ME 697:
INTELLIGENT SYSTEMS

OPTIMIZATION ALGORITHMS

Daniel McArthur
PhD Student

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EVOLUTIONARY STRATEGIES

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PROBLEM DEFINITION

2-INPUT NONLINEAR FITNESS FUNCTION TO MINIMIZE

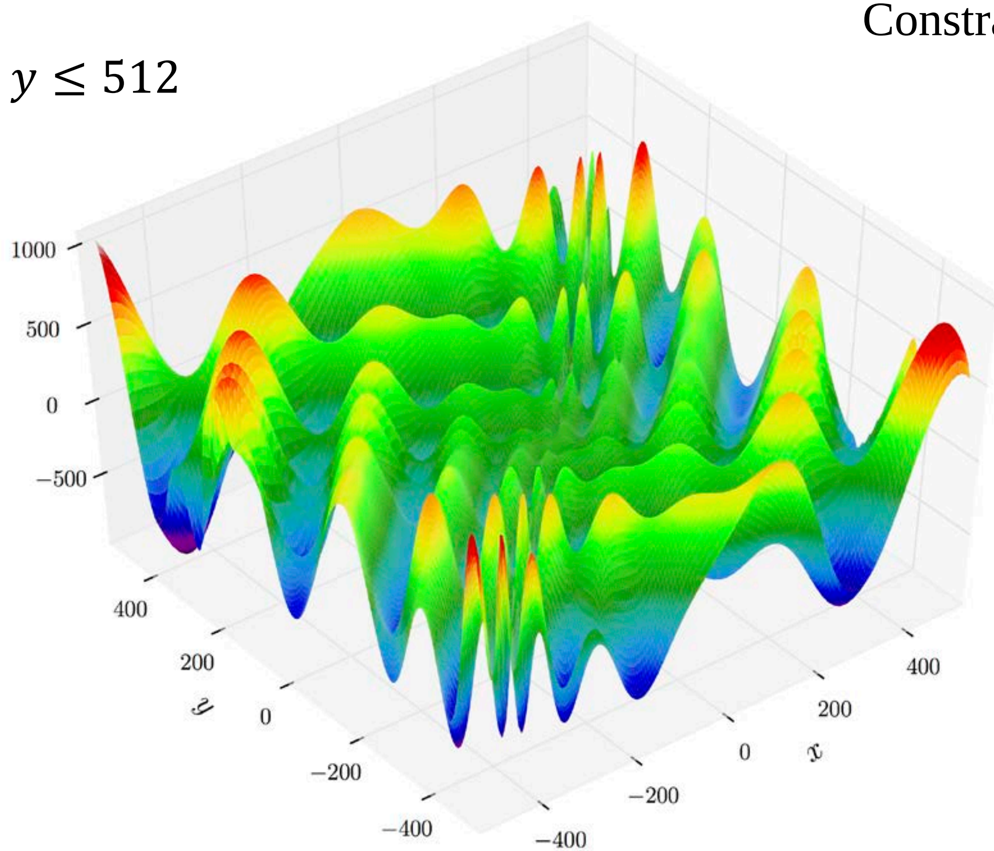
$$f = -(y+47) \cdot sin\left(\sqrt{\left|\frac{x}{2} + (y+47)\right|}\right) - x \cdot sin\left(\sqrt{|x-(y+47)|}\right)$$

Domain:

$$-512 \leq x, y \leq 512$$

Constraints:

$$g_1 = x + 512$$
$$g_2 = 512 - x$$
$$g_3 = y + 512$$
$$g_4 = 512 - y$$



"Eggholder Function"[1]

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EVOLUTIONARY STRATEGIES

ALGORITHM & PARAMETERS

Standard (μ,λ) Algorithm[2]:

$t := 0;$

initialize the initial population $\{\mathbf{a}_1(0), \ldots, \mathbf{a}_\mu(0)\} \in I^\mu$

where $I = \Re^n \times \Re^n_+$

and $\mathbf{a}_i(t) = (x_i(t), \sigma_i(t))$

$\forall i \in \{1, \ldots, \mu\};$

evaluate the initial population: $\{f(\mathbf{x}_1(0)), \ldots, f(\mathbf{x}_\mu(0))\}$

while stopping condition is not met, do

recombine μ parents to obtain λ offspring:

mutate the λ offspring:

evaluate the offspring: $\{f(\mathbf{x}_1(t)), \ldots, f(\mathbf{x}_\lambda(t))\}$:

select the best μ individuals out of λ offspring:

$t := t + 1;$

end

μ = Number of parents

λ = Number of children (offspring)

Parameters to Select

- Population Size
  - Number of parents
  - Number of children
- Maximum number of generations
- Elitism Factor
  - Which individuals reproduce?
  - Include parents in next population?
- Mutation Rate
- Cross-over Rate

Parameter Types

- Object Parameter (OP)
  - Individual = vector of real numbers
- Strategy Parameter (SP)
  - Standard Deviation for each input variable of each individual (controls individual mutation)

EVOLUTIONARY STRATEGIES

MATLAB IMPLEMENTATION

Random Population

- rng()
  - Set seed for repeatable tests
- rand()
  - Select (x,y)-pairs in domain
  - Set each σ between 0 & σ_{start}

Recombination

- randperm()
  - Select unique parent pairs
- Discrete
  - Select $OP_{\lambda}(\mathrm{i})$ from random parent
- Intermediate:
  - $OP_{\lambda}(\mathrm{i})$ = average of $OP_{\mu}(i)$

