ME697Y Intelligent Systems -- Project1

RBFN-based Observer

Bin Zhang

Problem Definition

- An inverted pendulum driven by an armature controlled DC motor :

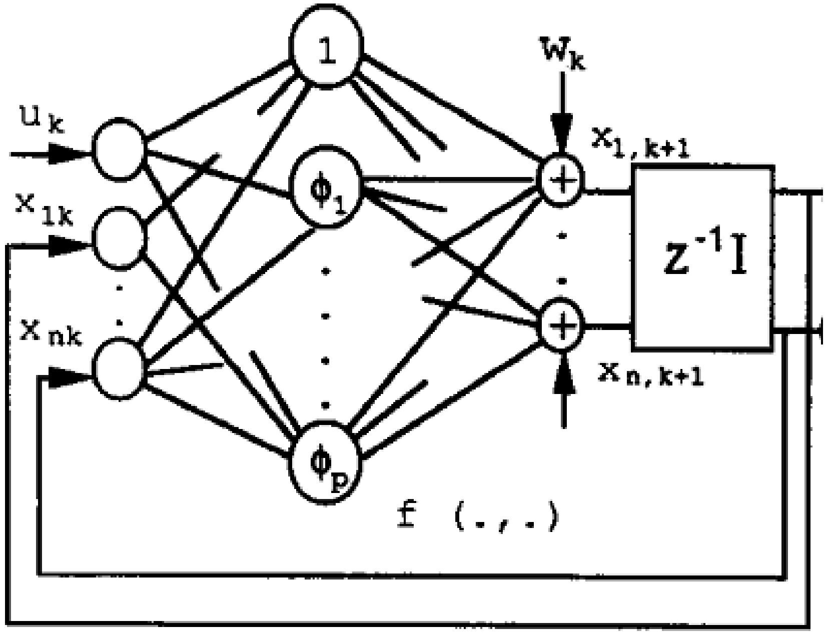
$$\mathbf{x}_{k+1} = \begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \\ x_{3,k+1} \end{bmatrix} = \mathbf{f}\left(\mathbf{x}_k\right) = \begin{bmatrix} x_{1,k} + Tx_{2,k} \\ x_{2,k} + T\left(9.8\sin x_{1,k} + x_{3,k}\right) \\ x_{3,k} - 10T\left(x_{2,k} + x_{3,k}\right) \end{bmatrix} + \mathbf{w}_k$$

$$y_k = x_{1,k} + v_k$$

- ***x1*** : angular position; ***x2*** : angular velocity ; ***x3*** : motor current;
- Sampling time ***T=0.01s***; ***w*** and ***v*** are additive white Gaussian noise;
- ***x1=π*** at the upright positon (stabilized);

