v \\ -\frac{\mu}{\|r\|_2^3} r \end{bmatrix}, \quad B = \begin{bmatrix} 0_{3\times 3} \\ \mathbb{I}_3 \end{bmatrix}, \quad a_d(x) = a_{J_2} + a_{drag}, \quad a_{J_2} = -\frac{3\mu J_2 r_0^2}{2r^5} \begin{bmatrix} \left(1 - 5\frac{z^2}{r^2}\right) x \\ \left(1 - 5\frac{z^2}{r^2}\right) y \\ \left(3 - 5\frac{z^2}{r^2}\right) z \end{bmatrix} \quad \vec{a}_{drag} = -\frac{1}{2}\rho \frac{C_D A}{m} v_{rel} \vec{v}_{rel}$$

Control Hamiltonian: $H = L + \lambda^T f = ||u||_2 + \lambda^T (f_0(x) + Ba_d(x) + Bu)$

Costate Dynamics: $\dot{\lambda}^T = -\frac{\partial H}{\partial x} = -\lambda^T \left(\frac{\partial f_0(x)}{\partial x} + \frac{\partial B a_d(x)}{\partial x} \right)$

$$\Longrightarrow \dot{\lambda}^* = -\left[\frac{\partial f_0(x)}{\partial x} + \frac{\partial B(a_{J_2} + a_{drag})}{\partial x} \right]^T \lambda^*$$

$$\min_{x,u,t_0,t_f} \int_{t_0}^{t_f} \|u\|_2 dt$$

such that: $\dot{x} = f_0(x) + Ba_d(x) + Bu$

$x_0, x_f, t_0, t_f = given, \quad \|u\|_2 \le u_{max}$

Optimal Control: $minH = \frac{\partial}{\partial u} argmin \left[||u||_2 + \lambda^T (f_0(x) + Bu) \right] = argmin[||u||_2 - \vec{p}^T u]$

$$u^* = \frac{\vec{p}}{||\vec{p}||_2} \Gamma^*, \quad (\Gamma^*, \hat{u}^*) = argmin \Gamma(1 - \vec{p}^T \hat{u})$$

Transversality Condition: $\Psi = 0 \longrightarrow$ all boundary conditions are fixed.

Approach: Convex Optimization

Clohessy-Wiltshire-Hill (CWH) Eq (Relative Frame Coordinates:

$$\ddot{x} = 3n^2 x + 2n\dot{y} + u_x$$
$$\ddot{y} = -2n\dot{x} + u_y$$
$$\ddot{z} = -n^2 z + u_z$$

Equation of Motion (EoM) → $\dot{x} = Ax + Bu$

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -n^2 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Convexified Optimal Control Problem w/ Time Discretization:

$minimize\ J = \int_{t_o}^{t_f} \|u\| dt$ — **Objective Function**

subject to $x_{k+1} = A_k x_k + B_k u_k + c_k$ — **Dynamic Function**

$\|u_k\| \le u_{max} = 0.1$ — **Control Constraint**

$\|x_k\| \le x_{max} = 2.5 \quad \text{atan}\left\{ \frac{\sqrt{r_1(t)^2 + r_3(t)^2}}{r_2(t)} \right\} \le \gamma_{max} = 30°$ — **State Path Constraints**

$x_o = [r_o^T, v_o^T] \quad x_f = [0, 0.08, 0, 0, 0, 0]$ — **Boundary Constraints**

Results: Convex Optimization

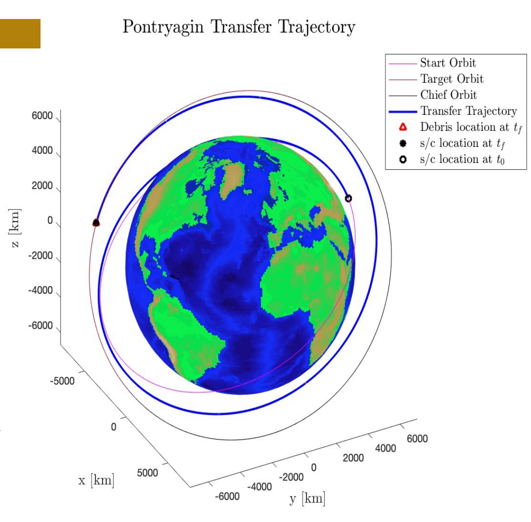


Fig. 1: Pontryagin Transfer Trajectory

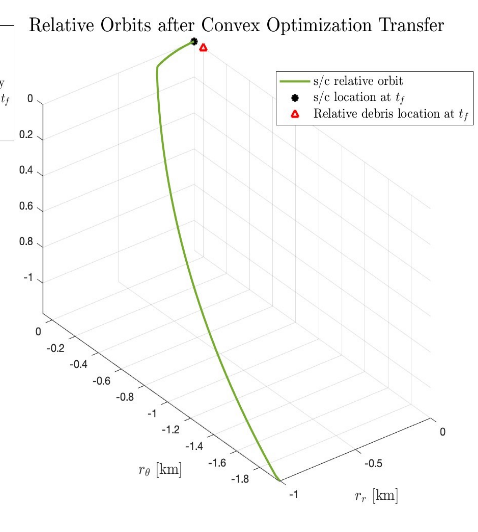


Fig. 2: Convex Optimization Transfer Trajectory

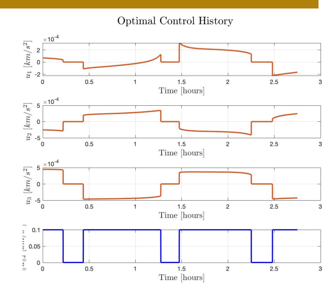


Fig. 3: Pontryagin Transfer Control Time History

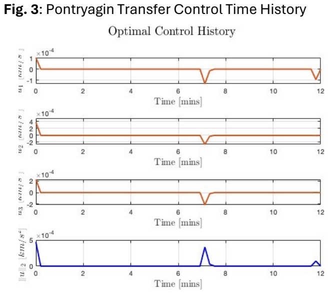


Fig. 4: Conv. Opt. Transfer Control Time History

Discussion

The combination of a Pontryagin transfer to approach the debris cloud and a convex optimization transfer to approach the specific piece of debris was successful at navigating the spacecraft to the desired location, as shown by Figures 1 and 2, while minimizing fuel usage. To this end, Figure 3 shows that the control limitation was effective at minimizing the controller's exertion, as the magnitude of the control vector never exceeds 0.1. Meanwhile, the significantly smaller scale of the convex optimization transfer meant that the control output was never close to approaching this limit, as seen in Figure 4. While the Level 1 objectives and most of the Level 2 objectives were accomplished, there are still further steps that were not accomplished due to time constraints. For one, there was not enough time to implement the de-orbiting impulse operation at the end of the convex optimization transfer, although its final position is optimal for such an action. Additionally, none of the Level 3 objectives could be properly addressed.

Conclusion

Overall, the use of a minimum fuel Pontryagin TPBVP was effective at accounting for non-linear EOMs and the dynamics of a system complicated by the LEO environment, allowing for the spacecraft to successfully arrive within close proximity of the debris object. Meanwhile, the use of convex optimization utilizing CWH equations with time discretization was found to be effective at determining a global solution to transfering to the target location while also adhering to several key constraints.

More work is needed to improve the product's performance, but current simulations show promise of a solution to the threat posed by space debris in low-Earth orbit.

Acknowledgements

The team would like to thank Professor Kenshiro Oguri, Aaron Liao, and Nicholas Craig for all of the assistance and guidance that they provided in navigating this project.

Optimal Control for Satellite Formation Reconfiguration

AAE 590 ACA
Masashi Nishiguchi
PURDUE UNIVERSITY

Objective

To find the optimal control profile for reshaping the formation of satellites from one configuration to another.

Approach

Case 1:
Minimize the total fuel cost

$$\min \sum_{j=1}^{N} \int_{0}^{t_f} \left\| \boldsymbol{u_j}(t) \right\|_1 dt$$

Case 2:
Balance the fuel usage for satellites

$$\min(\max_{j \in \{1,\ldots N\}} \int_{0}^{t_f} \left\| \boldsymbol{u_j}(t) \right\|_1 dt\ )$$

- **Constraints**

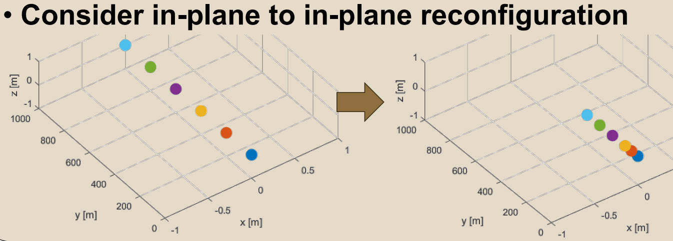
1. Terminal Condition
   $$\boldsymbol{x_j}(0) = \boldsymbol{x_{j0}}\ ,\ \boldsymbol{x_j}(t_f) = \boldsymbol{x_{jt_f}}$$
2. Maximum Thrust
   $$\left\| u_j(t) \right\|_\infty \le U_{max}$$
3. Discretized HCW equation
4. No Collisions (Non-Convex)
   $$\left\| C[x_j(t) - x_i(t)] \right\| \ge r_{safe}$$

- **Convexification of Collision Free Constraint (by approximation)**

$$\left(\boldsymbol{\bar{x}_j(k) - \bar{x}_i(k)}\right)^T \boldsymbol{C^T C}\left(\boldsymbol{x_j(k) - x_i(k)}\right) \ge \boldsymbol{r_{safe}} \| \boldsymbol{C(\bar{x}_j(k) - \bar{x}_i(k)}_{\|}$$

→ Requires Reference States, $\bar{x}$
→ **Sequential Convex Program (SCP)** → Solves for the control and offset

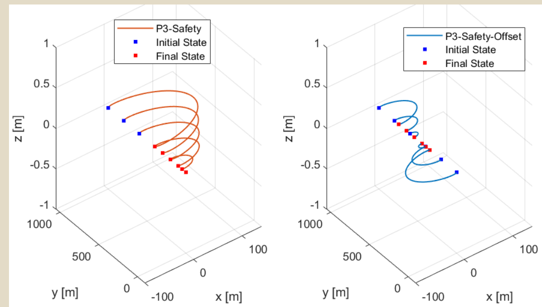
- **Introduced Offset to the final position, y_r**
- **Consider in-plane to in-plane reconfiguration**



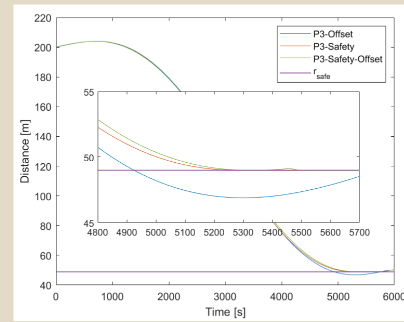
$r_{safe} = 49\ [m]$
$t_f = 6000\ [s]$,
$U_{max} = 25 \times 10^{-6} m/s^2$

Results

Case1: Only Safety (left), Safety and Offset (right)



Distance between Sat. 1 and 2



Fuel Usage for each sats., total fuel cost (J_t) and offset

Case 1	Safety	Offset	Safety-Offset
$\|\|u_1\|\|$ [10^{-4} m/s]	0.62	12.79	12.99
$\|\|u_2\|\|$ [10^{-4} m/s]	5.50	7.30	7.16
$\|\|u_3\|\|$ [10^{-4} m/s]	11.34	1.80	2.13
$\|\|u_4\|\|$ [10^{-4} m/s]	14.65	1.86	2.00
$\|\|u_5\|\|$ [10^{-4} m/s]	18.31	5.52	5.66
$\|\|u_6\|\|$ [10^{-4} m/s]	21.98	9.18	9.32
J_t [10^{-3} m/s]	7.24	3.85	3.93
yr [m]	0	349.23	345.47

Case 2	Safety	Offset	Safety-Offset
$\|\|u_1\|\|$ [10^{-4} m/s]	5.16	10.99	10.99
$\|\|u_2\|\|$ [10^{-4} m/s]	7.27	5.79	6.39
$\|\|u_3\|\|$ [10^{-4} m/s]	12.08	0.21	2.85
$\|\|u_4\|\|$ [10^{-4} m/s]	14.91	4.14	4.66
$\|\|u_5\|\|$ [10^{-4} m/s]	18.37	7.46	7.59
$\|\|u_6\|\|$ [10^{-4} m/s]	21.78	10.99	10.99
J_t [10^{-3} m/s]	7.96	3.95	4.35
yr [m]	0	300.00	300.00

Discussions

- The proposed SCP algorithm solves the fuel optimization problem for minimum total fuel cost and balancing out the fuel consumption among all satellites.
- The fuel usage among satellites can be balanced out in the cost of increasing the total fuel consumption.
- Introducing offset in the final position reduce the total fuel consumption and can still meet the finall geometric constraint.

