ENGINEERING PROFESSIONAL EDUCATION - EXAM COVER SHEET

EXAM INFO

Exam Number: 2  Scheduled Exam Date: Oct. 27, 2017

PROFESSOR (THE FOLLOWING MUST BE COMPLETED BY THE PROFESSOR.)

COURSE: AAE 508  PROFESSOR NAME: Prof. James M. Longuski  PROFESSOR’S PHONE: 765-494-5139

CONDITIONS (INDICATE ONE FOR EACH OF THE FOLLOWING):

1. Closed Textbook, Open Textbook, Take-Home, Other (Explain):  Closed Textbook
2. Notes Allowed:  YES  NO
3. Calculators Allowed:  YES  NO
4. Laptop Allowed:  YES  NO
5. Cell Phones are to be turned off:  YES  NO
6. Time Limit (Number of Minutes and/or Hours):  50 Minutes
7. Other Specifications (crib sheets, special tables, references, etc.) Two crib sheets allowed in original handwriting (no copies), on 8.5 x 11 inch sheets, with writing on both sides allowed. Crib sheets or scans of cribs should be turned in with the exam. Students may keep their original cribs.

PROCTORS - Please remind students to write clearly, press firmly, not to write on the back-side or to the very edge of the page.

EXAM PROCTOR (THE FOLLOWING MUST BE COMPLETED BY THE EXAM PROCTOR.)

CERTIFICATION (CHECK THE FOLLOWING TO INDICATE YOUR AGREEMENT):

☐ I have read and understand the exam regulations.
☐ I have verified the identity of the student(s) by their government issued ID.
☐ This examination has been administered in accordance with the specified regulations and conditions.
☐ This exam has been given on the date specified above, or...
☐ Prior arrangements have been made by student(s) with written approval of professor for different exam date.

STUDENT NAME:

EXAM SITE:

DATE AND TIME (START & STOP) EXAM ADMINISTERED:

SIGNATURE & PRINTED NAME OF PROCTOR:

PROCTOR PHONE & EMAIL:

EXAM PROCTOR COMMENTS TO PROED OR PROFESSOR:

* *IMPORTANT EXAM RETURN INSTRUCTIONS* *

- Scan the completed cover sheet and entire exam into a single PDF file (include ALL numbered pages as well as crib sheets if requested in #7 above). Immediately save the file and upload it to the secure exam portal.
- If unable to scan and upload, contact ProEd for alternate exam return arrangements.
- Retain the completed exam original until the end of the semester, then shred.

QUESTIONS? CONTACT US!

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Rev. 7/10/2014
Instructions:

The exam is worth a total of 100 points. Work as quickly and accurately as you can.

Write your name at the bottom of this page.

DO NOT TURN THE PAGE UNTIL

INSTRUCTED TO DO SO

Note: Partial Credit can only be given for

1. Correct partial steps toward the complete solution, which are

2. Clearly labeled in a logical and systematic manner.

Name _______________________________
I. **Minimum Time to Climb** (35 points)

In the minimum time to climb problem we have

\[ \text{Min } J = t_f \]

Subject to

\[ h_c = P(h_c, V) \]

where \( h_c(0) \) and \( h_c(t_f) \) are given.

The variable \( P \) is excess power \((P = (T - D)V/W)\) and \( h_c \) is the energy height \((h_c = h + \frac{V^2}{2g})\).

Ia. Define the Hamiltonian.

Ib. Apply the Euler-Lagrange and Legendre-Clebsch theorems to find expressions for \( H_v \) and \( H_{tv} \).

Ic. Since the Hamiltonian is not an explicit function of time it is a constant. What is the value of \( H \) throughout the optimal trajectory?

Id. Show that to minimize time to climb, we must pick \( V \) to maximize excess power.
I. Solution
I. Solution (continued)
I. Solution (continued)
II. Lagrange Problem (35 points)

A bead can move along a line, where its position is given by $x$. Assume that the bead is initially located at $x = 0$ and that its final position is $x = L$. Assume that the control of the bead is the velocity, $V$.

Set up the functional optimization as a Lagrange problem and work out the following steps:

IIa. Assume that we wish to minimize the following:

$$\min J = \int_{t_o}^{t_f} (V^2 + 1) dt$$

where $V$ is the control. Write the state equation and the initial and final conditions for the state. Assume that $t_o = 0$ and $t_f$ is free.

IIb. Write the Hamiltonian.

IIc. Write the costate equation, and give the form of its solution.

IId. Assuming unbounded control, state the control law (for $V$) in terms of the costate.

IIe. Show that your control satisfies the Legendre-Clebsch necessary condition.

IIf. Solve for the velocity

IIG. Solve for the final time.

IId. Solve for the minimum cost, $J_{min}$.
II. Solution
II. Solution (continued)
II. Solution (continued)
III. Fill in the Blanks  (20 points)

Unless otherwise stated, assume that all notation is the same as that of the lectures or handouts. No partial credit. All answers must be precise and complete.

Name the equations, conditions, or theorems (for questions 1-5):

1. $\lambda^T = -H_x, H_u = 0$ are called

2. The condition $H_{uu} \geq 0$ is named the

3. $H(t, x^*, \lambda, u^*) \leq H(t, x^*, \lambda, u)$ is referred to as the

4. When the Hamiltonian takes the form $H = H_0(t, x, \lambda) + H_1(t, x, \lambda)u$, then $H_1$ is referred to as the

5. If the problem of minimizing $\lambda_0 I$ can be solved with $\lambda_0 = 0$ and with all conditions of the Minimum Principle satisfied, the problem is referred to as
6. The contour in the h – V plane which describes the limits of aircraft performance is called the

7. In aircraft flight, the angle between the velocity vector and the zero-lift axis is called the

8. Give an equation which defines excess power:

9. Give an equation for energy height, \( h_e \):

10. Give the state constraint which usually governs aircraft flight:
IV. True or False  (10 points)

Answer questions in the context of what was assumed in AAE 508. Note: Penalty for wrong guesses is -1 point. 1 point for each correct answer and zero for unanswered. Circle the correct answer.

1. The transversality condition can be ignored if all of the initial and terminal boundary conditions are specified.
   
   True          False

2. In the aircraft performance analysis it was shown that to minimize the time to climb the following rule applies: at a given energy height pick the velocity which maximizes the excess power.
   
   True          False

3. For bounded control, the control law is found by setting $H_u$ to zero.
   
   True          False

4. Sometimes a supersonic aircraft may need to dive in order to achieve the minimum time to climb to a particular altitude. (Assume the aircraft trajectory begins at zero altitude with zero velocity.)
   
   True          False

5. From the Euler-Lagrange theorem, $H_u = 0$ provides m differential equations.
   
   True          False
6. The optimal trajectory for launch from the flat moon using $\ell/g = 3$ requires more propellant than a Hohmann transfer.
   
   True _______ False _______

7. In the Euler-Lagrange theorem, we assumed that $u^a$ is piecewise continuous.
   
   True _______ False _______

8. The step function, $h(t)$, is piecewise continuous.
   
   True _______ False _______

9. In our simplified analysis of the aircraft minimum time to climb problem we assumed that the time required for zoom climbs and dives is negligible.
   
   True _______ False _______

10. In general, the Legendre-Clebsch condition applies to a broader class of controls than does the Pontryagin Minimum Principle.

    True _______ False _______