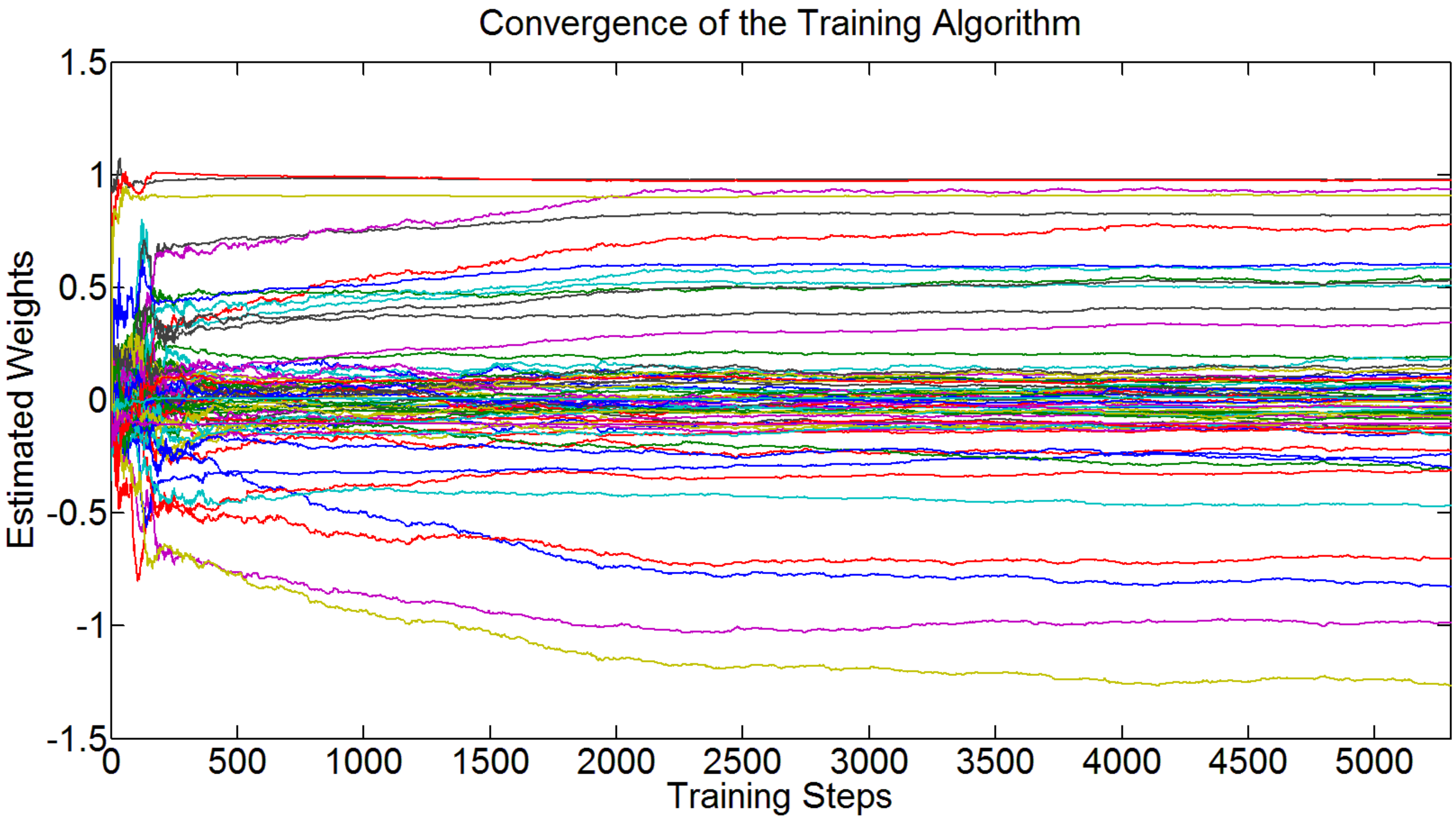
Dynamic System Approximation by RBFN

$$\mathbf{x_{k+1}} = [\mathbf{\Lambda} \quad \mathbf{\Lambda_0}] \begin{bmatrix} \mathbf{\Phi}(\mathbf{x}_k) \\ \mathbf{x}_k \end{bmatrix}$$



