1. Consider the classical electron oscillator model for the case where an external optical field of angular frequency $\omega$ acts on the atom. The atomic resonance has an angular resonance frequency of $\left(\omega_r / 2\pi c\right) = 20,000 \text{ cm}^{-1}$, a radiative transition rate of $\gamma_{rad} = 10^7 \text{ sec}^{-1}$, and a homogeneous linewidth of $\left(\Delta \omega_a / 2\pi c\right) = 0.15 \text{ cm}^{-1}$. The number densities of the upper and lower levels of the transition are $n_1 = 10^{24} \text{ m}^{-3}$ and $n_2 = 0 \text{ m}^{-3}$. The external optical field is linearly polarized in the $x$-direction and propagates in the $z$-direction. The electric field amplitude is $E_{0x} = E_{0x}^* = 400 V/m = 400 J/C - m$ and the frequency of the field is $\left(\omega / 2\pi c\right) = 20,000 + \delta \text{ cm}^{-1}$. For values of the parameter $\delta$ of -0.3, -0.03, 0.00, +0.03, and +0.8 cm$^{-1}$, plot the response $p_x(t)$ of the resonance for $t = 0$ to $t = 8\pi / \omega$.

Solution:

Basic Equations:

$$E(z,t) = E(0,t) = \frac{1}{2} \hat{x}E_{0x} \exp(+i\omega t) + \frac{1}{2} \hat{x}E_{0x} \exp(-i\omega t)$$

$$E_x(t) = E_{0x} \cos(\omega t)$$

$$p_x(t) = \frac{1}{2} P_{ox} \exp(+i\omega t) + \frac{1}{2} P_{ox}^* \exp(-i\omega t)$$

$$P_{ox} = \varepsilon_0 \chi_{res}(\omega) E_{0x}$$

$$\chi_{res}(\omega) = \chi'(\omega) + i \chi''(\omega)$$

$$\chi'(\omega) = -\chi'' \frac{\Delta x}{1 + (\Delta x)^2}$$

$$\chi''(\omega) = -\chi'' \frac{1}{1 + (\Delta x)^2}$$

$$\chi'' = \frac{1}{4\pi^2} \left( \frac{g_2}{g_1} n_1 - n_2 \right) \lambda_{21}^3 \gamma_{rad}$$

Assume $g_2 = g_1$, $\Delta x = \frac{2(\omega - \omega_a)}{\Delta \omega_a}$

Given:

Resonance: $\frac{\omega_a}{2\pi c} = \bar{v}_a = 20,000 \text{ cm}^{-1}$

$\lambda_{21} = \frac{1}{\bar{v}_a} = 5 \times 10^{-5} \text{ cm} = 5 \times 10^{-7} \text{ m} = 500 \text{ nm}$

$$\frac{\Delta \omega_a}{2\pi c} = \Delta \bar{v}_a = 0.15 \text{ cm}^{-1}$$

$\delta_{rad} = A_{21} = 10^7 \text{ s}^{-1}$

$n_1 = 10^{24} \text{ m}^{-3}$

$n_2 = 0 \text{ m}^{-3}$

Laser: $E_{0x} = E_{0x}^* = 400 \frac{J}{C \cdot m}$

$$\bar{v}_L = \frac{\omega}{2\pi c} = \bar{v}_a + \delta$$
Find: $p_x(t)$ for $\delta = -0.3, -0.03, 0.00, +0.03, and +0.8 \text{ cm}^{-1}$, $t = 0$ to $t = \frac{8\pi}{\omega}$