Mutation

- Modify OP by adding $N(\xi,\sigma)$ with ξ=0
  - $OP_{mut}(i) = OP(i) + N(0,\sigma(i))$
- Vary mutation step size (σ) by "1/5 success rule"
- After k iterations, reset σ by:

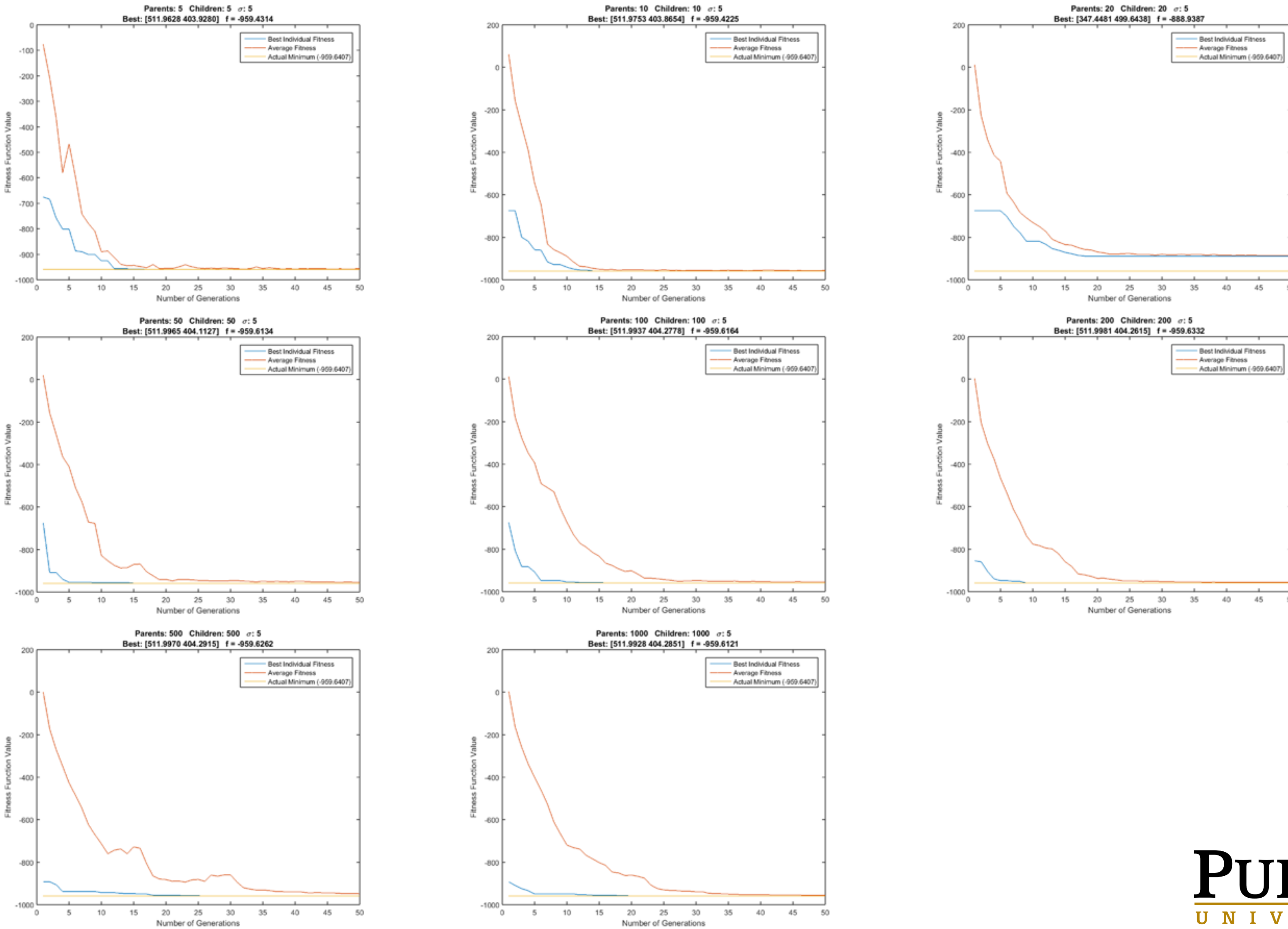
$$\begin{array}{lll} \sigma = \frac{\sigma}{c} & if & p_s > \frac{1}{5} \\ \sigma = \sigma * c & if & p_s < \frac{1}{5} \\ \sigma = \sigma & if & p_s = \frac{1}{5} \end{array}$$

Fitness & Elitism

- fit_fun.m & constr_fun.m
  - Evaluate fitness and constraint values for an individual
- Individuals are sorted by fitness value using min(), and only the best individuals reproduce
  - Parents included in selection pool