Sequential Convex Programming for Minimum Fuel Soft-Asteroid Landing

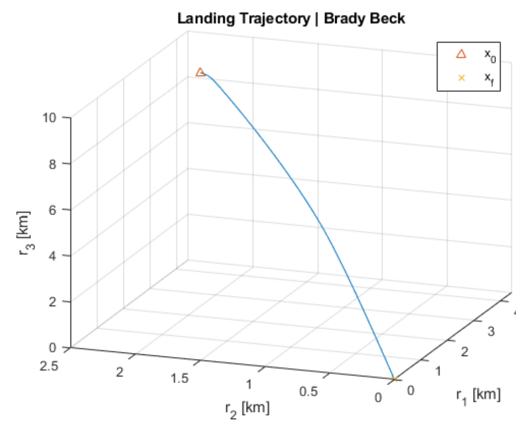
- Optimization problem defined, convexified, discretized to yield:

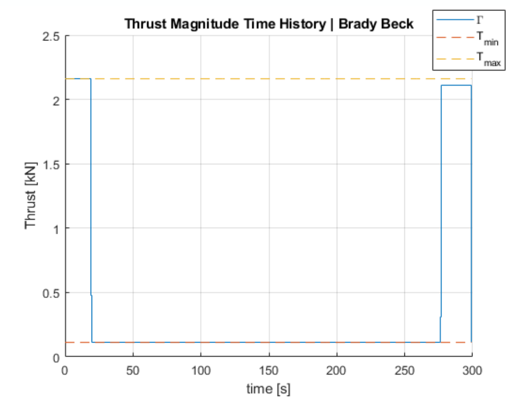
$$
\begin{aligned}
\min_{\vec{x}_k, \vec{u}_k} \quad & \sum_{k=1}^{N} \sigma_{k-1} \Delta t \\
\text{s.t.} \quad & \vec{x}_{k+1} = A_k \vec{x}_k + B_k \vec{u}_k + \vec{c}_k \\
& 0 \leq T_{\min} e^{-z_k} \leq \sigma_k \leq T_{\max} e^{-z_{\text{lb}_k}} (1 - (z_k - z_{\text{lb}_k})) \\
& z_{\text{lb}_k} \leq z_k \\
& \|\vec{a}_k\|_2 \leq \sigma_k \\
& \|S_1 \vec{x}_k\| + \vec{d}_1^T \vec{x}_k \leq 0 \\
& \vec{d}_2^T \vec{x}_k \leq 0 \\
& \vec{x}_0 = \begin{bmatrix} \vec{r}_0^T & \vec{v}_0^T & m_0 \end{bmatrix}^T \\
& \vec{x}_{N_{1:6}} = \begin{bmatrix} \vec{r}_f & \vec{v}_f \end{bmatrix}^T
\end{aligned}
$$

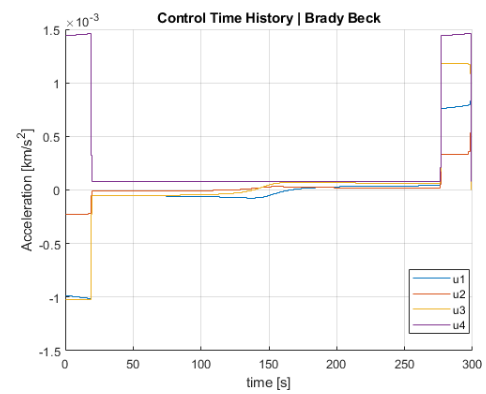
where

$$
\begin{aligned}
& \vec{x}_k = \begin{bmatrix} \vec{r}_k^T & \vec{v}_k^T & z_k \end{bmatrix}^T \\
& \vec{u}_k = \begin{bmatrix} \vec{a}_k^T & \sigma_k \end{bmatrix}^T && z_{\text{lb}_k} = \ln(m_0 - \alpha T_{\max} t_k) \\
& A_k = \Phi(t_{k+1}, t_k) && S_1 = \begin{bmatrix} I_2 & \mathbf{0}_{2\text{x}5} \end{bmatrix} \\
& B_k = \int_{t_k}^{t_{k+1}} \Phi(t_{k+1}, \tau) B(\tau) d\tau && \vec{d}_1 = \begin{bmatrix} 0 & 0 & -\tan(\gamma_{\max}) & 0 & 0 & 0 & 0 \end{bmatrix}^T \\
& \vec{c}_k = \int_{t_k}^{t_{k+1}} \Phi(t_{k+1}, \tau) \vec{c}(\tau) d\tau && \vec{d}_2 = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}^T
\end{aligned}
$$

- Implemented scenario based on OSIRIS-REx Bennu landing
- Optimal Result: 59.4506 kg fuel







- Sequential Convex Programming Algorithm Implemented for Solution:
  - Give initial guess, Z(1) (constant gravity optimal trajectory)
  - Convexify with current trajectory, Z(i)
  - Solve optimization problem to find Z(i)*
  - Use Z(i)* as the reference for Z(i+1)
  - Repeat steps 2-4 until mean-squared error <= 1.5e-4

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Optimal Orbit Transfer Using Sequential Convex Programming

Austin Aden and India Hutson

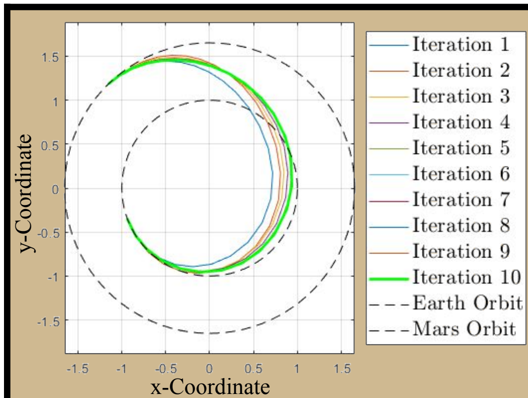
PURDUE UNIVERSITY
School of Aeronautics and Astronautics

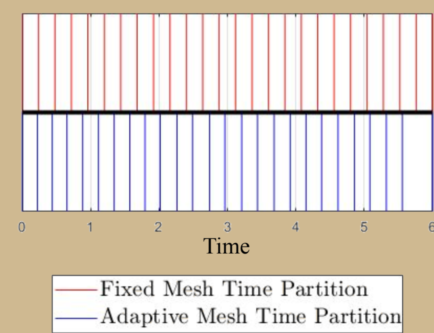
Objectives

- Generate minimum-fuel low-thrust orbit transfer
- Compare performance of fixed and adaptive mesh SCP algorithms

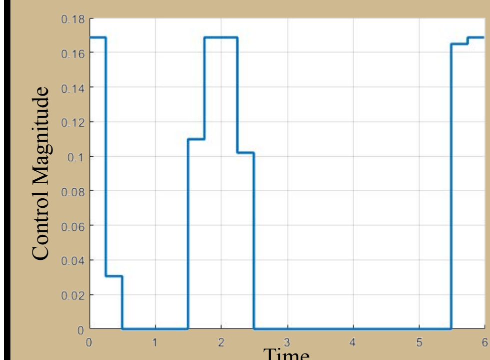
Approach

- Used sequential convex programming with trust region and Augmented Lagrangian to improve convergence
- Used fixed mesh (uniform discrete time partitions), and adaptive mesh (varying, optimized discrete time partitions)

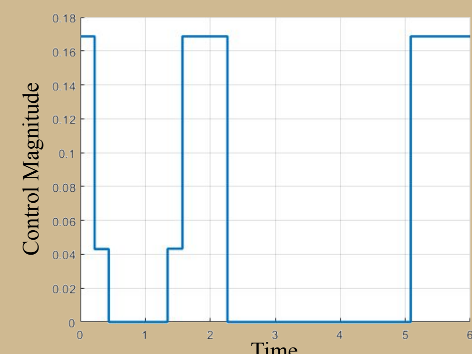




Fixed Mesh Control



Adaptive Mesh Control



Discussion

- Similar trajectory to those generated via indirect optimization
- Adaptive mesh control profile more closely resembles bang-bang control, indicating improved optimality
- Further improvements to approach needed to obtain more optimal solution
- Method applicable to more complex multi-phase transfers incorporating path constraints

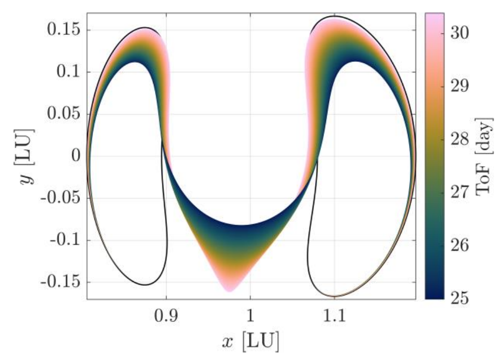
Uncooperative Maneuvering Target Tracking during Optimal Transfers between Cislunar Periodic Orbits

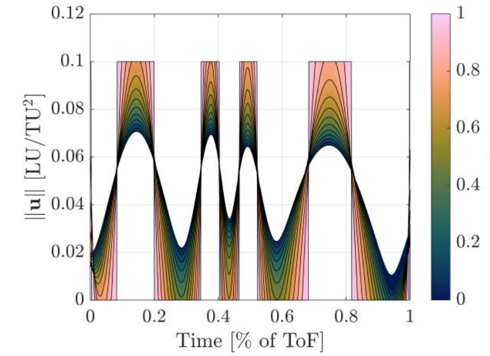
John L. Iannamorelli and Aneesh V. Khilnani

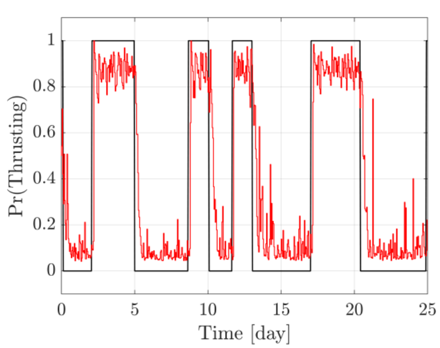
Increasing presence in Cislunar space requires development of optimal trajectories between periodic orbits.