Solution:

$$\chi''_0 = \frac{1}{4\pi^2} \left( \frac{g_2}{g_1} n_1 - n_2 \right) \frac{\lambda_2^3 \gamma_{rad}}{\Delta \omega} = \frac{n_1 \lambda_2^3 A_{21}}{4\pi^2 (2\pi c \Delta \tilde{v}_a)} = \frac{(10^{24} \text{ m}^{-3})(500 \times 10^{-9} \text{ m}^3)(10^7 \text{ s}^{-1})}{8\pi^3 (2.998 \times 10^{10} \text{ cm} \text{ s}^{-1})(0.15 \text{ cm}^{-1})} = 1.121$$

$$px(t) = \frac{1}{2} P_{ox} \exp(+i\omega t) + \frac{1}{2} P_{ox}^* \exp(-i\omega t)$$

$$P_{ox} = \varepsilon_0 \chi_{res}^{(1)}(\omega) E_{ox} = \varepsilon_0 \chi^{(1)}(\omega) E_{ox} + i \varepsilon_0 \chi^{(2)}(\omega) E_{ox}$$

$$P_{ox}^* = \varepsilon_0 \chi_{res}^{(1)}(\omega) E_{ox}^* = \varepsilon_0 \chi^{(1)}(\omega) E_{ox}^* - i \varepsilon_0 \chi^{(2)}(\omega) E_{ox}$$

$$px(t) = \frac{1}{2} \varepsilon_0 E_{ox} \left[ \chi^{(1)}(\omega) + i \chi^{(2)}(\omega) \right] \exp(+i\omega t) + \left[ \chi^{(1)}(\omega) - i \chi^{(2)}(\omega) \right] \exp(-i\omega t)$$

$$= \frac{1}{2} \varepsilon_0 E_{ox} \left[ \chi^{(1)}(\omega) + i \chi^{(2)}(\omega) \right] \left[ \cos(\omega t) + i \sin(\omega t) \right] + \left[ \chi^{(1)}(\omega) - i \chi^{(2)}(\omega) \right] \left[ \cos(\omega t) - i \sin(\omega t) \right]$$

$$= \varepsilon_0 E_{ox} \left[ \chi^{(1)}(\omega) \cos(\omega t) - \chi^{(2)}(\omega) \sin(\omega t) \right]$$

$$\delta = -0.3 \text{ cm}^{-1}$$

$$\Delta x = \frac{2(\tilde{v} - \tilde{v}_0)}{\Delta \tilde{v}_a} = \frac{2(-0.3 \text{ cm}^{-1})}{0.15 \text{ cm}^{-1}} = -4.00$$

$$\chi^{(1)}(\omega) = -\chi^{(2)}(\omega) \frac{\Delta x}{1 + (\Delta x)^2} = -1.121 \frac{-4.00}{1 + 16.00} = 0.2638$$

$$\chi^{(2)}(\omega) = -\chi^{(2)}(\omega) \frac{1}{1 + (\Delta x)^2} = -1.121 \frac{1}{1 + 16.00} = -0.06594$$

$$px(t) = \varepsilon_0 E_{ox} \left[ \chi^{(1)}(\omega) \cos(\omega t) - \chi^{(2)}(\omega) \sin(\omega t) \right]$$

$$px(t) = \left[ 8.854 \times 10^{-12} \frac{C^2}{J \text{ m}} \right] \left[ 400 \frac{J}{C \text{ m}} \right] \left[ 0.2638 \cos(\omega t) + 0.06594 \sin(\omega t) \right]$$

$$px(t) = \left[ 3.542 \times 10^{-9} \frac{C \text{ m}}{m^3} \right] \left[ 0.2638 \cos(\omega t) + 0.06594 \sin(\omega t) \right]$$
Electron oscillation in phase with applied electric field.

$$\delta = -0.03 \text{ cm}^{-1}$$

$$\Delta x = \frac{2(\tilde{v} - v_0)}{\Delta \tilde{v}_a} = \frac{2(-0.03 \text{ cm}^{-1})}{0.15 \text{ cm}^{-1}} = -0.40$$

$$\chi'(\omega) = -\chi'_0 \frac{\Delta x}{1 + (\Delta x)^2} = -1.121 \frac{(-0.40)}{1 + 0.16} = 0.3866$$

$$\chi''(\omega) = -\chi''_0 \frac{1}{1 + (\Delta x)^2} = -1.121 \frac{1}{1.16} = -0.9664$$

$$p_x(t) = \left[3.542 \times 10^{-9} \frac{C m}{m^3}\right] \left[0.3866 \cos(\omega t) + 0.9664 \sin(\omega t)\right]$$