PARAMETER SELECTION

POPULATION SIZE: NUMBER OF PARENTS



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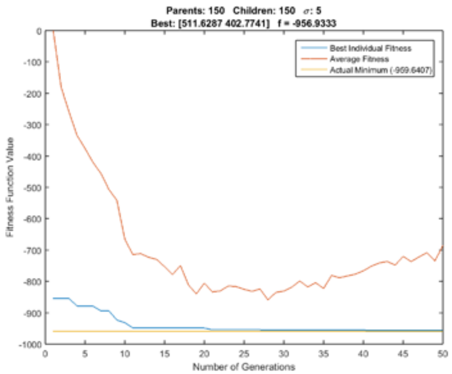
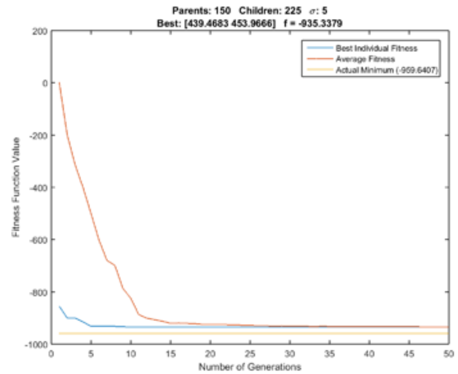
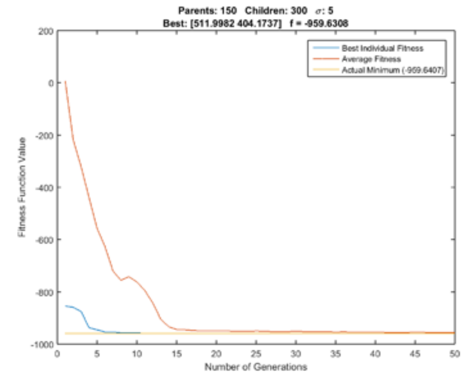
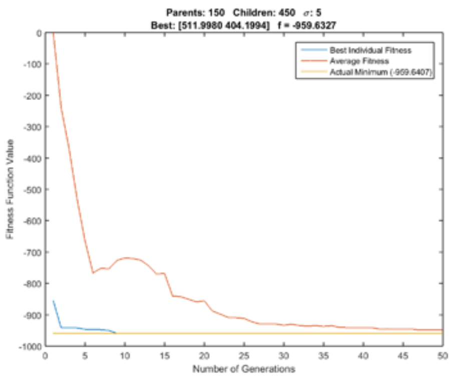
PARAMETER SELECTION