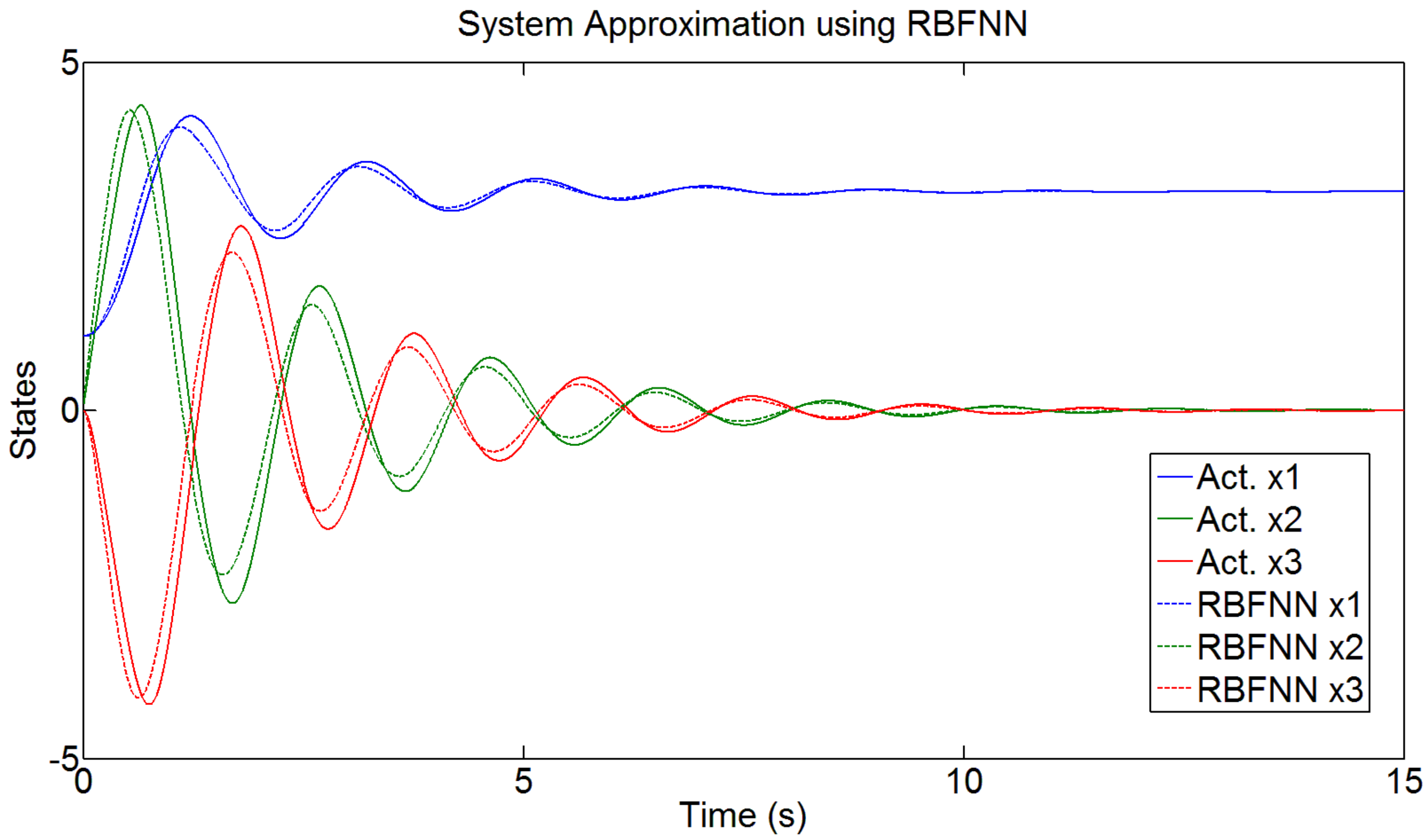
- 20 neurons in the hidden layer
- Gaussian basis function
- The centers and widths are pre-determined
- Only the weights in $[\mathbf{\Lambda} \quad \mathbf{\Lambda_0}]$ are tuned by the training algorithm
- 5 experiments with randomly generated initial conditions
- During training, all state variables are assumed to be measurable with additive noise

Training Results: Convergence



Training Results: System Approximation

- Initial Condition: $\mathbf{x}_0 = \begin{bmatrix} \frac{\pi - 1}{2} & 0 & 0 \end{bmatrix}^T$