Electron oscillation about 90 degrees out of phase with applied electric field, amplitude of oscillation increases.
\[\delta = 0 \text{ cm}^{-1}\]

\[
\Delta x = \frac{2(\ddot{v} - \ddot{v}_0)}{\Delta \ddot{v}_\omega} = 0
\]

\[\chi'(\omega) = -\chi'^{0} \frac{\Delta x}{1 + (\Delta x)^2} = 0\]

\[\chi''(\omega) = -\chi''^{0} \frac{1}{1 + (\Delta x)^2} = -1.121 \left(\frac{1}{1}\right) = -1.121\]

\[p_x(t) = \left[3.542 \times 10^{-9} \frac{Cm}{m^3}\right] \left[1.121 \sin(\omega t)\right]\]

Electron oscillation exactly 90 degrees out of phase with applied electric field, amplitude of oscillation reaches a maximum.

\[\delta = 0.03 \text{ cm}^{-1}\]

\[
\Delta x = \frac{2(\ddot{v} - \ddot{v}_0)}{\Delta \ddot{v}_\omega} = \frac{2(0.03 \text{ cm}^{-1})}{0.15 \text{ cm}^{-1}} = +0.4
\]

\[\chi'(\omega) = -\chi'^{0} \frac{\Delta x}{1 + (\Delta x)^2} = -1.121 \left(+0.40\right) \frac{1}{1.16} = -0.3866\]

\[\chi''(\omega) = -\chi''^{0} \frac{1}{1 + (\Delta x)^2} = -1.121 \frac{1}{1.16} = -0.9664\]

\[p_x(t) = \left[3.542 \times 10^{-9} \frac{Cm}{m^3}\right] \left[-0.3866(\omega t) + 0.9664 \sin(\omega t)\right]\]
Electron oscillation about 90 degrees out of phase with applied electric field, amplitude of oscillation decreases.

\[
\delta = 0.8 \text{ cm}^{-1}
\]

\[
\Delta x = \frac{2(\tilde{v} - \tilde{v}_0)}{\Delta \tilde{v}_a} = \frac{2(0.8 \text{ cm}^{-1})}{0.15 \text{ cm}^{-1}} = 10.67
\]

\[
\chi'(\omega) = -\chi_0' \frac{\Delta x}{1 + (\Delta x)^2} = -1.121 \frac{10.67}{114.8} = -0.1042
\]

\[
\chi''(\omega) = -\chi_0'' \frac{1}{1 + (\Delta x)^2} = -1.121 \frac{1}{114.8} = -0.009765
\]

\[
p_x(t) = \left[ 3.542 \times 10^{-9} \frac{Cm}{m^3} \right] \left[ -0.1042 \cos(\omega t) + 0.009765 \sin(\omega t) \right]
\]

Electron oscillation about 180 degrees out of phase with applied electric field, amplitude of oscillation continues to decrease.
2. The power absorbed by the classical electron oscillator (CEO) is given by the product of the force on the electron and the velocity of the electron. Show that the power absorbed per unit volume by an ensemble of CEOs is given by
\[ P_{\text{abs}} = \frac{dU_a}{dt} = E_x(t) \frac{dp_x(t)}{dt} \]
for an incident electric field polarized in the x-direction. Starting from the expressions for the incident electric field and induced polarization,
\[ E_x(t) = \frac{1}{2} E_0 \exp(+i \omega t) + \frac{1}{2} E_0 \exp(-i \omega t), \quad p_x(t) = \frac{1}{2} P_0 \exp(+i \omega t) + \frac{1}{2} P_0^* \exp(-i \omega t) \]
show that the induced absorption rate \( W_{12} \) is given by
\[ W_{12} = \frac{A_{21} I_L \lambda_{21}^3}{4 \pi^2 \hbar c \Delta \omega_a} \left( \frac{g_2}{g_1} \right) \frac{1}{1 + \left[ 2(\omega - \omega_a)/\Delta \omega_a \right]^2} \]
where \( I_L \) is the laser irradiance (W/m^2). Hints: use the relation \( P_0 = \varepsilon_0 \chi(\omega) E_0 \) and note that the power absorbed per unit volume is given by \( P_{\text{abs}} = n_i W_{12} \hbar \omega \). In your expression for power absorbed you will encounter terms which contain \( \exp(\pm 2i \omega t) \). Neglect these terms because their contribution to the power absorbed will average to zero rapidly, after only a few optical cycles. Also note that \( P_0 = P_{0r} + iP_{0i} \).