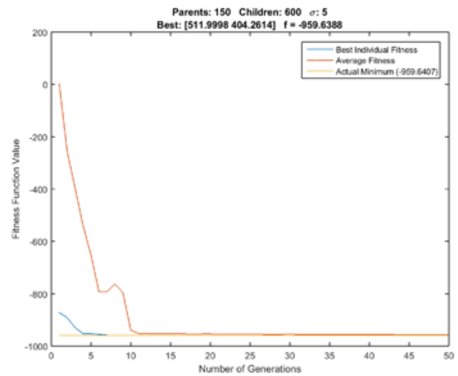
POPULATION SIZE: NUMBER OF CHILDREN

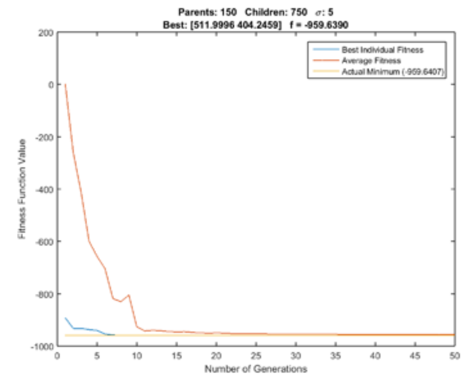


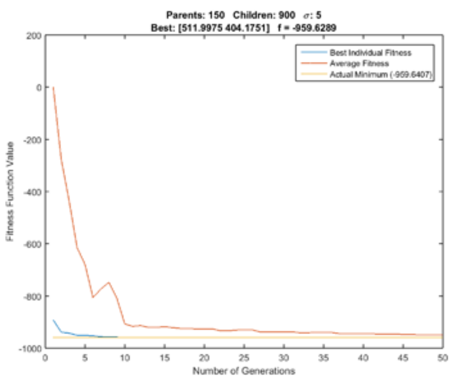


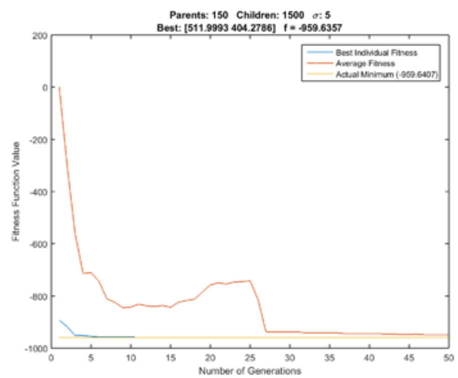












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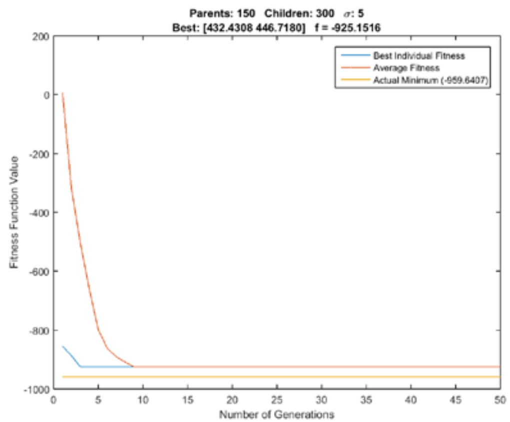
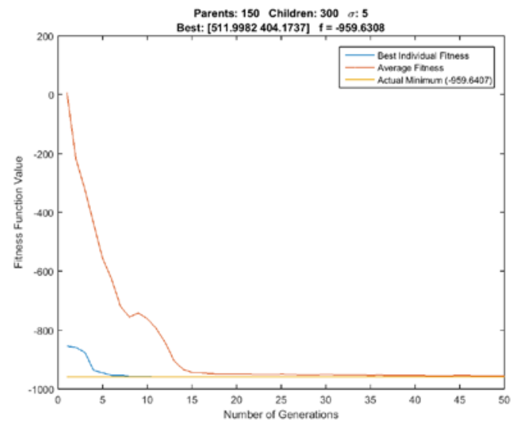
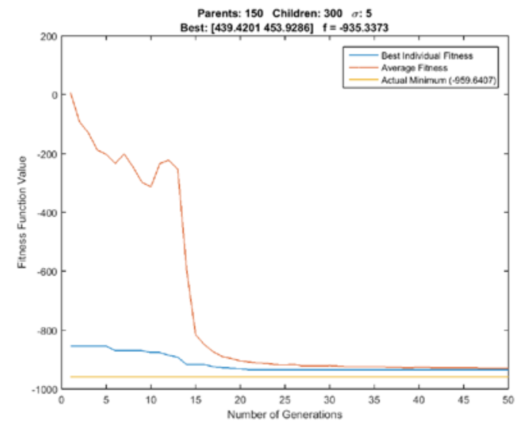
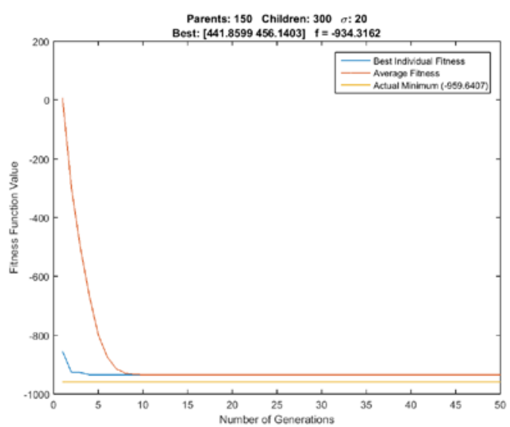
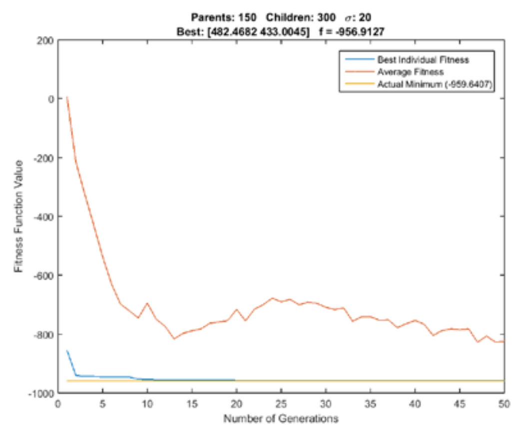
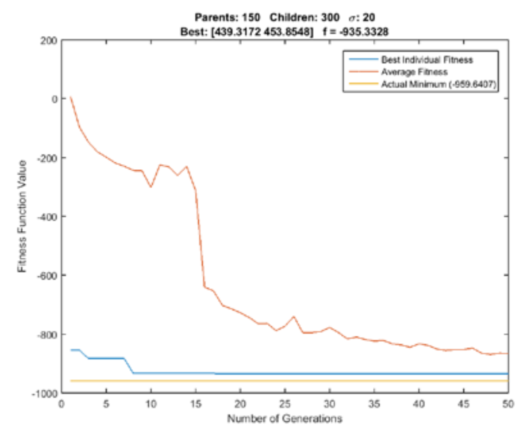
PARAMETER SELECTION

RECOMBINATION

	Mutation Only	Discrete Recombination	Intermediate Recombination
σ = 5			
σ = 20			

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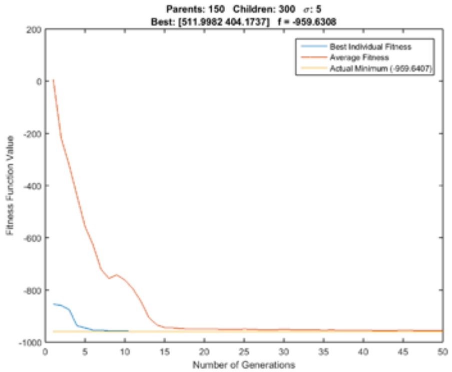
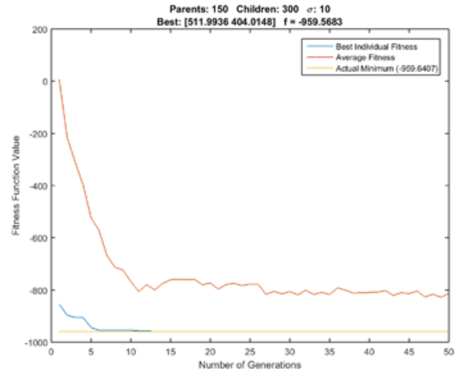
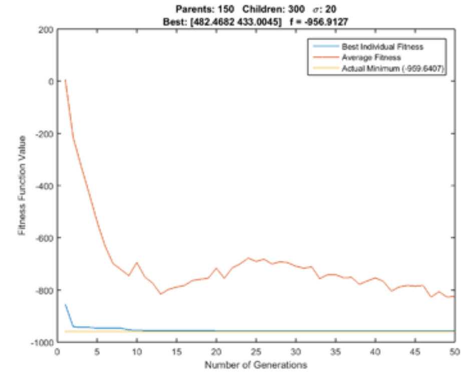
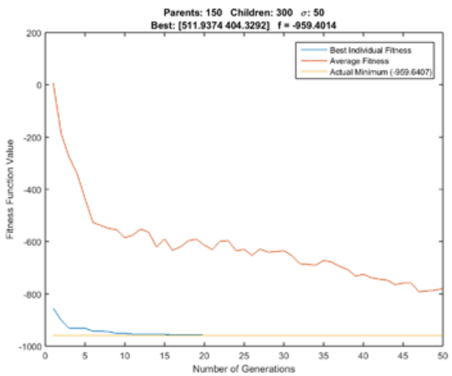
PARAMETER SELECTION