Observer Design

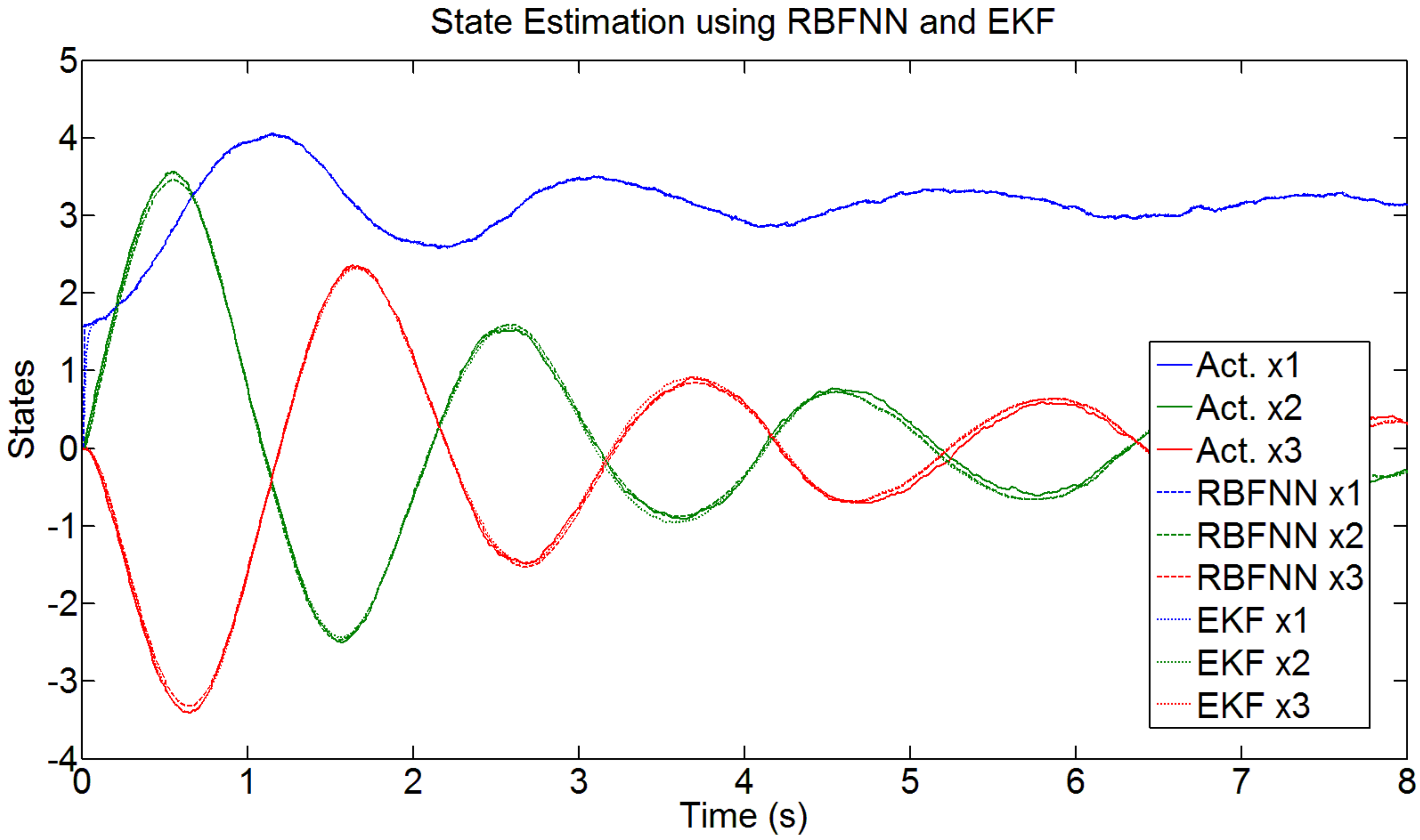
RBFN-based Observer	Extended Kalman Filter (EKF)
$\hat{\mathbf{x}}_{\mathbf{k+1}} = [\boldsymbol{\Lambda} \quad \boldsymbol{\Lambda}_{\mathbf{0}}] \begin{bmatrix} \boldsymbol{\Phi}(\hat{\mathbf{x}}_{\mathbf{k}}) \\ \hat{\mathbf{x}}_{\mathbf{k}} \end{bmatrix} + \mathbf{K}_k (y_k - \mathbf{H}\hat{\mathbf{x}}_k)$ $\mathbf{H} = [1 \quad 0 \quad 0]$	$\hat{\mathbf{x}}_{\mathbf{k+1}} = \mathbf{f}_0(\hat{\mathbf{x}}_{\mathbf{k}}) + \mathbf{K}_k (y_k - \mathbf{H}\hat{\mathbf{x}}_k)$ $\mathbf{H} = [1 \quad 0 \quad 0]$ $\mathbf{f}_0 \approx \mathbf{f}$
Nonlinear Basis Function	13 Linearizations ***x1= [60, 80, 100, … , 180, …, 300]***
The covariance of both the state noise ***w*** and measurement noise ***v*** are known	

Reference:

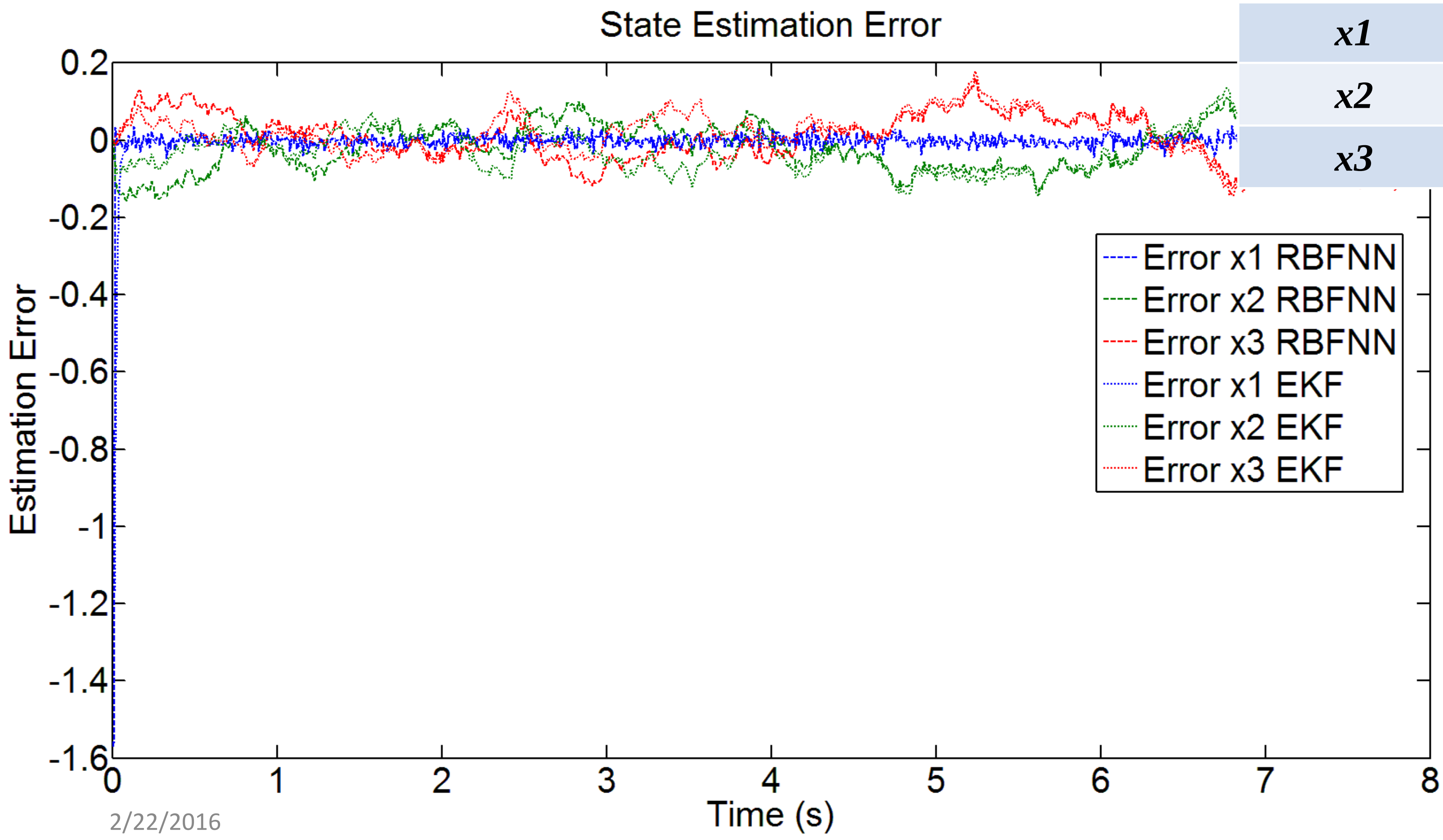
Elanayar, V. T., and Yung C. Shin. "Radial basis function neural network for approximation and estimation of nonlinear stochastic dynamic systems.“ *Neural Networks, IEEE Transactions on* 5.4 (1994): 594-603.

State Estimation Results

$$\mathbf{f}_0(\mathbf{x}_k) = \mathbf{f}(\mathbf{x}_k) \quad \mathbf{x}_0 = \begin{bmatrix} \frac{\pi}{2} & 0 & 0 \end{bmatrix}^T \quad \hat{\mathbf{x}}_0 = [0 \quad 0 \quad 0]^T$$