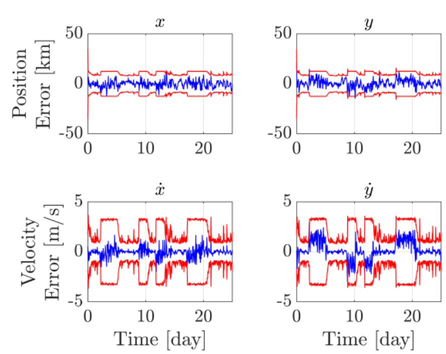
Mission requirements require stable continuation methods for time-of-flight (ToF) and fuel consumption.

The Interacting Multiple Model (IMM) estimation algorithm provides insight on a thrusting/maneuvering phases of a target, without knowledge of the onboard control via a probabilistic approach.









Dynamical systems theory-informed initial trajectory generation for optimal initial costate guess.

Natural parameter continuation to reduce time-of-flight and step from a minimum-energy to a minimum-fuel solution.

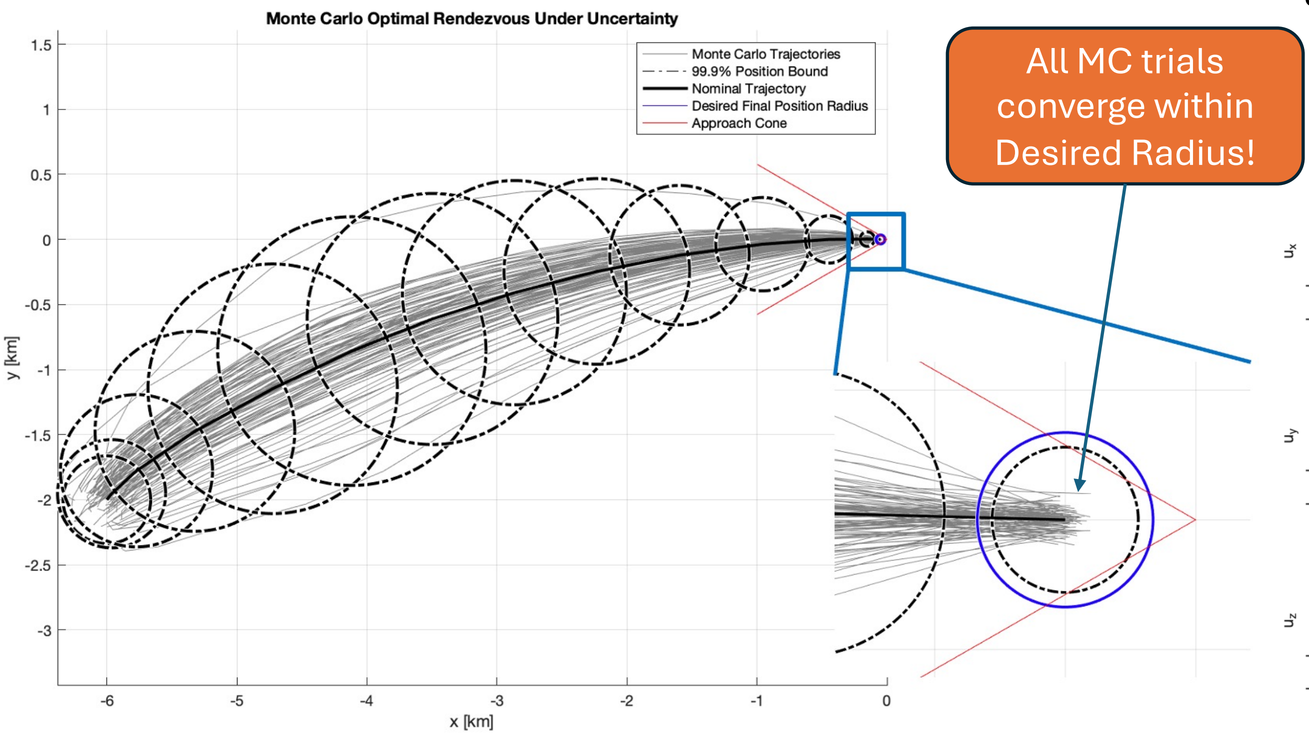
State estimation of the target's state within 20 [km] and 4 [m/s] using the IMM-UKF.

Thrusting mode is predicted without any knowledge of the spacecraft's onboard control system.

A Convex Optimization Approach for Fuel Optimal Spacecraft Rendezvous under Stochastic Uncertainty - By Connor Plaks

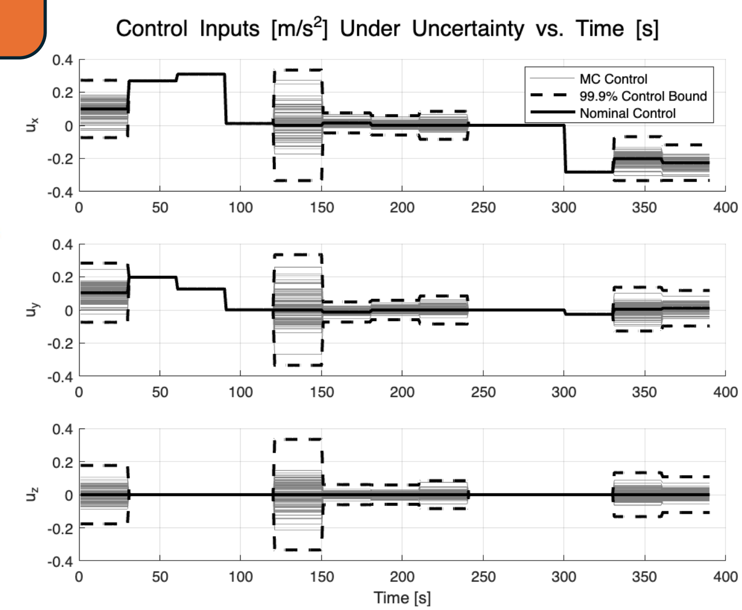
Objectives:

1. Minimize the maximum control cost for rendezvous with worst case uncertainty corrections with a 99% confidence.
2. Arrive at final state with a state dispersion covariance below a threshold
3. Fulfill probabilistic fuel magnitude and safe approach cone constraints to a 99.9% confidence



Approach

1. Define a linear feedback control policy to correct state estimation errors.
2. Solve the convex problem based on the Kalman filter for optimal nominal control and error feedback gain matrices for each time step.
3. Assess control robustness with Monte Carlo analysis.



Lyapunov-based 6 DoF control for rendezvous and docking

Objectives

- Control of 6 DoF relative states of chaser w.r.t. target in LEO.
- Avoid the unwinding problem in the attitude maneuver.

Approach

- Attitude error representation that is free from unwinding problem :

$$e_{\tilde{R}} \triangleq \frac{1}{2\sqrt{1+\mathrm{tr}(\tilde{R})}}(\tilde{R}-\tilde{R}^T)^\vee$$ where $\tilde{R}$: relative rotation matrix

- Dynamics of relative error states in target's body frame:

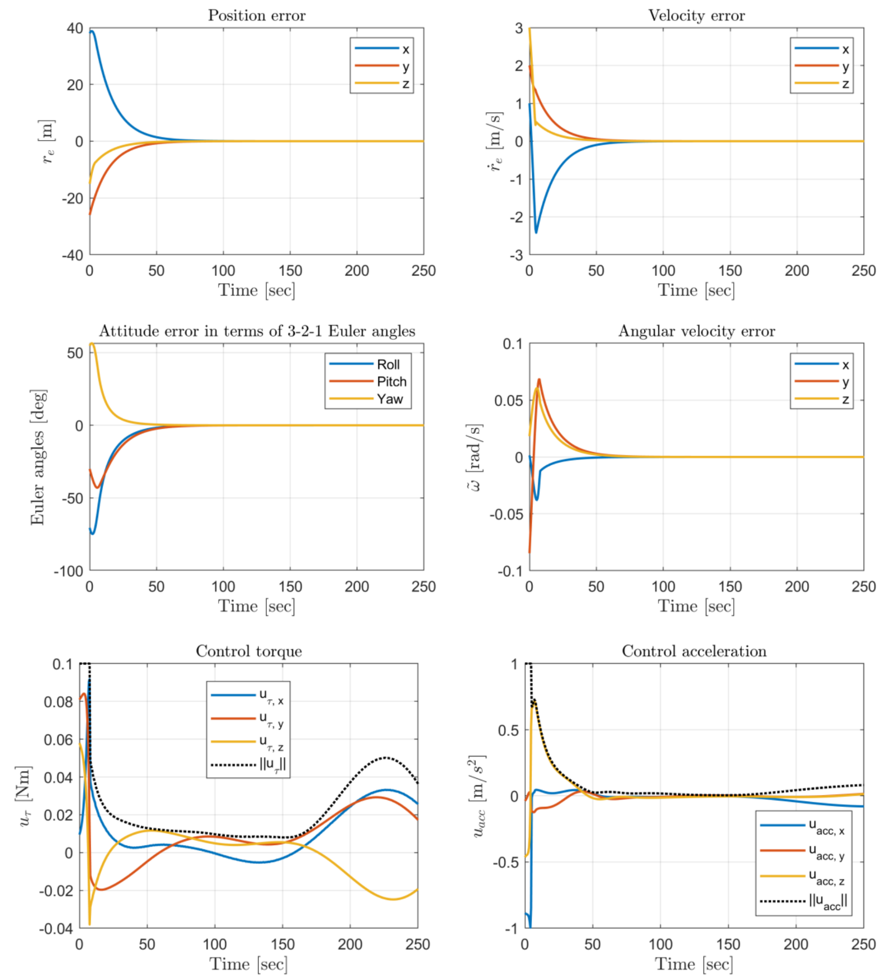
$$\ddot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{c}$$

$$\boldsymbol{x} = \begin{bmatrix} e_{\tilde{\boldsymbol{R}}} \\ r_e \end{bmatrix}, \quad \boldsymbol{u} = \begin{bmatrix} \boldsymbol{u}_\tau \\ \boldsymbol{u}_{\mathrm{acc}} \end{bmatrix} = \boxed{B^{-1}\{-(A+P)\dot{\boldsymbol{x}} - K\boldsymbol{x} + \boldsymbol{c}\}}$$

Error state — Control — Lyapunov controller

Results and discussion

- Attitude error has been reduced almost monotonically despite of a large initial error. → Effectiveness of $e_{\tilde{R}}$
- Control magnitude remains nonzero even after error convergence. → Due to the non-zero docking distance & coupled dynamics.



Continuous-Thrust Station-keeping in Earth-Moon L2 Halo Orbit with Pulsar Navigation

Jillian Ross

Cislunar Missions

Why is station-keeping important for future cislunar missions?

- Orbits inherently dynamically unstable
- Provide means of observation and communication to Earth and/or the Moon
  - Artemis
  - Lunar Gateway

Why is pulsar-based navigation appealing for cislunar station-keeping?