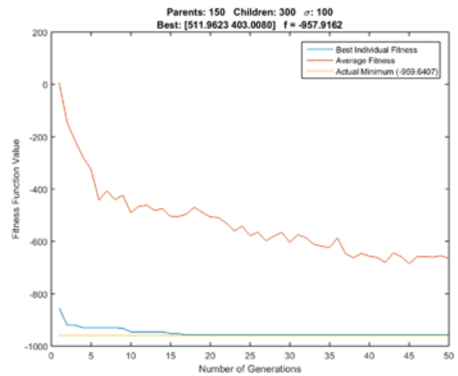
MUTATION: STANDARD DEVIATION

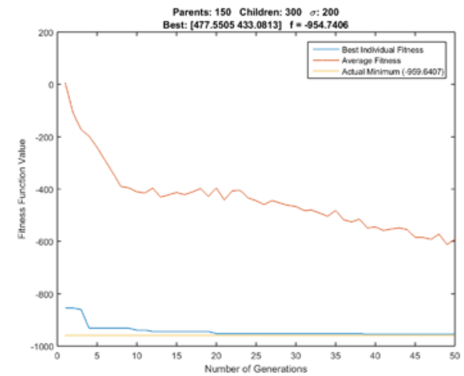


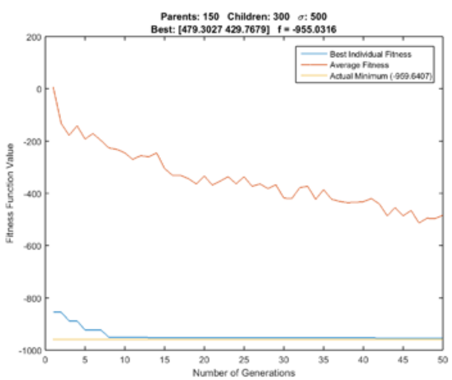


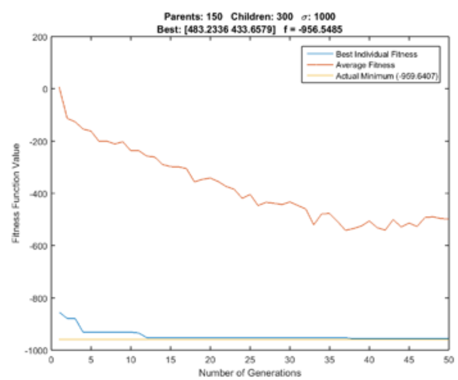








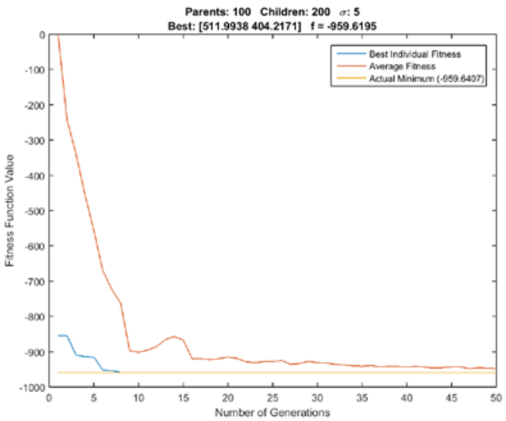


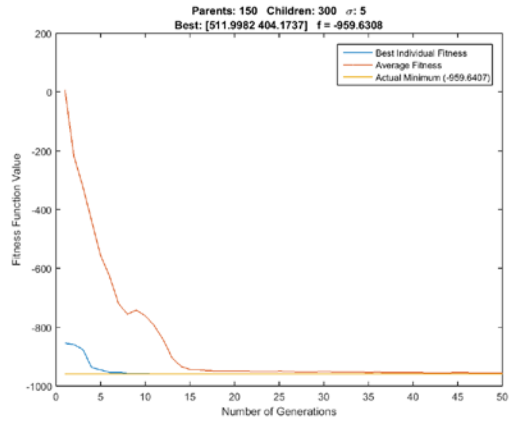


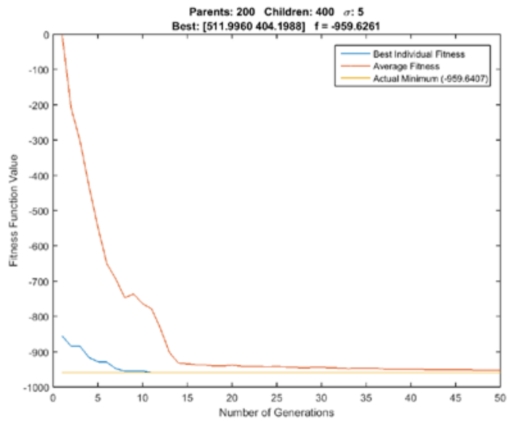
PARAMETER SELECTION

MUTATION: STRATEGY PARAMETER DISTRIBUTION

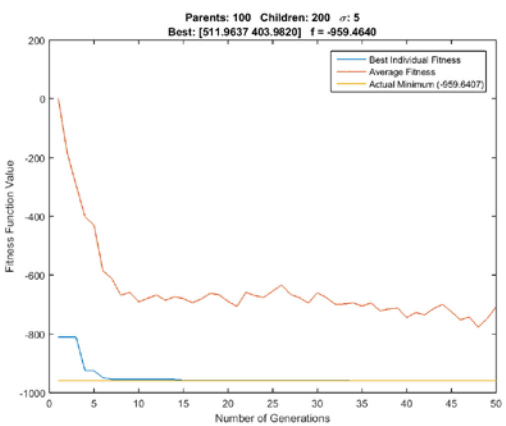
σ Randomly Distributed at Start
$(0 \leq \sigma \leq \sigma_{max})$

