Design and Modeling of Fluid Power Systems
ME 597/ABE 591  Lecture 4

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Displacement machines – design principles & scaling laws

Power density comparison between hydrostatic and electric machines

Volumetric losses, effective flow, flow ripple, flow pulsation

Steady state characteristics of an ideal and real displacement machine

Torque losses, torque efficiency
Historical Background

Hydrostatic transmission

Archimedes

Pascal

Bramah

230

0

1500 1600 1700 1800 1900 2000

Kepler

Ramelli

Vane pump

Gear pump

Axial Piston Pump

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Displacement machine

due to compressibility of a real fluid

\[ V_{\text{min}} = V_T \quad \text{with} \quad V_T \quad \text{dead volume} \]

Adiabatic expansion

Adiabatic compression

K_A, adiabatic bulk modulus
Displacement machine
due to viscosity & compressibility of a real fluid

Pressure drop between displacement chamber and port

Port pressure

Pressure in displacement chamber

Pump

Motor
Power Density

Electric Motor

\[ F_e = I \cdot B \cdot L \cdot \sin \alpha \]

with \( I \) current [A]

\( B \) … magnetic flux density [T] or [Vs/m²]

Hydraulic Motor

\[ F_h = p \cdot L \cdot h \]

\[ T = I \cdot B \cdot L \cdot r \cdot \sin \alpha \]

I = J \cdot b \cdot h

J … current density [A/m²]

\( r \) … radius [m]

\( h \) … height [m]

\( b \) … width [m]

\( \alpha \) … angle [°]

\( \omega \) … angular speed [rad/s]

\( p \) … pressure [Pa]

\( L \) … length [m]

\( r \) … radius [m]

\( h \) … height [m]

Torque:

\[ T = p \cdot L \cdot h \cdot r \]
Example

Power: \( P = T \omega = T 2\pi n \)

For electric motor follows: \( P = I B L r 2\pi n \) assuming \( \alpha = 90^\circ \)

For hydraulic motor follows: \( P = p \cdot L \cdot h \cdot r \cdot 2\pi \cdot n \)

Force density:

- **Electric Motor**

\[
\frac{F_e}{L \cdot h} = \frac{J \cdot b \cdot h \cdot B \cdot L}{L \cdot h} = J \cdot b \cdot B
\]

\[
7.6 \cdot 10^6 \text{ A} \cdot \text{m}^2 \cdot 1.8 \text{Vs} \cdot \text{m}^{-3} \cdot 3 \cdot 10^{-3} \text{ m} = 4.1 \cdot 10^4 \text{ Pa}
\]

- **Hydraulic Motor**

\[
\frac{F_h}{L \cdot h} = p
\]

\[
\text{up to } 5 \cdot 10^7 \text{ Pa}
\]

with a cross section area of conductor: \( 9 \cdot 10^{-6} \text{ m}^2 \)
Mass / Power Ratio

Electric Machine

Positive displacement machine

\[
\frac{\text{mass}}{\text{power}} = 1 \ldots 15 \text{ kg/kW} \quad \quad 0.1 \ldots 1 \text{ kg/kW}
\]

Positive displacement machines (pumps & motors) are:

- 10 times lighter
- min. 10 times smaller
- much smaller mass moment of inertia (approx. 70 times)

much better dynamic behavior of displacement machines
Displacement Machines

- Piston Machines
  - Axial Piston Machines
  - In-line Piston Machines
  - Radial Piston Machines

- Gear Machines
  - External Gear
  - Internal Gear
  - Annual Gear

- Vane Machines

- Swash Plate Machines
  - Bent Axis machines
    - with external piston support
    - with internal piston support

- Screw Machines

- Others

Fixed displacement machines

Variable displacement machines

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Axial Piston Pumps

- Cylinder block
- Pitch radius R
- Outlet
- Inlet
- Swash plate
- Piston
- Valve plate (distributor)
- Cylinder block

- Variable displacement pump
- Requires continuous change of $\beta$
- Piston stroke $= f(\beta, R)$

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Bent Axis & Swash Plate Machines

Torque generation on cylinder block

Torque generation on "swash plate"

Swash plate design

Radial force $F_R$ exerted on piston!

