PURDUE UNIVERSITY | School of Nuclear Engineering

Graduate Research Seminar

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Wednesday, January 17, 2024

3:30pm | PHYS 112

Application of Symmetries for Calculating Space-Charge-Limited Current

Abstract

Many fundamental problems based on deceptively simple differential equations are not amenable to analytic solutions, necessitating numerical approaches that may be computationally expensive and hinder the derivation of scaling laws that may provide important physical insight. This seminar reports methods for solving such problems by leveraging geometric principles and symmetries and applies them to general problems in electrostatics, focusing on space-charge limited current (SCLC), which is the maximum current that may be emitted into a diode. SCLC is a critical quantity for numerous applications, including fusion, high-power microwave, thermionic converters, and semiconductors.

We first present a simple problem from basic electric circuit theory to demonstrate that Kirchhoff's circuit laws are incomplete without a symmetry axiom. We next apply conformal mapping to solve for SCLC in various one-dimensional (1D) geometries. Gauss's *Theorema Egregium* (the remarkable theorem) of differential geometry is used to show that the conformal map of a hyperboloid tip onto a plane creates a non-Euclidean disk (Poincaré disk). Identifying nontrivial symmetries using Lie symmetries allowed us to solve for SCLC with monoenergetic emission of electrons from the cathode for any orthogonal 1D geometry. We next apply this knowledge to derive a general equation for SCLC that is true for any multidimensional diode as long as one can calculate the vacuum capacitance for many geometries. The exact closed form solution for a 2D planar diode has elliptical integrals because of the elliptical fringing fields that form on the electrode edges, which increase the current densities at those locations and damage the cathodes, as demonstrated experimentally by the Air Force. Particle-in-cell simulations agreed with the theoretical predictions in 2D diode with a maximum error of 13% for field emission and 7% for monoenergetic emission. Future work will assess relativistic diodes, non-vacuum conditions, and complicated cathode geometries.

Harsha received the B. E. degree in electrical engineering from the Rashtreeya Vidyalaya College of Engineering, India, in 2015, and the M.Sc. degree in nuclear science and engineering from the University of Bristol, U.K., in 2017. For his master's thesis, he worked with Culham Center for Fusion Energy (CCFE) to develop novel techniques to measure tritium for ITER. He is currently pursuing the Ph.D. degree in nuclear engineering with Prof. Garner in the BioElectrics and ElectroPhysics Laboratory with a focus on developing mathematical techniques to analyze space-charge limited current in various geometries of practical interest.

He is the principal author of the book The Foundations of Electric Circuit Theory published by the IOP Publishing Ltd., in 2016. He is a graduate student member of the Nuclear and Plasma Sciences Society (NPSS) of IEEE.