- Stable millisecond pulsar stars act as navigation beacons, generating X-ray signals that can be used analogously to GPS signals [10]
- Options exist for GNSS and DNS, but best to have redundancy for mission safety

[2,4]

State Dynamics

Linearization

1. Define state deviation with respect to nominal orbit: $\Delta \boldsymbol{x} = \boldsymbol{x} - \boldsymbol{x}_N$
2. Define error dynamics: $\Delta \dot{\boldsymbol{x}} = \boldsymbol{A}(\boldsymbol{x}_N, \boldsymbol{t})\Delta \boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}(\boldsymbol{t})$
3. Update state via State-Transition Matrix update

$$\Delta \dot{x} = \begin{pmatrix} 0_{3\times3} & I_{3\times3} \\ \frac{\partial f(x_N, t)}{\partial r} & 0_{3\times3} \end{pmatrix} \Delta x + \boldsymbol{B}\boldsymbol{u}(\boldsymbol{t})$$

CR3BP Dynamics

Goal: Provide optimal station-keeping, balancing system performance with control effort while meeting navigation requirement.

Define cost via Linear Quadratic Regulator

$$J(\Delta x(t), u(t)) = \frac{1}{2}\int_0^\infty \{\Delta \boldsymbol{x}^T(t)Q(t)\Delta \boldsymbol{x}(t) + \boldsymbol{u}^T(t)R(t)\boldsymbol{u}(t)\}\, dt$$

$Q(t) \in \mathbb{R}^{6\times6}$ Positive semi-definite

$R(t) \in \mathbb{R}^{3\times3}$ Positive definite

$$\boldsymbol{u}^*(t) = -R^{-1}(t)B^T K(t)\Delta \boldsymbol{x}(t)$$

$$K(t)A(t) + A^T(t)K(t) - K(t)BR^{-1}(t)B^T K(t) + Q(t) = 0$$

Navigation Constraint:

Occultation Model:

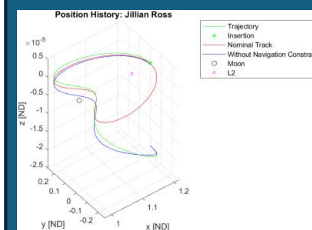
$$\pi - \cos^{-1}\frac{\sqrt{r_{B,S}^2 + R_B^2}}{r_{B,S}} \le \cos^{-1}\hat{n}\cdot\hat{r}_{B,S} \le \pi + \cos^{-1}\frac{\sqrt{r_{B,S}^2 + R_B^2}}{r_{B,S}}$$

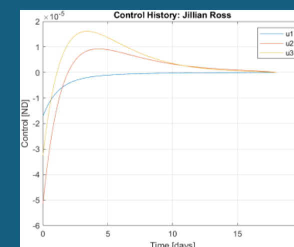
Treat binary nature of occultation with sigmoid and apply to control:

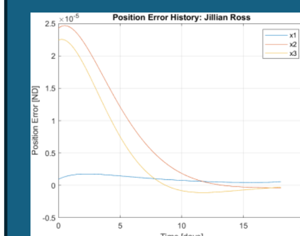
$$S(x) = \frac{1}{1 + e^{-x}}$$

Fine-tune to achieve convergence!

Results:





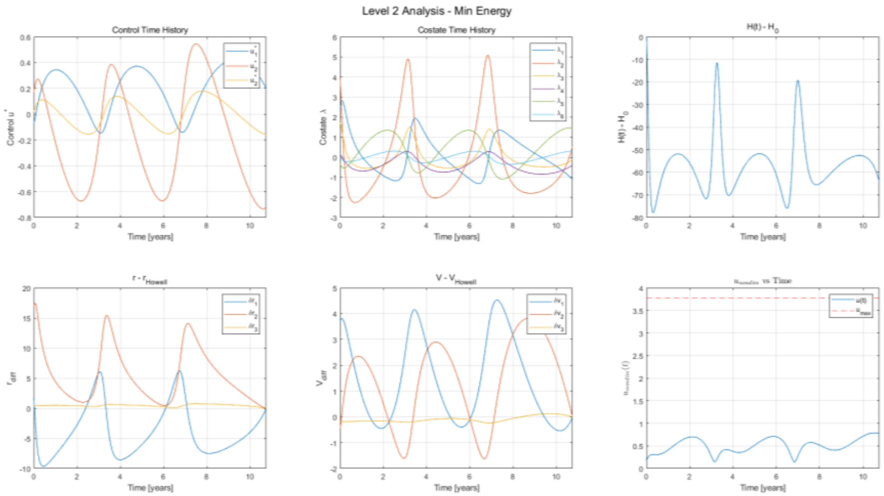


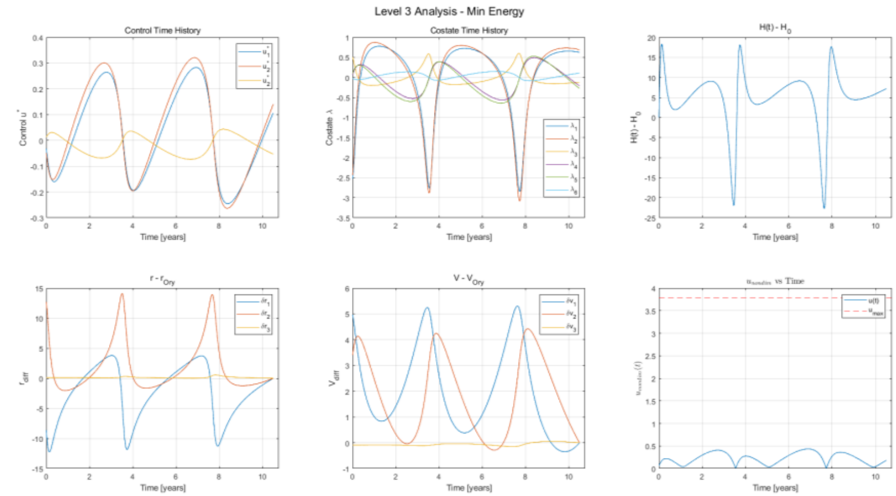
Outcomes:

- An LQR controller balances system performance and low-thrust control for insertion into an L2 Halo orbit
- Treating the thruster to only fire when the line-of-sight to a pulsar is available allows for a navigation constraint
- Fine-tuning a smoothing parameter to treat the binary nature of the thrust allows for improved convergence under this constraint

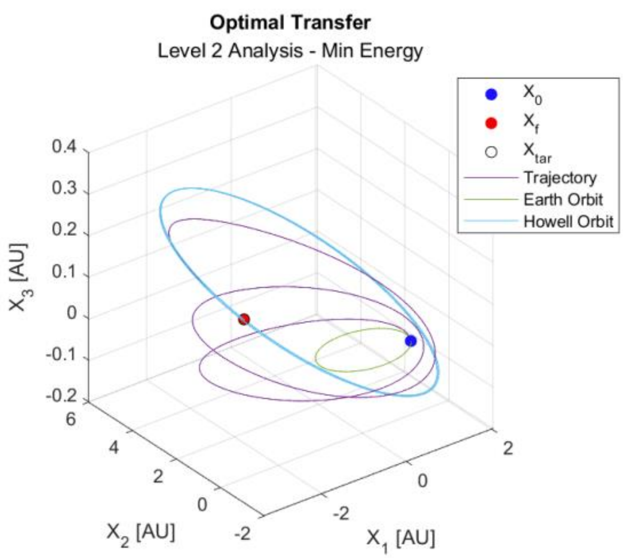
Low-Thrust Trajectory Optimization to Comet

By: Andrew Harrison & Landon Abboud



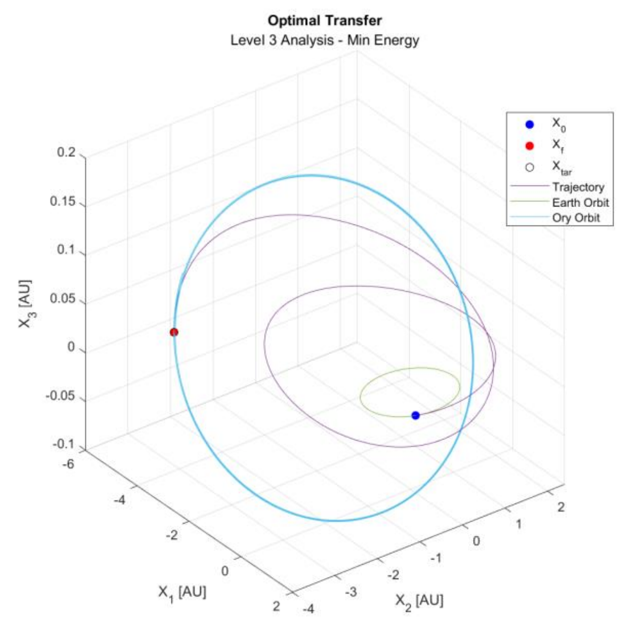


Optimal Transfer to Comet Howell



Find minimum ΔV transfer from Earth to comets 88P/Howell and 304P/Ory using Pontryagin's Minimum Principle for minimum energy

Optimal Transfer to Comet Ory



Summary of Results

Level	Minimize	ΔV ($\frac{km}{s}$)	TOF (days)
1	Fuel	11.266	2915
1	Energy	3.700	2549
1	Energy	1.728	3280
2	Energy	1.492	3939
3	Energy	1.979	2979
3	Energy	0.680	3846

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Sequential Convex Programming for Enceladus Landing

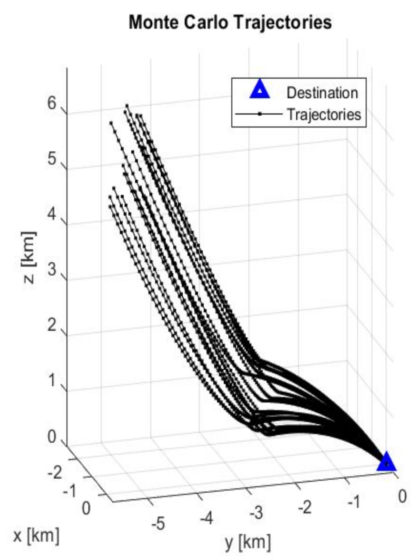
Drew Siciliano and Lucea Larest

Objective: Soft landing on Enceladus using Sequential Convex Programming (SCP) to aid in search for life

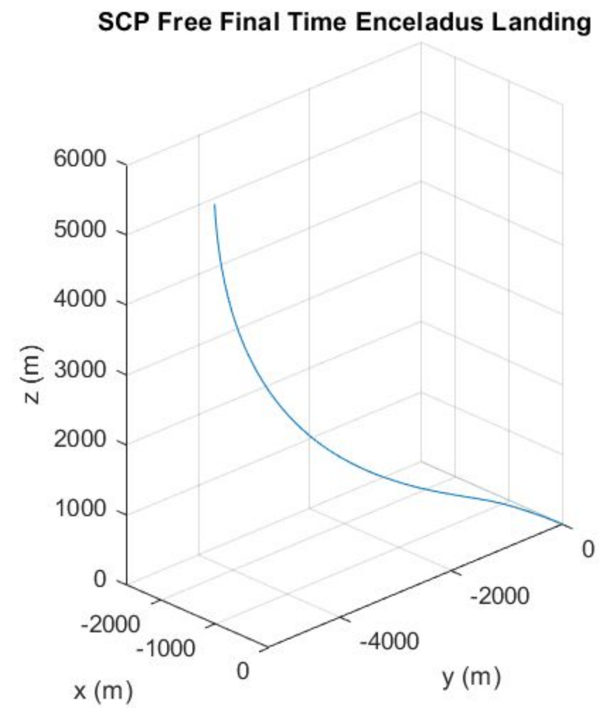
Approaches:
- Non-Constant Gravity
  - Sequential Convex Programming
  - Piecewise Affine
- Free Time of Flight (ToF)
  - Sequential Convex Programming
  - Golden Search