Solution:

The power absorbed by an individual oscillator is given by \( \vec{F} \cdot \vec{V} \)
\[ \vec{F} \cdot \vec{V} = \left[ -eE_x(t) \right] \left[ \frac{dx(t)}{dt} \right] = E_x(t) \frac{d\mu_x(t)}{dt} \]
For a collection of oscillators
\[ P_{\text{abs}} = n \left[ E_x(t) \frac{d\mu_x(t)}{dt} \right] = E_x(t) \frac{dp_x(t)}{dt} \]
\[ E_x(t) = \frac{1}{2} E_0 e^{i\omega t} + \frac{1}{2} E_0^* e^{-i\omega t} \]
(We can assume without loss of generality that \( E_0 = E_0^* \))
\[ p_x(t) = \frac{1}{2} P_0 e^{i\omega t} + \frac{1}{2} P_0^* e^{-i\omega t} \]
note: \( P_0 \neq P_0^* \) in general
\[ P_0 = \varepsilon_0 \chi(\omega) E_0 = \varepsilon_0 \chi'(\omega) E_0 + iE_0 \chi''(\omega) E_0 \]
\[
P_{\text{abs}} = E_x(t) \frac{dp_x(t)}{dt} = \frac{1}{4} \left[ E_0 e^{i\omega t} + E_0 e^{-i\omega t} \right] \left[ i\omega P_0 e^{\omega t} - i\omega P_0^* e^{-\omega t} \right]
\]

\[
= \frac{1}{4} i\omega \left[ E_0 P_0 e^{2i\omega t} - E_0 P_0^* + E_0 P_0 - E_0 P_0^* e^{-2i\omega t} \right]
\]

The \( e^{2i\omega t} \) and \( e^{-2i\omega t} \) terms average to zero rapidly, oscillating at optical frequencies of \( \sim 10^{15} \) Hz.

\[
P_{\text{abs}} = \frac{1}{4} i\omega \left[ E_0 P_0 - E_0 P_0^* \right]
\]

\[
= \frac{1}{4} i\omega \epsilon_0 E_0^2 \left[ \chi'(\omega) + i\chi''(\omega) - \chi'(\omega) + i\chi''(\omega) \right]
\]

\[
= -\frac{1}{2} \omega \epsilon_0 E_0^2 \chi''(\omega)
\]

\[
= \frac{1}{2} \omega \epsilon_0 E_0^2 \chi''_0 \frac{1}{1 + (\Delta \omega)^2} \quad \Delta \omega = \frac{2(\omega - \omega_a)}{\Delta \omega_a}
\]

\[
= \left( \frac{1}{2} \omega \epsilon_0 E_0^2 \right) \frac{1}{4\pi^2} \frac{\lambda_{21}^3 A_{21}}{\Delta \omega_a} \left( \frac{g_2 n_1 - n_2}{g_1} \right) \frac{1}{1 + (\Delta \omega)^2}
\]

Assume \( n_1 >> n_2 \) (we want the absorption rate, note that \( I_L = \frac{1}{2} c \epsilon_0 E_0^2 \))

\[
P_{\text{abs}} = \left( \frac{\omega}{c} I_L \right) \frac{1}{4\pi^2} \frac{\lambda_{21}^3 A_{21}}{\Delta \omega_a} \left( \frac{g_2}{g_1} n_1 \right) \left[ \frac{1}{1 + (\Delta \omega)^2} \right]
\]

\[
W_{12} = \frac{P_{\text{abs}}}{n_1 \hbar \omega}
\]