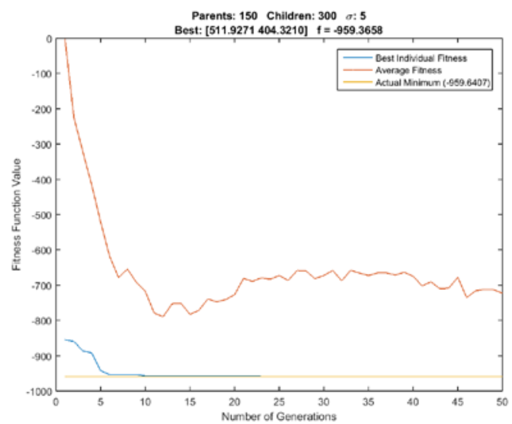


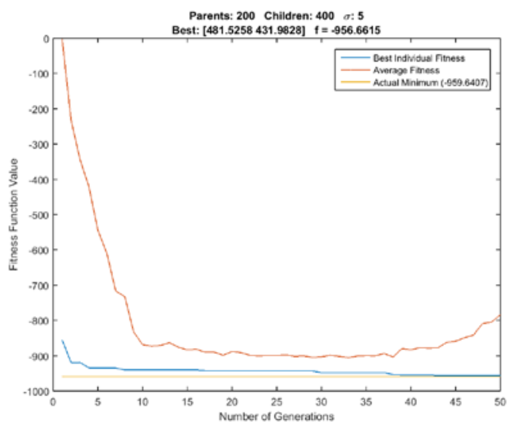




σ Same at Start
$(\sigma = \sigma_{max})$







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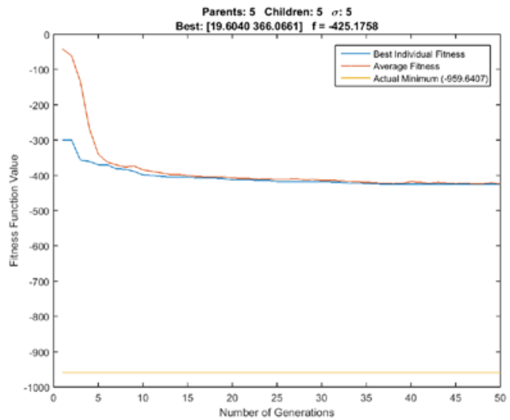
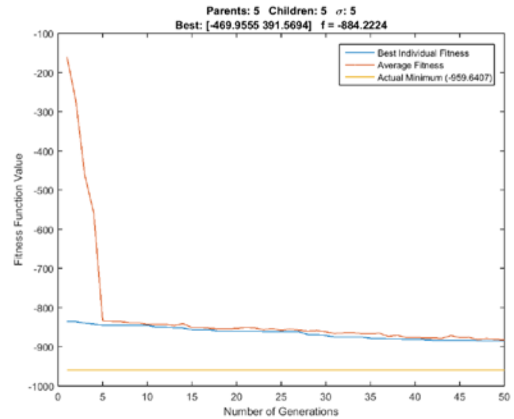
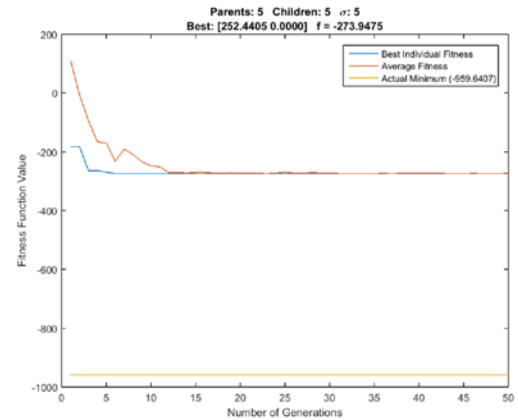
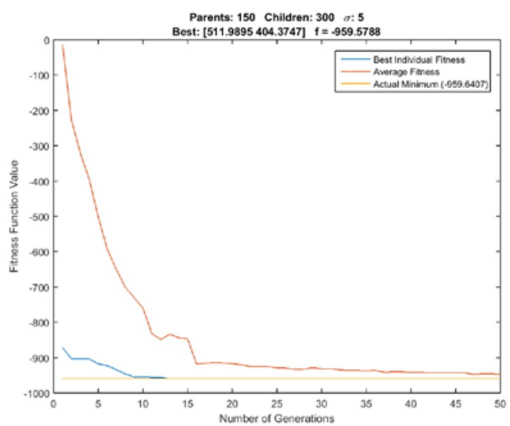
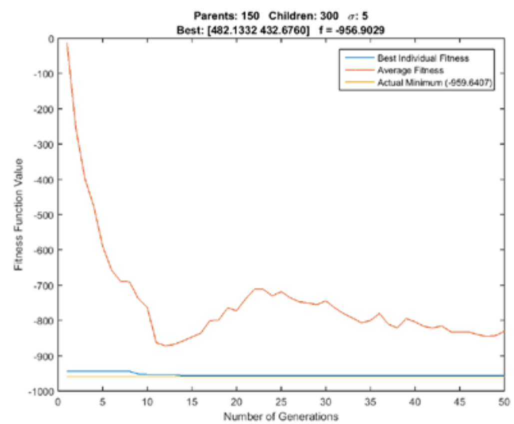
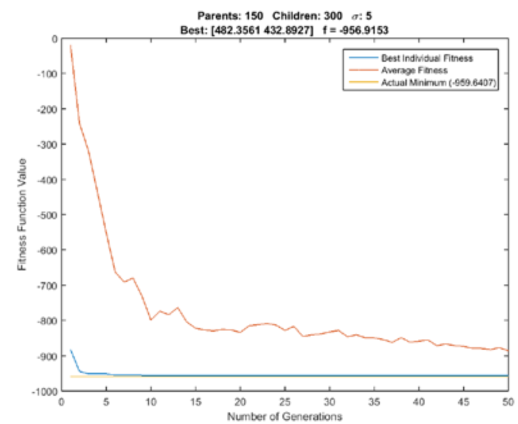
PARAMETER SELECTION