Driving flange must cover radial force

Bent axis machines
Axial Piston Pumps

Openings in cylinder bottom

In case of plane valve plate

In case of spherical valve plate
Axial Piston Pumps

Connection of displacement chambers with suction and pressure port

Plane valve plate

Inlet opening  Outlet opening

Inlet  Outlet

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Axial Piston Pumps

Kinematic reversal: pump with rotating swash plate

Check valves fulfill distributor function

Suction valve

Pressure valve for each cylinder

Outlet

Inlet

can only work as pump
Comparison of axial piston pumps

- Simple and compact design
- Short response time, high bandwidth
- Through going shaft
- Long service life, low loaded bearings
- Limited swash plate angle $\beta_{\text{max}}$ ca. 21°
- High radial piston forces

- higher max. speed
- Angle $\beta$ up to 45°
- Less losses
- High loaded bearings
- Expensive design
- Synchronisation required
Steady state characteristics
ideal displacement machine

Displacement volume of a variable displacement machine:

\[ V = \alpha \, V_{\text{max}} \]
Scaling laws

The pump size is determined by the displacement volume \( V \) [cm\(^3\)/rev]. Usually a proportional scaling law, conserving geometric similarity, is applied, resulting in stresses remaining constant for all sizes of units.

\[
T = \frac{\Delta p \cdot V}{2 \cdot \pi} \quad Q = V \cdot n \quad \Delta p = p_2 - p_1
\]

First Order Scaling Laws: \( \lambda \) ... linear scaling factor

- \( L = \lambda \cdot L_0 \)
- \( T = \lambda^3 \cdot T_0 \)
- \( P = \lambda^2 \cdot P_0 \)
- \( V_i = \lambda^3 \cdot V_{i0} \)
- \( m = \lambda^3 \cdot m_0 \)
- \( n = \lambda^{-1} \cdot n_0 \)

Assuming same maximal operating pressures for all unit sizes and a constant maximal sliding velocity!
Example

The maximal shaft speed of a given pump is 5000 rpm. The displacement volume of this pump is $V = 40 \text{cm}^3/\text{rev}$. The maximal working pressure is given with $40 \text{ MPa}$. Using first order scaling laws, determine:
- the maximal shaft speed of a pump with $90 \text{ cm}^3/\text{rev}$
- the torque of this larger pump
- the maximal volume flow rate of this larger pump
- the power of this larger pump

For the linear scaling factor follows: 
$$\lambda = \sqrt[3]{\frac{V}{V_0}} = \sqrt[3]{\frac{90}{40}} = 1.31$$

Maximal shaft speed of the larger pump:  
$$n = \lambda^{-1} \cdot n_0 = 1.31^{-1} \cdot 5000 \text{ rpm} = 3816.8 \text{ rpm}$$

Torque of the larger pump:  
$$T = \frac{\Delta p \cdot V}{2 \cdot \pi} = \frac{40 \cdot 10^6 \text{Pa} \cdot 90 \cdot 10^{-6} \text{m}^3}{2 \cdot \pi} = 573.25 \text{ Nm}$$

Maximal volume flow rate:  
$$Q_{\text{max}} = V \cdot n_{\text{max}} = 90 \cdot 10^{-6} \text{ m}^3/\text{rev} \cdot 3816.8 \text{ rpm} = 0.3435 \text{ m}^3/\text{min} = 343.51/\text{min}$$

Power of the larger pump:  
$$P = \Delta p \cdot Q = 40 \cdot 10^6 \text{Pa} \cdot 0.3435 \text{ m}^3 \cdot \frac{1}{60} \text{ s}^{-1} = 229 \text{ kW}$$
**Real Displacement Machine**

![Diagram of a real displacement machine with labels for different parts: Cylinder, Distributor, Inlet, Outlet, Piston, and arrows indicating flow directions and pressures.]

**Effective Flow rate:**

\[ Q_e = \alpha V_{max} n - Q_s \]

**Effective torque:**

\[ T_e = \frac{\Delta p \cdot \alpha \cdot V_{max}}{2 \cdot \pi} + T_s \]

\[ \Delta p = p_2 - p_1 \]

- **Q_{Se}**: external volumetric losses
- **Q_{Si}**: internal volumetric losses
- **Q_s**: volumetric losses
- **T_s**: torque losses
Volumetric Losses

\[ Q_S = \sum_{i=1}^{n} Q_{Sei} + \sum_{j=1}^{m} Q_{Sij} + Q_{SK} + Q_{Sf} \]

- \( \text{external volumetric losses} \)
- \( \text{internal volumetric losses} \)
- \( \text{losses due to incomplete filling} \)
- \( \text{losses due to compressibility} \)

\( Q_{SL} \) external and internal volumetric losses = flow through laminar resistances:

\[ Q_{SL} = C_{\mu} \cdot \frac{\Delta p}{\mu} \]

Assuming const. gap height

\[ \mu = f(\theta, p) \]