Results:
- Minimum ToF for soft landing from SCP
- Monte Carlo simulation of ToF to minimize fuel consumption

Discussion:
- SCP guarantees convergence for any initial reference trajectory and ToF guess
- Monte Carlo shows linear relationship between fuel consumption and ToF



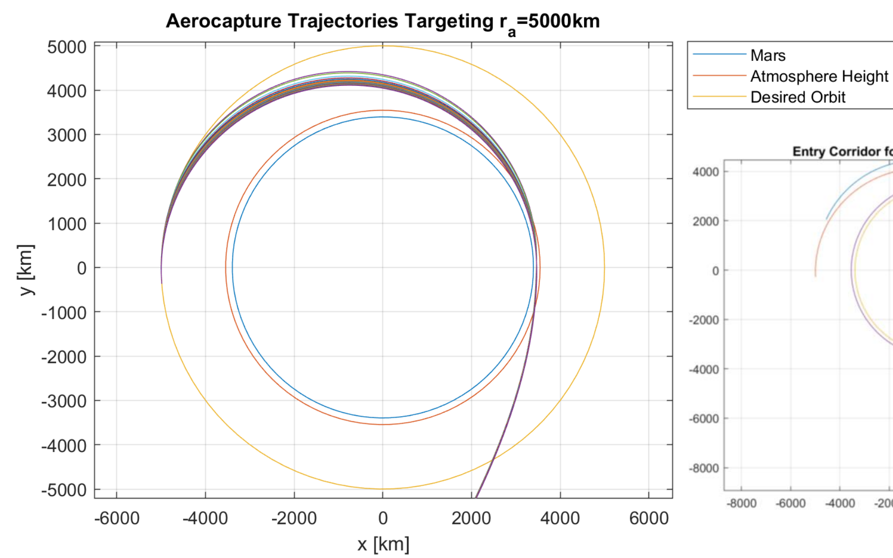
Monte Carlo



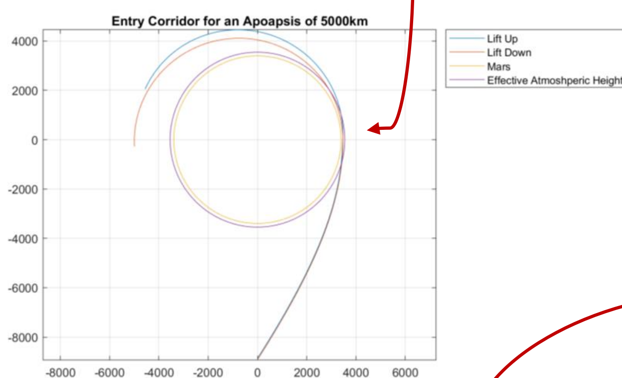
Sequential Convex Programming

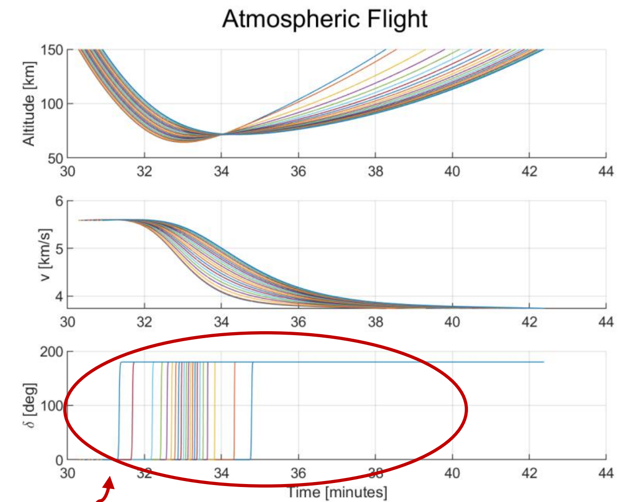
Indirect Optimization for Mars Aerocapture

$$\min J = \sqrt{\frac{-2\mu}{r_a^* + r_p^*} + \frac{\mu}{r_a^*}} - v(t_f) \ \text{: Minimize } \Delta V$$

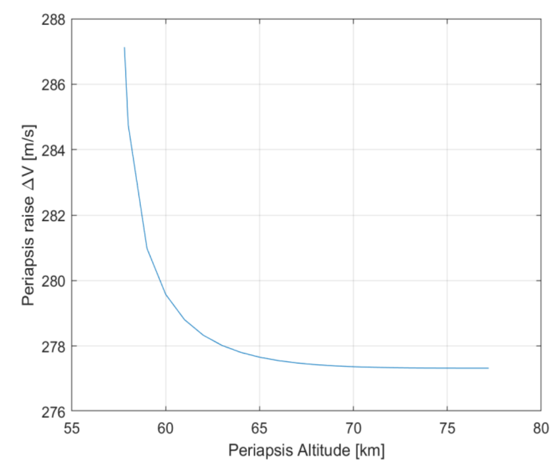


Entry corridor set by final apoapsis





Bang-Bang Optimal Bank Angle



ΔV Savings Increase with Higher Passes Through the Atmosphere

Control Derived via Pontryagin's Minimum Principle

$$H = \boldsymbol{\lambda}^T \begin{bmatrix} v \sin\gamma \\ -\frac{D}{m} - \frac{\mu}{r^2}\sin\gamma \\ -\frac{1}{v}\left(\frac{\mu}{r^2} - \frac{v^2}{r}\right)\cos\gamma + \frac{L}{mv}\cos\delta \end{bmatrix}$$

$$\delta^* \approx \frac{180^\circ}{2}\left[1 + \tanh\left(\frac{\lambda_3}{p}\right)\right]$$

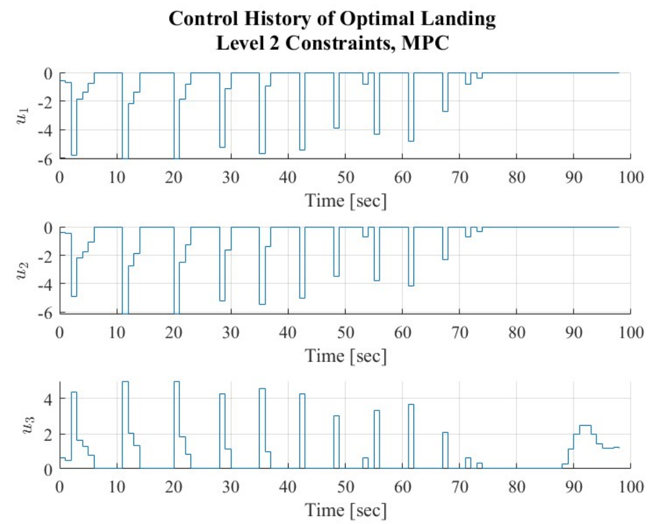
$$\dot{\boldsymbol{\lambda}} = -\begin{bmatrix} 0 & \sin\gamma & v\cos\gamma \\ -\frac{D_r}{m} + \frac{2\mu}{r^3}\sin\gamma & -\frac{D_v}{m} & -\frac{\mu}{r^2}\cos\gamma \\ \left(\frac{2\mu}{vr^3} - \frac{v}{r^2}\right)\cos\gamma + \frac{L_r}{mv}\cos\delta & \left(\frac{\mu}{v^2r^2} + \frac{1}{r}\right)\cos\gamma - \frac{L}{mv^2}\cos\delta + \frac{L_v}{mv}\cos\delta & \frac{1}{v}\left(\frac{\mu}{r^2} - \frac{v^2}{r}\right)\sin\gamma \end{bmatrix}^T \boldsymbol{\lambda}$$

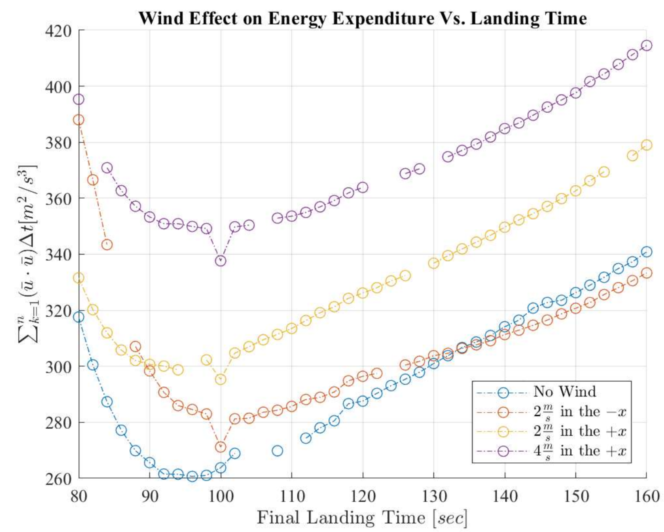
Convex Optimization of Dragonfly Landing on Titan

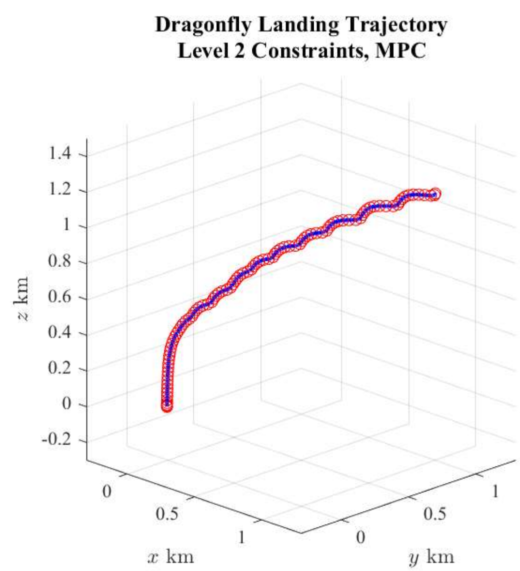
Charlotte Bennett, Brandon Castillo

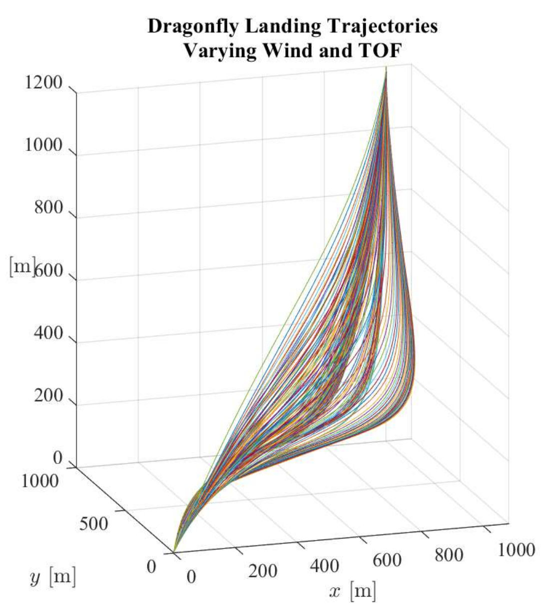
Goal: Design optimal entry trajectory for Dragonfly lander via convex optimization

- **Titan: Target Moon**
  - Low Gravity
  - Thick Atmosphere
- **Dragonfly Lander:**
  - Quad-copter craft design
  - Trajectory covers mid-air release to ground
- **Relevant constraints:**
  - Floor collision
  - Control pointing direction
  - Drag
  - Atmospheric Wind
  - Velocity pointing direction
- **Results:**
  - Demonstrated influence of drag on optimal trajectory energy