SENSITIVITY TO RANDOMIZATION

	Run 1	Run 2	Run 3
μ = 5			
μ = 150			

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FINAL PARAMETERS

BEST COMBINATION

Population Size

- Number of Parents (μ) = 150
- Number of Children (λ) = 2 * μ

Number of Generations

- 50 Generations
  - Typically converged much sooner (e.g. < 30 generations)

Recombination

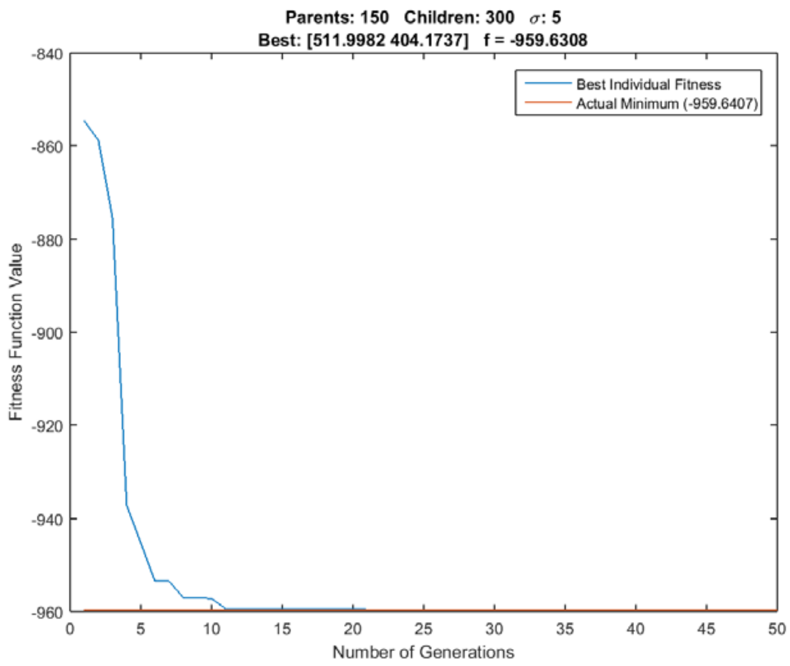
- Discrete outperforms Intermediate

Mutation

- σ = 5
- Reset after every 5 iterations

Elitism

- Include parents in fitness competition

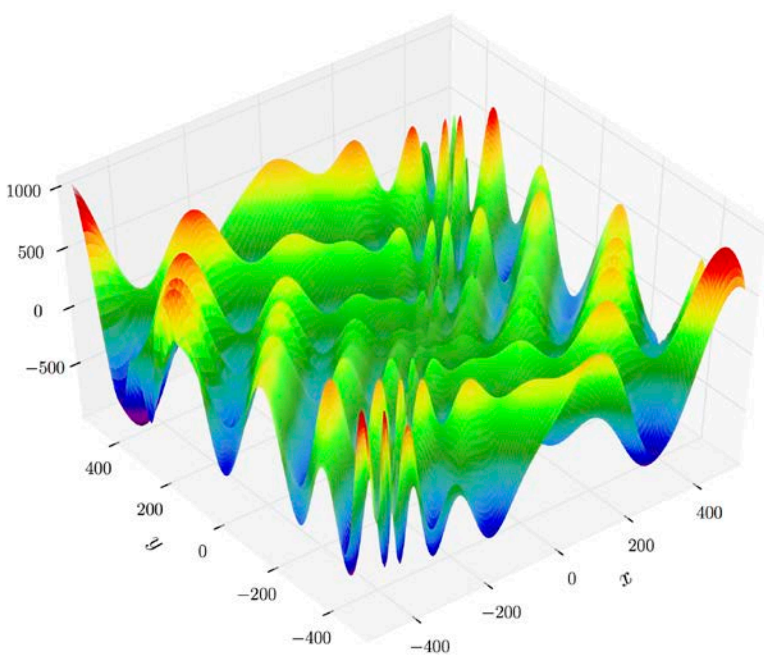


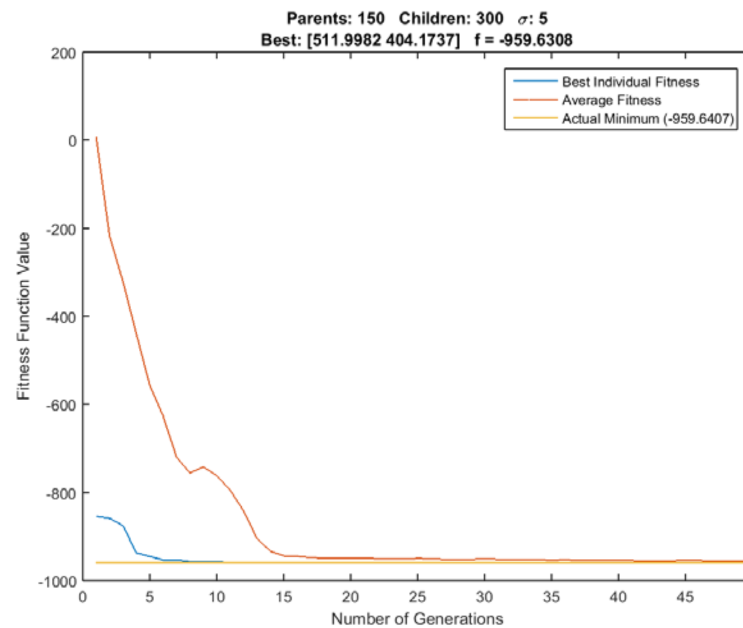
TEST RESULTS

EVOLUTIONARY STRATEGIES OPTIMIZATION VS. ACTUAL VALUES

$$f = -(y+47)\cdot sin\left(\sqrt{\left|\frac{x}{2}+(y+47)\right|}\right) - x\cdot sin\left(\sqrt{|x-(y+47)|}\right)$$