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Volumetric Losses

Effective volume flow rate is reduced due to compressibility of the fluid

\[
\int_{B}^{C} \frac{dV}{V} = \int_{B}^{C} - \frac{1}{K_A} \, dp
\]

\[
\ln V_C - \ln V_B = - \frac{1}{K_A} (p_C - p_B)
\]

\[
\Delta V_B = V_B \left(1 - e^{-\frac{1}{K_A} (p_C - p_B)}\right)
\]

simplified

\[
\Delta V_B = V_B \frac{\Delta p}{K_A}
\]

\[
Q_{SK} = n \, \Delta V_B
\]

with \(n\) … pump speed
Steady state characteristics of a real displacement machine \( Q_i = V \ n = \alpha \ V_{\text{max}} \ n \)

Effective volumetric flow rate \( Q_e = Q_i - Q_S \)

\[ Q_S = f(\Delta p, n, V, \theta) \]  
\( \theta \) ... temperature
Steady state characteristics

Effective mass flow at pump outlet $Q_{me}$

Loss component due to compressibility does not occur!
Instantaneous Pump Flow

Instantaneous volumetric flow $Q_a$

$$Q_a = \frac{dV}{dt} = f(\varphi)$$

Volumetric flow displaced by a displacement chamber

$$Q_{ai} = f(\varphi_i)$$

The instantaneous volumetric flow is given by the sum of instantaneous flows $Q_{ai}$ of each displacement element:

$$Q_a = \sum_{i=1}^{k} Q_{ai}$$

$k$ … number of displacement chambers, decreasing their volume, i.e. being in the delivery stroke

$z$ is an **even** number \quad $k = \frac{z}{2}$

$z$ is an **odd** number \quad $k = \frac{z}{2} + 0.5$ \quad or \quad $k = \frac{z}{2} - 0.5$

Flow pulsation of pumps \quad Pressure pulsation
Flow pulsation

Non-uniformity grade of volumetric flow is defined:

\[ \delta_Q = \frac{Q_{\text{max}} - Q_{\text{min}}}{Q_{\text{mi}}} \]

\[ Q_{\text{mi}} = \frac{Q_{\text{max}} + Q_{\text{min}}}{2} \]

\[ \delta_Q = 2 \frac{Q_{\text{max}} - Q_{\text{min}}}{Q_{\text{max}} + Q_{\text{min}}} \]
Torque Losses

\[ T_s = T_{S\mu} + T_{Sp} + T_{Sp} + T_{Sc} \quad \rightarrow \quad \text{constant value} \]

Torque loss due to viscous friction in gaps (laminar flow)

\[ T_{S\mu} = k_{T\mu} \cdot \frac{\mu}{h} \cdot n = C_{T\mu} \cdot \mu \cdot n \]

\[ h \ldots \text{gap height} \]

Torque loss to overcome pressure drop caused in turbulent resistances

\[ T_{Sp} = C_{Tp} \cdot \rho \cdot n^2 \]

\[ \Delta p_s = \frac{\lambda}{d} \cdot \rho \cdot \frac{V^2}{2} + \xi \cdot \rho \cdot \frac{V^2}{2} \]

Torque loss linear dependent on pressure

\[ T_{Sp} = C_{Tp} \cdot \Delta p \]

\[ \xi \ldots \text{drag coefficient} \]

\[ \lambda \ldots \text{flow resistance coefficient} \]

\[ \lambda_{\text{turbulent}} = \frac{0.3164}{\sqrt[4]{Re}} \]

\[ T_e = \frac{\Delta p \cdot \alpha \cdot V_{\text{max}}}{2 \cdot \pi} + T_s \]

effective torque required at pump shaft
Steady state characteristics

Torque losses

of a real displacement machine

\[ T_s = f(n, \Delta p, V, \theta) \]
Steady state characteristics

Effective Torque

\[ T_e = T_i + T_s = \frac{\Delta p \cdot \alpha \cdot V_{\text{max}}}{2 \cdot \pi} + T_s \]

Effective torque \( T_e \)

\[ T_S = f(\Delta p, n, V, \theta) \]

\( \Delta p = \text{const} \)

\( n = \text{const} \)
Axial Piston Machine

Kinematics

Piston displacement: \( s_p = -z \)

\[ s_p = -R \cdot \tan \beta \cdot (1 - \cos \varphi) \]

Piston stroke:

\[ H_p = 2 \cdot R \cdot \tan \beta \]

\( R \) … pitch radius

Outer dead point AT

\( \varphi = 0 \)

Inner dead point IT

\[ z = b \cdot \tan \beta \]
\[ b = R - y \]
\[ y = R \cdot \cos \varphi \]
Kinematic Parameters

Piston velocity in z-direction:

\[ v_P = \frac{ds_P}{dt} = \frac{ds_P}{d\phi} \cdot \frac{d\phi}{dt} = -\omega \cdot R \cdot \tan \beta \cdot \sin \varphi \]