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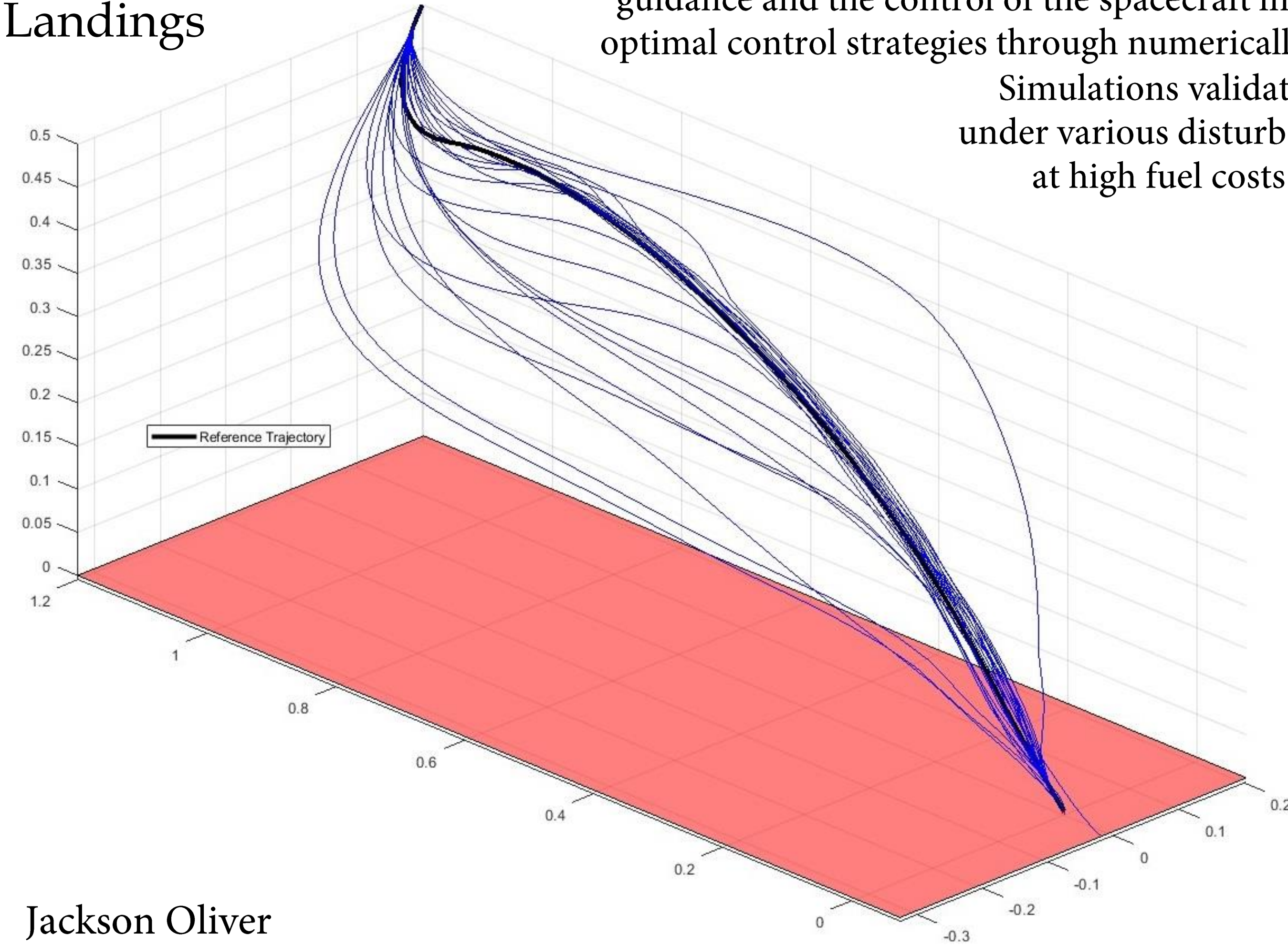
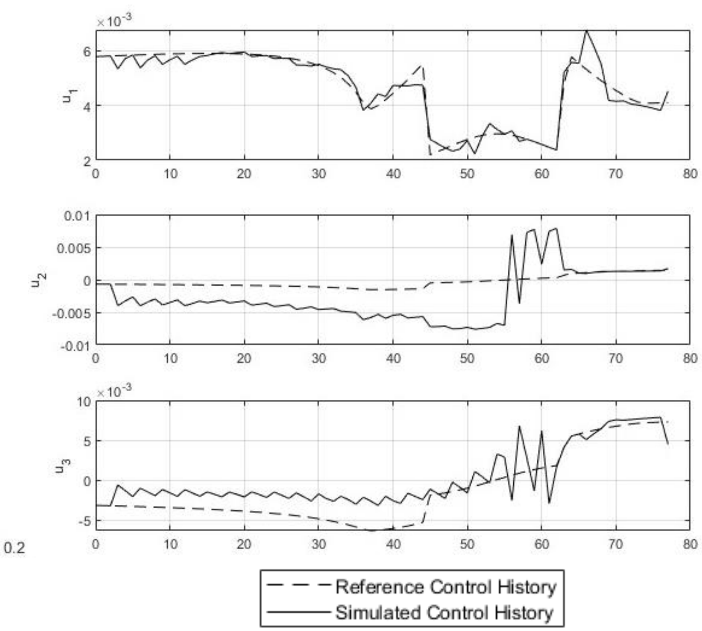
Model Predictive Control for Propulsive Landings

A model predictive controller is developed, simulated, and validated, to support retro-propulsive landing of spacecraft in disturbed environments. This controller separates the guidance and the control of the spacecraft into two separate controllers, both develop optimal control strategies through numerically solving convex optimization problems. Simulations validate successful achievement of soft landings under various disturbances using the developed controller, but at high fuel costs and often through difficult to unrealistic control actuations.





Jackson Oliver

MPC FOR DIRECT TRANSFERS FROM LPO TO THE MOON SURFACE

Author: Moacir Fonseca Becker

OBJECTIVE

The project aims to establish a Model Predictive Control (MPC) framework for a direct transfer from a Halo orbit to any point on the Moon's surface, using the Circular Restricted Three Body Problem (CR3BP) model. The primary objective is focused on implementing a Convex MPC to simulate direct transfers to the lunar surface from periodic Libration Points Orbits (LPOs). The secondary goal of the project is the integration of real-time data adaptability to account for perturbations and test thereby the controller's responsiveness to uncertainties.

APPROACH

The convex MPC uses a linearized version of the CR3BP dynamics to perform a prediction its future states over a finite-horizon and to solve an optimization problem in discrete time. The cost function used encompasses both state deviation over the predicted state with respect to a reference trajectory and the sum of the predicted control inputs. The optimization problem is run until the spacecraft reaches the desired target within 1e-4 of the norm of the final desired state vector, or until max simulation time is reached. A rotating hyperplane collision constraint is implemented conditionally until some threshold distance during the simulation to avoid over constraining the problem initially.

Constraints
Control input, start/end state of prediction window and collision

Reference Trajectories
Pontryagin's min time and via differential corrections

MPC Formulation

$$
\begin{aligned}
\underset{\delta x, u_k}{\text{minimize}} \quad & \frac{1}{2}\delta x_N^{\top} Q_f \delta x_N + \sum_{i=0}^{N-1}\left(\delta x_i^{\top} Q \delta x_i + u_{ki}^{\top} K u_{ki}\right) \\
\text{subject to} \quad & x_{k+1} = A_k x_k + B_k u_k + c_k, \quad i = 1,\dots,N, \\
& x(t_0) = [x_0, y_0, \dot{x}_0, \dot{y}_0, \dot{z}_0]^{\top}, \\
& x(t_f) = [x_f, y_f, \dot{x}_f, \dot{y}_f, \dot{z}_f]^{\top}, \\
& \|u_{ki}\|_2 \le u_{\max}, \\
& \eta_k^{\top}(r_{23,k} - r_{0,k}) \ge 0, \quad i = 1,\dots,N, \quad \text{(convex)}.
\end{aligned}
$$

Cost function
Predicted states deviation and control inputs

Dynamics Model
CR3BP

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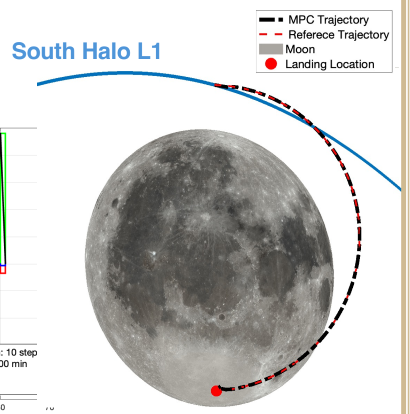
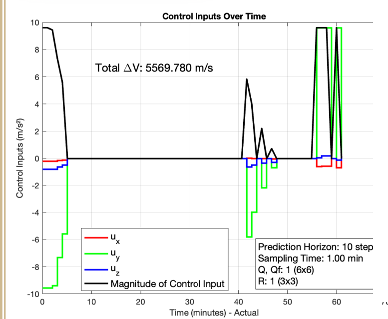
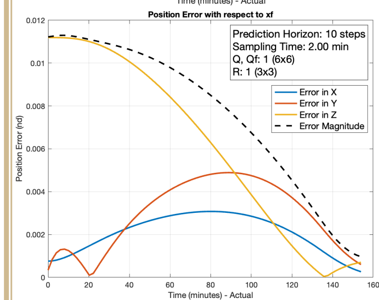
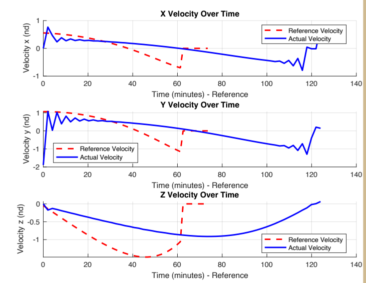
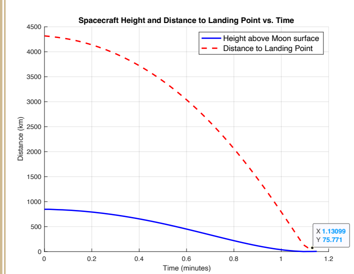
RESULTS

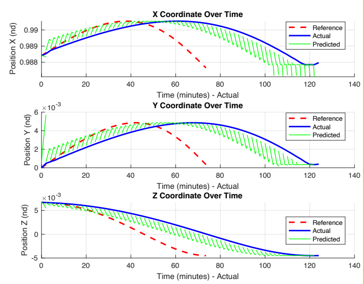












Total ΔV	ToF
MPC = 5.6 km/s	MPC = 60 min
Ref = 4.1049 km/s	Ref = 15 min

The S/C shows coasting phases where the control input is zero. This is indicative of a good following of the reference trajectory. The ΔV values and ToF, albeit higher than the reference, are not infeasible. Also, the prediction is adequately follows the actual trajectory of the S/C. The distance to the landing point decreases monotonically during the entire transfer.

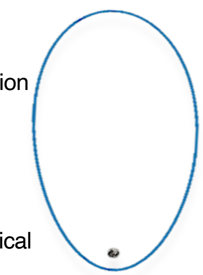
DISCUSSION

The MPC, hereby formulated with a conditional activation of the collision constraint, is shown to be capable of closely following the bi-impulsive reference trajectory as long as the penalty on the control matrix is not too high.