Actual		**Evolutionary Strategies**	
Global Minimum:	-959.6407	Global Minimum:	-959.6308
Location of Minimum:	(512,404.2319)	Location of Minimum:	(511.9982,404.1737)





Absolute Error: 0.001 %

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TEST RESULTS

EVOLUTIONARY STRATEGIES OPTIMIZATION VS. ACTUAL VALUES

$$f = -20e^{-0.2\sqrt{0.5(x^2+y^2)}} - e^{0.5(cos(2\pi x)+cos(2\pi y))} + e + 20$$

Actual

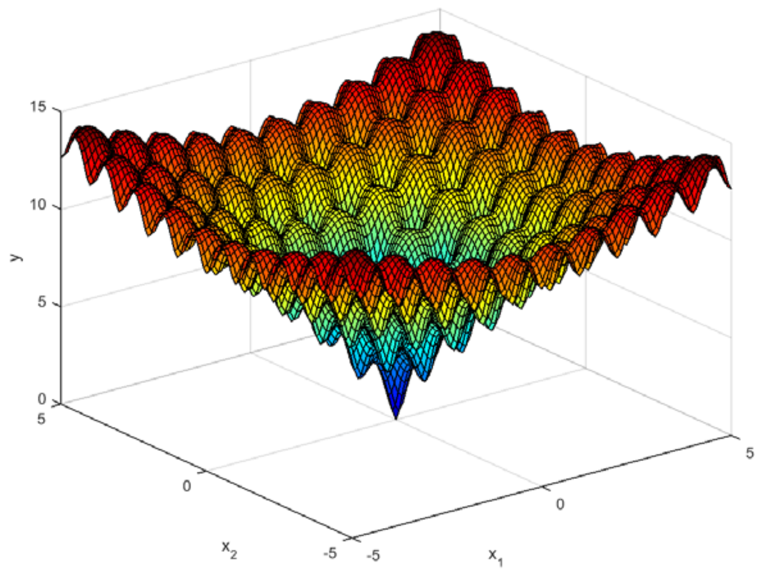
Global Minimum: 0

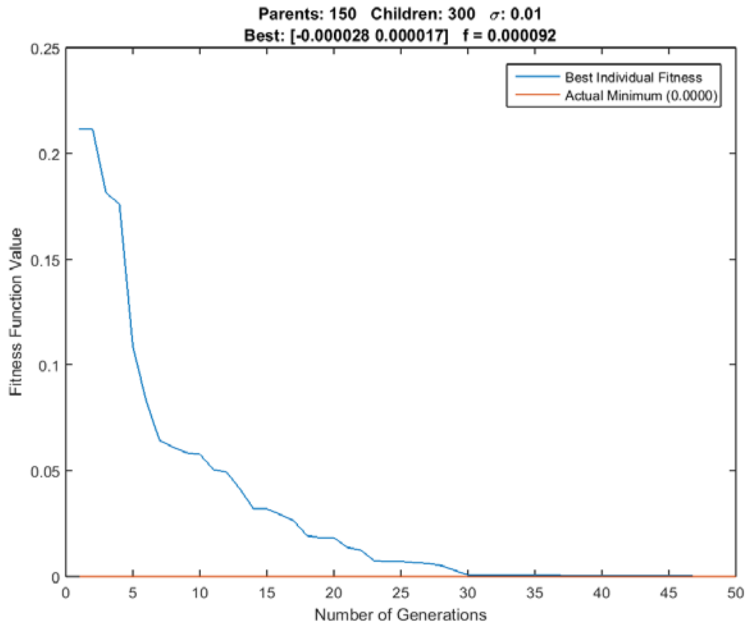
Location of Minimum: $(0, 0)$

Evolutionary Strategies

Global Minimum: 0.000092

Location of Minimum: $(-0.000028, 0.000017)$





Absolute Error: 0.0092 %

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CONCLUSIONS

EVOLUTIONARY STRATEGIES

- ES is able to optimize complex nonlinear functions with very little error
- Each parameter in the ES method requires careful tuning, and will vary depending on the function being optimized
    - The performance of ES can vary significantly due to randomization, but proper selection of population size (number of parents/children) can reduce this variation
    - Mutation parameters can affect the efficiency of the algorithm and the average fitness of each population, but within a reasonable range, the ES method will still come close to the global solution
- Thus, some experimentation may be required to get the parameters into a useable range, but the parameters generally don’t need to be perfect to get a reasonable solution

REFERENCES

1. Test Functions for Optimization. (n.d.). Retrieved March 04, 2016, from https://en.wikipedia.org/wiki/Test_functions_for_optimization
2. Shin, Yung C., and Chengying. Xu. *Intelligent Systems Modeling, Optimization, and Control*. Automation and Control Engineering. Hoboken: Taylor and Francis, 2008.