Piston acceleration in z-direction:

\[ a_P = \frac{dv_P}{dt} = \frac{dv_P}{d\phi} \cdot \frac{d\phi}{dt} = -\omega^2 \cdot R \cdot \tan \beta \cdot \cos \varphi \]

Circumferential speed

\[ v_u = R \cdot \omega \]

Centrifugal acceleration:

\[ a_u = R \cdot \omega^2 \]

Coriolis acceleration \( a_c \) is just zero, as the vector of angular velocity \( \omega \) and the piston velocity \( v_P \) run parallel.
**Instantaneous Volumetric Flow**

Geometric displacement volume:

\[ V_g = z \cdot A_p \cdot H_p \]

\( z \) ... number of pistons

In case of pistons arranged parallel to shaft axis:

\[ V_g = z \cdot \frac{\pi \cdot d_p^2}{2} \cdot R \cdot \tan \beta \]

Geometric flow rate:

\[ Q_g = n \cdot z \cdot \frac{\pi \cdot d_p^2}{2} \cdot R \cdot \tan \beta \]

Mean value over time

Instantaneous volumetric flow:

\[ Q_a = \sum_{i=1}^{k} Q_{ai} \]

\( k \) ... number of pistons, which are in the delivery stroke

with \( Q_{ai} \) \( i \) instantaneous volumetric flow of individual piston

\[ Q_{ai} = f(\varphi_i) \]

\[ v_p = \omega \cdot R \cdot \tan \beta \cdot \sin \varphi \]

\[ Q_{ai} = v_p \cdot A_p = \omega \cdot A_p \cdot R \cdot \tan \beta \cdot \sin \varphi_i \]
Instantaneous Volumetric Flow

In case of even number of pistons: \( k = 0.5 \cdot z \)

In case of odd number of pistons:

\[
k_1 = \frac{z}{2} + 0.5 \quad \text{for} \quad 0 < \varphi \leq \frac{\pi}{z}
\]

and

\[
k_2 = \frac{z}{2} - 0.5 \quad \text{for} \quad \frac{\pi}{z} < \varphi \leq 2 \cdot \frac{\pi}{z}
\]
Flow & Torque Pulsation

kinematic flow and torque pulsation due to a finite number of piston

Flow Pulsation: Non-uniformity grade:

\[ \delta_Q = \frac{Q_{\text{max}} - Q_{\text{min}}}{Q_{mi}} \quad \text{with} \quad Q_{mi} = \frac{Q_{\text{max}} + Q_{\text{min}}}{2} \]

Even number of pistons:

\[ \delta_Q = \frac{\pi}{z} \tan \frac{\pi}{2z} \]

Odd number of pistons:

\[ \delta_Q = \frac{\pi}{2z} \tan \frac{\pi}{4z} \]

Torque Pulsation

\[ \delta_T = \frac{T_{\text{max}} - T_{\text{min}}}{T_{mi}} \quad \text{with} \quad T_{mi} = \frac{T_{\text{max}} + T_{\text{min}}}{2} \]
Flow & Torque Pulsation

kinematic flow and torque pulsation due to a finite number of piston $z$... number of pistons

Non-uniformity

Even number of pistons:  $f = z \cdot n$

Odd number of pistons:  $f = 2 \cdot z \cdot n$

## Flow & Torque Pulsation

Flow and torque pulsation frequency $f$:

<table>
<thead>
<tr>
<th>NUMBER OF PISTONS</th>
<th>NON-UNIFORMITY of FLOW / TORQUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.1403</td>
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<tr>
<td>4</td>
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<td>0.0253</td>
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<th>NON-UNIFORMITY of FLOW / TORQUE</th>
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<td>26</td>
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</tbody>
</table>
Flow Pulsation

Non-uniformity grade:

\[ \delta_Q = \frac{Q_{\text{max}} - Q_{\text{min}}}{Q_{\text{mi}}} \quad \text{with} \quad Q_{\text{mi}} = \frac{Q_{\text{max}} + Q_{\text{min}}}{2} \]

Kinematic non-uniformity grade for piston machines:

<table>
<thead>
<tr>
<th>Number of pistons z</th>
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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-uniformity grade ( \delta )</td>
<td>0.140</td>
<td>0.325</td>
<td>0.049</td>
<td>0.140</td>
<td>0.025</td>
<td>0.078</td>
<td>0.015</td>
<td>0.049</td>
<td>0.010</td>
</tr>
</tbody>
</table>

Volumetric losses \( Q_s = f(\phi) \) and \( Q_s = f(\Delta p, n, V_i, \theta) \)

Flow pulsation of a real displacement machine is much larger than the flow pulsation given by the kinematics.
Flow Pulsation

Flow pulsation leads to pressure pulsation at pump outlet.