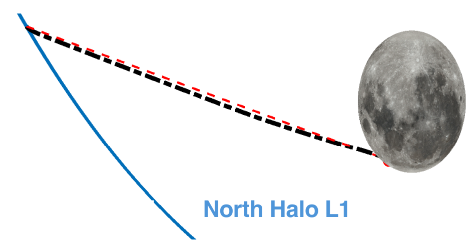
Given that, in general, MPC controllers are better at following or staying nearby references rather than generating trajectories, it becomes crucial to pay special attention to the trajectory generation methodology in the CR3BP problem. Pontryagin's or similar methods can be explored for trajectory design, which adds an additional layer of complexity. As long as the reference trajectory designed has been optimized according to the desired parameters, the MPC controller will do a good job of following the reference trajectory even under a highly non-linear environment, as can be seen in these results. On the other hand, when a reference trajectory is not available, the MPC might be able to find its own reference trajectory, provided it is close enough to the desired landing point.

The *prediction horizon* and the *sampling time* of the controller are parameters that are crucial to pay attention to. When the spacecraft is farther from the Moon, a higher sampling time (e.g., lower time discretization resolution) might be acceptable. As the spacecraft approach the near landing stage, a finer time resolution becomes necessary to react to the fast changing dynamics and low time. It's important to strike the right balance between the prediction horizon and the sampling time. Adaptability on these parameters might be worth exploring as well.

Other parameters that are important to pay attention to are the control and state deviation *penalty weights* for the convex optimization problem. More research is required to find the right heuristic when defining then.



Overall, the MPC here developed shows promising results that require further analytical work and iterations to be optimized.



Sample of another trajectory

Dynamically-Informed Optimal Low-Thrust Transfers Between Earth-Moon Libration Point Orbits

Drew Langford*, Juan-Pablo Almanza-Soto*
*Ph.D. Student, School of Aeronautics and Astronautics, Purdue University

Introduction

Gravitational dynamics of Earth-Moon system are highly non-linear and sensitive in the vicinity of Lagrange points. Recent literature regarding cislunar trajectory design split between distinct approaches: optimization and dynamical systems theory. Despite recent advances, there remains a knowledge gap regarding how mission designers can leverage dynamical information to inform the optimization process.

Project Outline and Framework

Goal: construct fuel-optimal transfers between L1 and L2 Lyapunov orbits of different energy levels

Key Insight

The costate history of a ballistic connection between Lyapunov orbits of the same energy is zero

Leading question

Do initial costates associated with optimal trajectories evolve smoothly as the energy of the arrival Lyapunov orbit is increased?

Energy- Optimal Control Problem

$$\min_{\mathbf{u}} \int_{t_0}^{t_f} ||\mathbf{u_2}||^2 dt$$

$$\text{subj. to } \dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}, \mathbf{t})$$

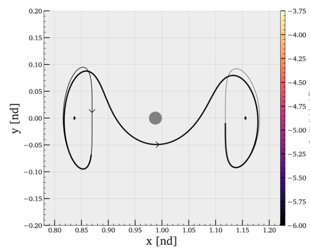
$$\mathbf{r(t_0)} = \mathbf{r_0} \quad \mathbf{v(t_0)} = \mathbf{v_0}$$

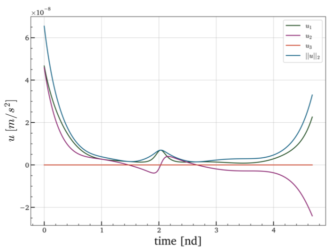
$$\mathbf{r(t_f)} = \mathbf{r_f} \quad \mathbf{v(t_f)} = \mathbf{v_f}$$

$$\mathbf{x} \in \mathbb{R}^4 \quad \mathbf{u} \in \mathbb{R}^2$$

Initial Guess Generation

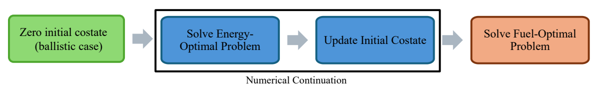
The stable and unstable invariant manifolds associated with the departure L_1 and arrival L_2 Lyapunov orbits define the state initial conditions in each case. Mixed-time multiple shooting enables the exploitation of dynamical information on both the departure and arrival legs of each transfer.

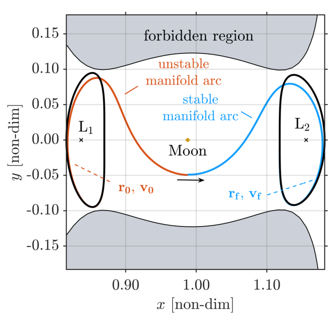


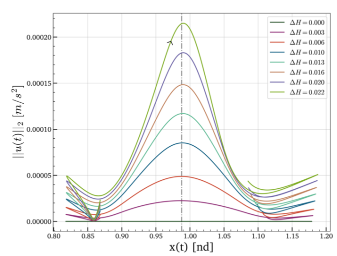


Methodology

Leverage numerical continuation in initial costates to enable construction of optimal transfers between orbits of increasingly different energies. Utilize energy-optimal solutions in computing more sensitive fuel-optimal trajectories.

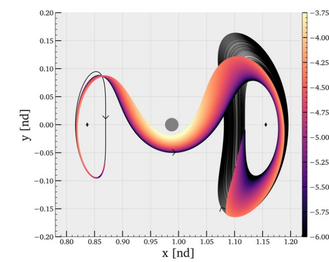


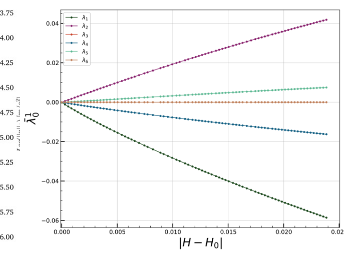




Results

A continuum of energy-optimal trajectories between L_1 and L_2 Lagrange points of increasingly different energies is computed via numerical continuation.

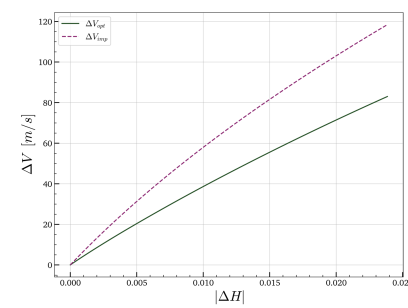




Analysis and Discussion

The initial costates associated with optimal transfers evolve smoothly from zero as the energy difference of the Lyapunov orbits is increased. The corresponding control history of each transfer illustrates that the largest burns are performed near the Moon. It follows that the optimal control leverages the gravitational well to reduce the value of the cost index.

Here, low-thrust optimal control outperforms the best case minimum impulsive delta-v. Note that velocity change is smooth across the continuation, reflecting the smooth deformation of the costate history for each transfer.



Launch Vehicle Propulsive Landings via Sequential Convex Programming

Objective

- Fuel-optimal landing trajectory of launch vehicle propulsive landing
- Falcon booster landing as motivating example
- Considering free final time, grid fin and thrust vectoring control, planar motion and attitude dynamics

Methodology

- Sequential convex programming
- Discretize state and control over timespan; compute optimal trajectory satisfying dynamics and state and control constraints
- To address nonlinear dynamics, repeatedly linearize dynamics about reference trajectory

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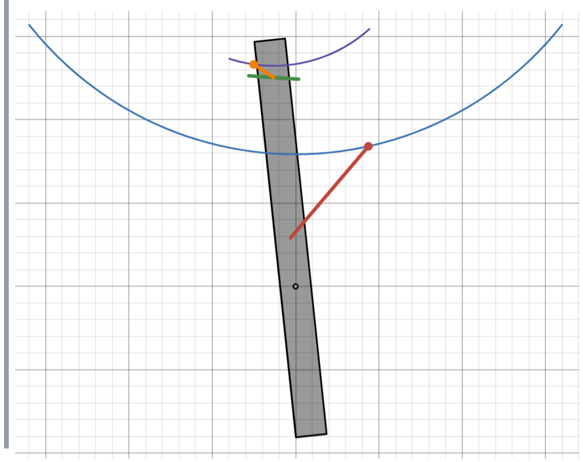
Aerodynamics model

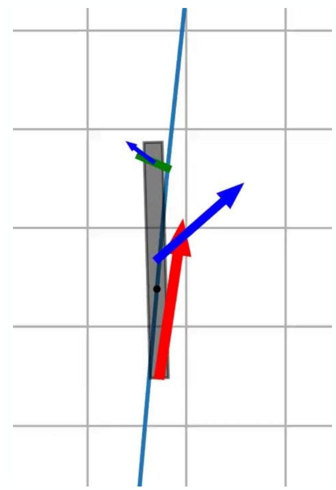
- Grid fin and body aerodynamics modelled with aerodynamic coefficient matrices
- Convexifiable framework
- Models both lift and drag



Model of body aerodynamics (red vector, blue locus) and grid fin aerodynamics (orange vector, purple locus) in inertial frame with 6° angle of attack and -10° grid fin deflection. Relative wind is directly upward.

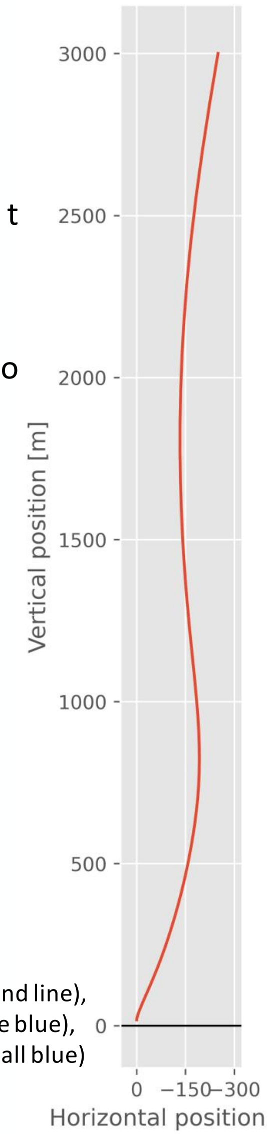
Results

- 6.5 t propellant consumed, of 16.3 t available
- Flight time: 30.94 sec
- Simulation convergence sensitive to problem parameters



Above: vehicle model with trajectory (blue background line), thrust vector (red), body aerodynamics vector (large blue), grid fin (green) and grid fin aerodynamics vector (small blue)

Right: scale view of computed optimal trajectory



MPC for Spacecraft Rendezvous with Collision Avoidance
By: Veronica Rankowicz

Objectives:

- Level 1: MPC model for spacecraft rendezvous optimal control problem
- Level 2: Collision avoidance of one stationary obstacle in 2D space
- Level 2: Collision avoidance of one moving obstacle in 2D space
- Level 3: Collision avoidance of one moving obstacle in 3D space

Approach:

$$\min_{\vec{x}_k,\vec{u}_k} \sum_{k=0}^{N-1} \vec{x}_k^T Q \vec{x}_k + \sum_{k=0}^{N-1} \vec{u}_k^T S \vec{u}_k + \vec{x}_N^T Q_N \vec{x}_N$$

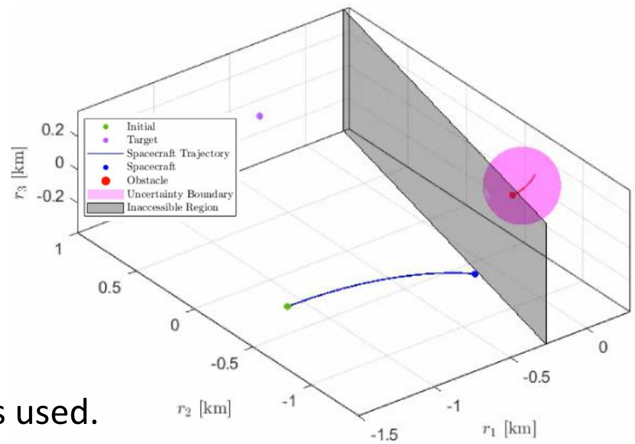
s. t.