State Estimation Results



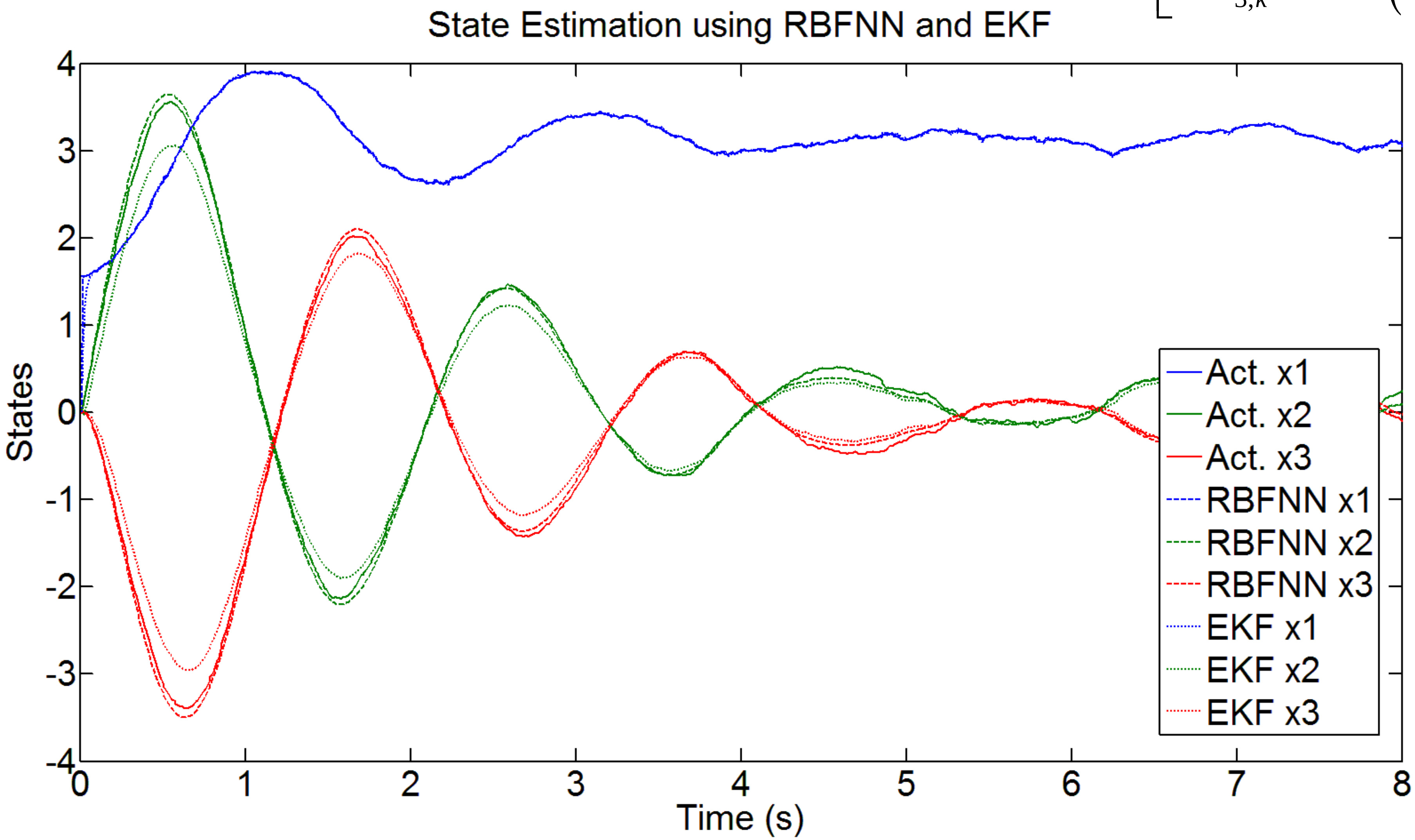
Root Mean Square Error

RMSE	RBFN	EKF
x1	0.0574	0.0648
x2	0.0669	0.0650
x3	0.0612	0.0609

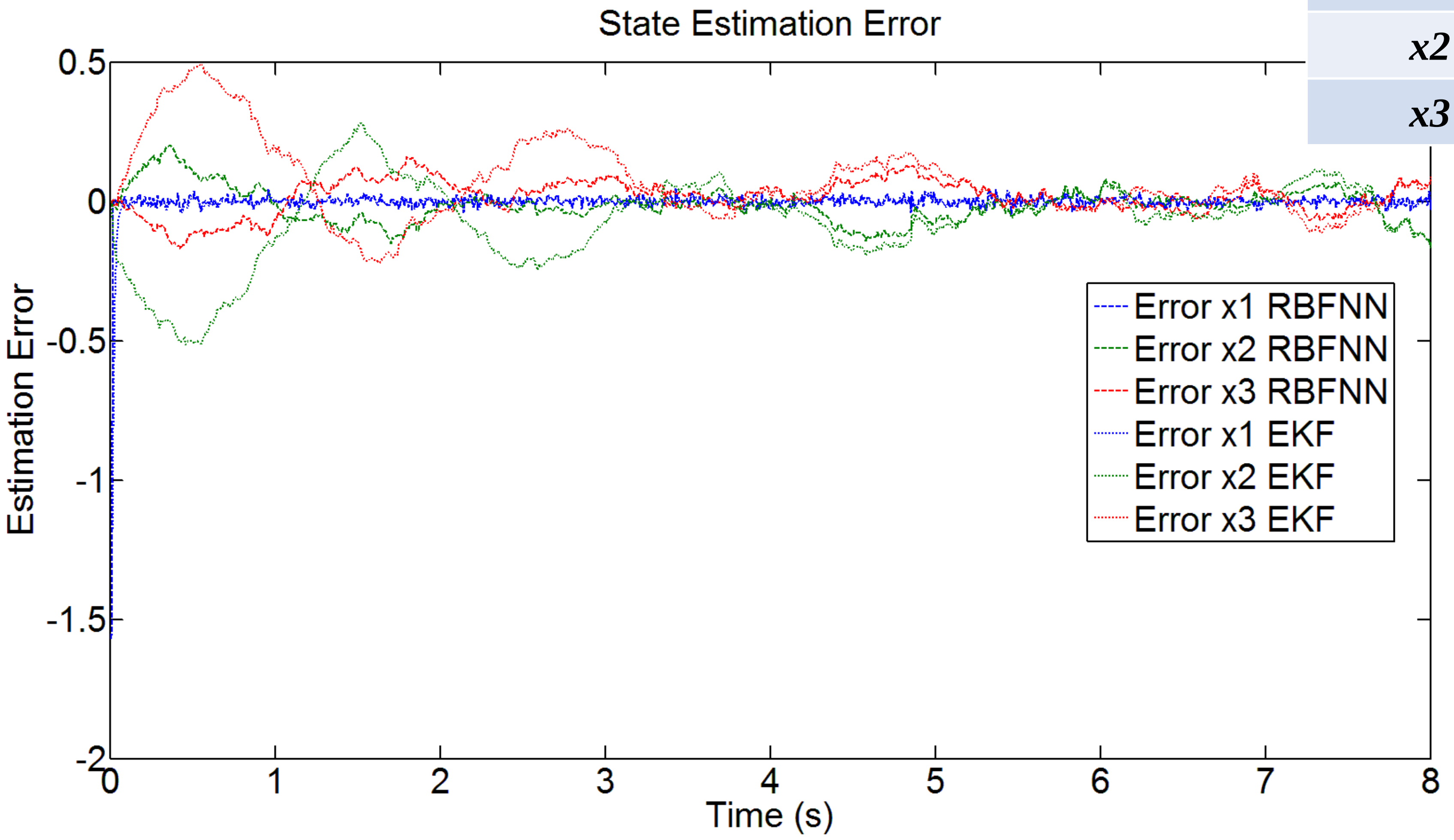
$$\mathbf{f}_0\left(\mathbf{x}_k\right) = \mathbf{f}\left(\mathbf{x}_k\right)$$

State Estimation Results

$$\mathbf{f}_0(\mathbf{x}_k) = \begin{bmatrix} x_{1,k} + T x_{2,k} \\ x_{2,k} + T\left(8.5 \sin x_{1,k} + x_{3,k}\right) \\ x_{3,k} - 10T\left(x_{2,k} + x_{3,k}\right) \end{bmatrix}$$



State Estimation Results



Root Mean Square Error

RMSE	RBFN	EKF
x1	0.0574	0.0652
x2	0.0627	0.1641
x3	0.0629	0.1561

$$\mathbf{f}_0\left(\mathbf{x}_k\right) \approx \mathbf{f}\left(\mathbf{x}_k\right)$$

Conclusion

- Both the RBFN-based observer and extended Kalman filter can accurately estimate the states of nonlinear dynamic systems with white Gaussian noise.
- In the presence of system uncertainty (nominal system model ≠ real system model), the performance of the extended Kalman filter will be degraded.
- As system uncertainty always exist in real systems, RBFN-based observer is a better choice than extended Kalman filter.