RBFN-based Observer

Bin Zhang
Problem Definition

• An inverted pendulum driven by an armature controlled DC motor:

\[
x_{k+1} = \begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \\ x_{3,k+1} \end{bmatrix} = f(x_k) = \begin{bmatrix} x_{1,k} + Tx_{2,k} \\ x_{2,k} + 10T(x_{2,k} + x_{3,k}) \\ x_{3,k} - 9.8\sin x_{1,k} + x_{3,k} \end{bmatrix} + w_k
\]

\[
y_k = x_{1,k} + v_k
\]

• \( x1 \) : angular position; \( x2 \) : angular velocity; \( x3 \) : motor current;
• Sampling time \( T=0.01s \); \( w \) and \( v \) are additive white Gaussian noise;
• \( x1=\pi \) at the upright positon (stabilized);
Dynamic System Approximation by RBFN

\[ x_{k+1} = \begin{bmatrix} \Lambda & \Lambda_0 \end{bmatrix} \begin{bmatrix} \Phi(x_k) \\ x_k \end{bmatrix} \]

- 20 neurons in the hidden layer
- Gaussian basis function
- The centers and widths are pre-determined
- Only the weights in \([\Lambda \quad \Lambda_0]\) are tuned by the training algorithm
- 5 experiments with randomly generated initial conditions
- During training, all state variables are assumed to be measurable with additive noise
Training Results: Convergence
Training Results: System Approximation

- Initial Condition: \( x_0 = \begin{bmatrix} \pi - 1/2 & 0 & 0 \end{bmatrix}^T \)
## Observer Design

<table>
<thead>
<tr>
<th>RBFN-based Observer</th>
<th>Extended Kalman Filter (EKF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{x}_{k+1} = [\Lambda \ \Lambda_0] \begin{bmatrix} \Phi(\hat{x}_k) \ \hat{x}_k \end{bmatrix} + K_k \left( y_k - H\hat{x}_k \right) )</td>
<td>( \hat{x}_{k+1} = f_0(\hat{x}_k) + K_k \left( y_k - H\hat{x}_k \right) )</td>
</tr>
<tr>
<td>( H = \begin{bmatrix} 1 &amp; 0 &amp; 0 \end{bmatrix} )</td>
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</tr>
<tr>
<td>Nonlinear Basis Function</td>
<td>13 Linearizations</td>
</tr>
<tr>
<td>( x_1 = [60, 80, 100, \ldots, 180, \ldots, 300] )</td>
<td></td>
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</tbody>
</table>

The covariance of both the state noise \( w \) and measurement noise \( v \) are known.

Reference:
State Estimation Results

\[ f_0(x_k) = f(x_k), \quad x_0 = \begin{bmatrix} \frac{\pi}{2} & 0 & 0 \end{bmatrix}^T, \quad \hat{x}_0 = [0 \ 0 \ 0]^T \]
State Estimation Results

Root Mean Square Error

<table>
<thead>
<tr>
<th></th>
<th>RMSE</th>
<th>RBFN</th>
<th>EKF</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>0.0574</td>
<td>0.0648</td>
<td></td>
</tr>
<tr>
<td>x2</td>
<td>0.0669</td>
<td>0.0650</td>
<td></td>
</tr>
<tr>
<td>x3</td>
<td>0.0612</td>
<td>0.0609</td>
<td></td>
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</tbody>
</table>

\[ f_0(x_k) = f(x_k) \]
State Estimation Results

\[
f_0(x_k) = \begin{bmatrix} x_{1,k} + T x_{2,k} \\ x_{2,k} + T (8.5 \sin x_{1,k} + x_{3,k}) \\ x_{3,k} - 10T (x_{2,k} + x_{3,k}) \end{bmatrix}
\]

State Estimation using RBFNN and EKF
State Estimation Results

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<tr>
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<th>RBFN</th>
<th>EKF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>0.0574</td>
<td>0.0652</td>
<td></td>
</tr>
<tr>
<td>$x_2$</td>
<td>0.0627</td>
<td>0.1641</td>
<td></td>
</tr>
<tr>
<td>$x_3$</td>
<td>0.0629</td>
<td>0.1561</td>
<td></td>
</tr>
</tbody>
</table>

Root Mean Square Error

$$f_0(x_k) \approx f(x_k)$$
Conclusion

• Both the RBFN-based observer and extended Kalman filter can accurately estimate the states of nonlinear dynamic systems with white Gaussian noise.

• In the presence of system uncertainty (nominal system model ≠ real system model), the performance of the extended Kalman filter will be degraded.

• As system uncertainty always exist in real systems, RBFN-based observer is a better choice than extended Kalman filter.