$$\vec{x}_{k+1} = A_k \vec{x}_k + B_k \vec{u}_k \quad \leftarrow \text{Equations of motion}$$

$$\|\vec{u}_i(t)\| \leq u_{max} \quad \leftarrow \text{Maximum thrust}$$

$$R_k \vec{y}_k \leq M_k \vec{x}_k + \vec{b}_k \quad \leftarrow \text{Collision avoidance}$$

$$\left.\begin{array}{l} \vec{r}(t_0) = \vec{r}_0, \quad \vec{v}(t_0) = \vec{v}_0 \\ \vec{r}(t_f) = \vec{r}_f \end{array}\right\} \leftarrow \text{Boundary constraints}$$

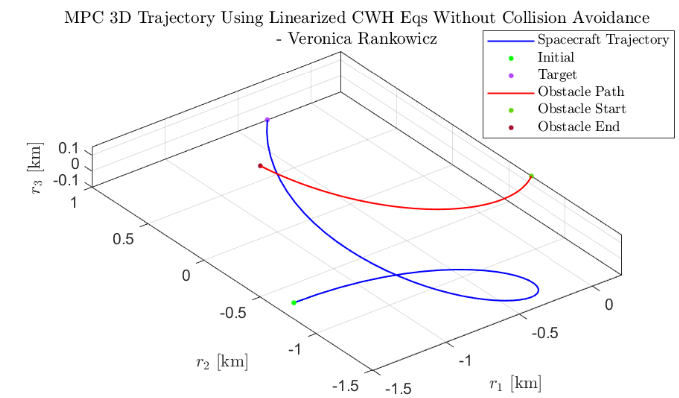


A single rotating hyperplane technique was used.

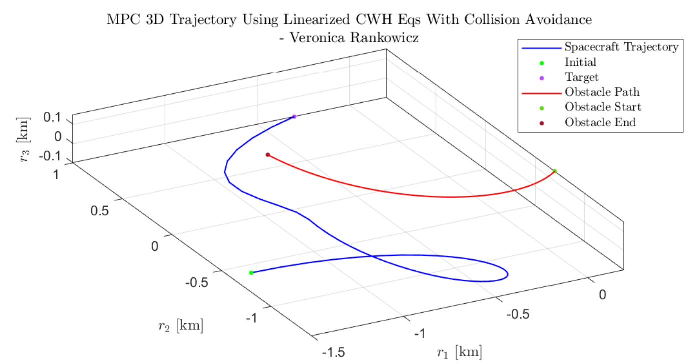
Discussion:

Using a single rotating hyperplane in 2D and 3D space worked effectively to prevent the spacecraft from colliding into the obstacle on its path to the target position as the hyperplane successfully guided the spacecraft around the obstacle and to the target.

3D Moving Obstacle Results



No Collision Avoidance Applied



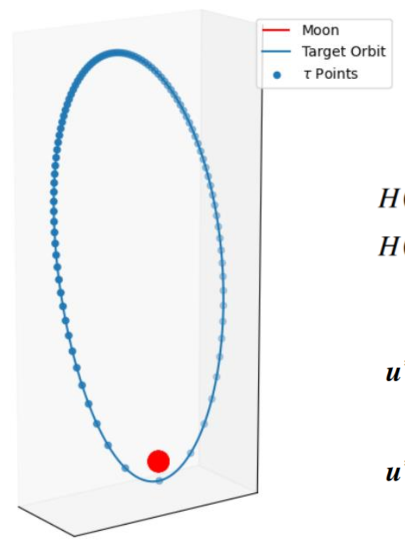
Collision Avoidance Applied

Objective and Methods

- Increased interest in periodic three-body orbits in cislunar space
- Prevalence of low-thrust propulsion
- Objective is to find low-thrust transfers between these orbits with minimum fuel
- Pontryagin's maximum principle used with continuation to find families of orbits

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{f}_0(\boldsymbol{x}) + B\boldsymbol{u}$$

$$f_0(\boldsymbol{x}) = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \frac{-(1-\mu)(x+\mu)}{d^3} - \frac{\mu(x-1+\mu)}{r^3} + 2n\dot{y} + n^2 x \\ \frac{-(1-\mu)y}{d^3} - \frac{\mu y}{r^3} - 2n\dot{x} + n^2 y \\ \frac{-(1-\mu)z}{d^3} - \frac{\mu z}{r^3} \end{bmatrix}, \quad B = \begin{bmatrix} 0_{3\times 3} \\ I_3 \end{bmatrix}$$



$$L(\boldsymbol{u}) = \|\boldsymbol{u}\|_2^2, \quad \text{(minimum energy)}$$
$$L(\boldsymbol{u}) = \|\boldsymbol{u}\|_2, \quad \text{(minimum fuel)}$$

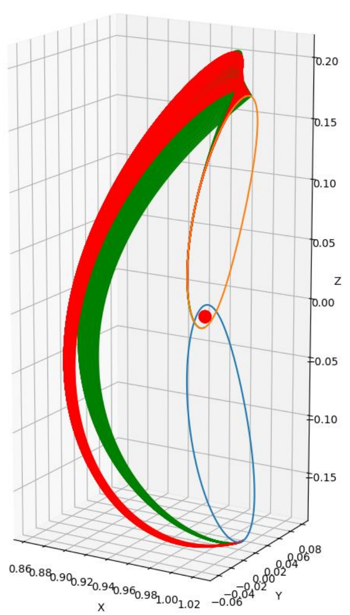
$$\boldsymbol{u}^* = \arg\min_{\boldsymbol{u}\in\mathcal{U}} H(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\lambda})$$

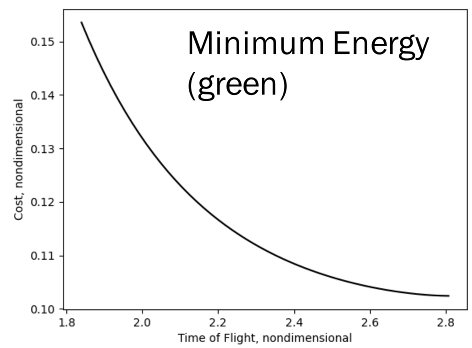
$$H(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\lambda}) = \|\boldsymbol{u}\|_2^2 + \boldsymbol{\lambda}^T(\boldsymbol{f}_0(\boldsymbol{x}) + B\boldsymbol{u}), \quad \text{(minimum energy)}$$
$$H(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\lambda}) = \|\boldsymbol{u}\|_2 + \boldsymbol{\lambda}^T(\boldsymbol{f}_0(\boldsymbol{x}) + B\boldsymbol{u}), \quad \text{(minimum fuel)}$$

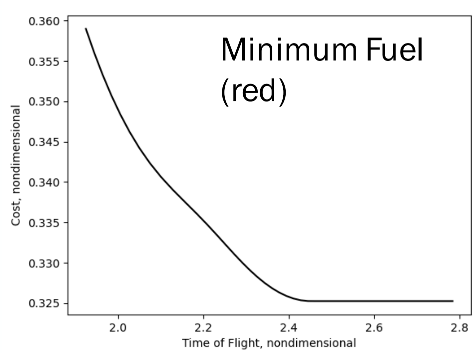
$$\boldsymbol{u}^* = \begin{cases} \frac{p}{2}, & \|p\|_2 \le 2u_{\max} \\ \frac{p}{\|p\|_2} u_{\max}, & \|p\|_2 > 2u_{\max} \end{cases} \quad \text{(minimum energy)}$$

$$\boldsymbol{u}^* = \begin{cases} 0, & \|p\|_2 \le 1 \\ \frac{p}{\|p\|_2} u_{\max}, & \|p\|_2 > 1 \end{cases} \quad \text{(minimum fuel)}$$

Results

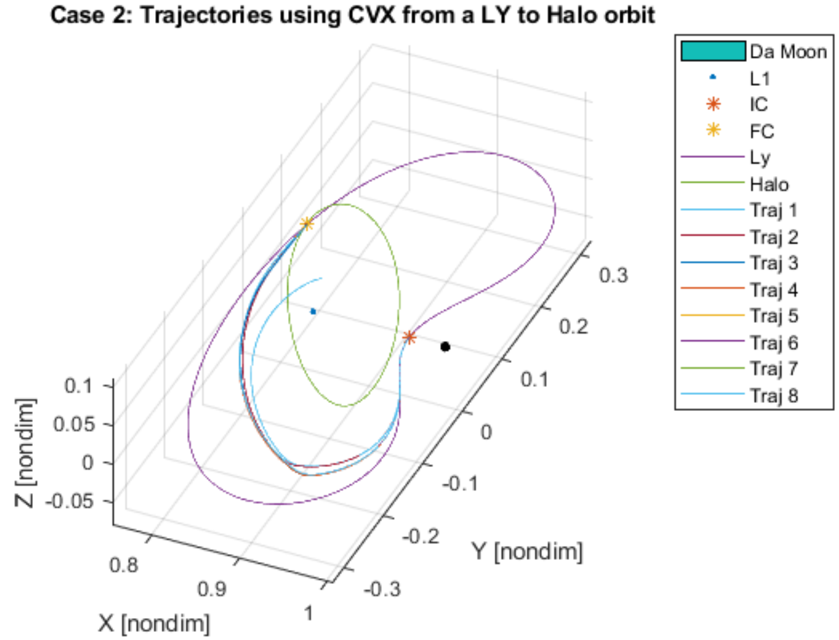


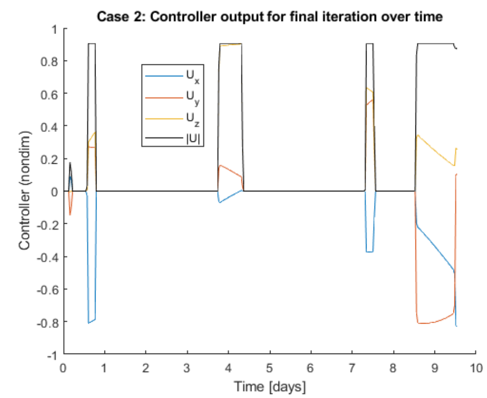


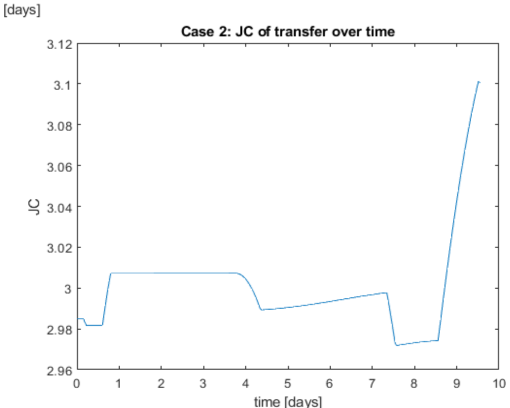


Discussion

- Continuation is highly successful in obtaining low-thrust, minimum fuel transfers
- Longer times of flight decreases cost, but has diminishing gains with minimum energy, and reaches a minimum with minimum fuel
- Bang-bang profile successfully attained for minimum fuel solutions
- Numerical difficulty with continuation when changing problem too much, so smaller steps are advantageous







Lyapunov and Halo Transfers in the Earth-Moon L1 via Sequential Convex Programming

By Steven Allen Williams Jr. and Nathaniel Phillip Sailor

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