\[
W_{12} = \frac{1}{4\pi^2} \frac{I_L^2 \lambda_{21}^3 A_{21}}{\hbar c \Delta \omega_a} \left( \frac{g_2}{g_1} \right) \frac{1}{1 + \left[ \frac{2(\omega - \omega_a)}{\Delta \omega_a} \right]^2}
\]
3. A 0.2 m-long gas cell contains nitric oxide (NO) at a partial pressure of 0.1 kPa and a temperature of 700 K. A monochromatic laser beam is directed through the cell and its frequency is tuned near the \(^{0}\text{P}_{12}(2)\) transition \((N'' = 2, J'' = 1.5; N' = 0, J' = 0.5)\) in the \((v'' = v' = 0)\) vibrational band of the \(A^2\Sigma^+ - X^2\Pi\) electronic transition. The resonance frequency \(\tilde{\nu}_{21} = \tilde{\nu}_{a0} = 44,074 \text{ cm}^{-1}\), and \(A_{21} = 4.852 \times 10^5 \text{ s}^{-1}\) for this transition. For NO, the rotational constant \(B_a = 1.704 \text{ cm}^{-1}\), the vibrational frequency \(\omega_v = 1904 \text{ cm}^{-1}\), and the electronic degeneracy of the ground state is 4. (Hint: the ratio of NO molecules in the given ground energy level to the total number of NO molecules is 3.35 \times 10^{-3}). The cell also contains helium buffer gas at a pressure of 50 kPa and the collisional broadening of the resonance is due almost entirely to the helium buffer gas. At a He pressure of 50 kPa, the homogeneous linewidth of the NO resonance is 0.165 \text{ cm}^{-1}, and it can be assumed that the broadening is due to collisions with helium alone (neglect NO-NO collisions).

(a) What is the peak value of the imaginary part of the susceptibility, 
\[ \chi''_0 = \left( \frac{g_2}{g_1} n_1 - n_2 \right) \frac{\lambda^2_{21} A_{21}}{4 \pi^2 \Delta \omega_a} \]?

(b) For a monochromatic laser beam tuned to line center, what is the fractional transmission through the 0.2 m-long cell? What is the fractional transmission for \(\Delta x = 0.1\)? Repeat the calculations for a He pressure of 400 kPa and an NO partial pressure of 0.10 kPa.

**For a helium pressure of 50 kPa**

\[ \Delta \tilde{\nu}_c = 0.165 \text{ cm}^{-1}, \quad \Delta \tilde{\nu}_D = \tilde{\nu}_{a0} \sqrt{\frac{8 \ln 2 k_B T}{m_a c^2}} \]

\[ g_f(\omega) = \sqrt{\frac{\ln 2}{\pi}} \frac{2}{\Delta \omega_D} V(a, x) \quad a = \sqrt{\ln 2} \frac{\Delta \tilde{\nu}_c}{\Delta \tilde{\nu}_D}, \quad x = 2 \sqrt{\ln 2} \frac{\tilde{\nu} - \tilde{\nu}_{a0}}{\Delta \tilde{\nu}_D} \]

\[ K_{21} = \left( \frac{g_2}{g_1} n_1 - n_2 \right) \frac{\lambda^2_{21} A_{21}}{4} = \text{line strength} \]

We need to find \(K_{21}\) for the \(^{0}\text{P}_{12}(2)\) transition:

\[ \lambda_{21} = \frac{c}{\nu_{21}} = \frac{1}{\tilde{\nu}_{21}} = \frac{2 \pi c}{\omega_a} = \frac{1}{44,074 \text{ cm}^{-1}} = \frac{1}{4,407,400 \text{ m}^{-1}} = 2.269 \times 10^{-7} \text{ m} = 226.9 \text{ nm} \]
Find the number density of NO. For NO:

\[ B_e = 1.704 \, \text{cm}^{-1}, \quad \omega_e = 1904 \, \text{cm}^{-1}, \quad g_{0,\text{elec}} = 4 \]

\[ P \forall = N_{\text{rot}} k_B T \quad \Rightarrow \quad n_{\text{NO}} = \frac{N_{\text{NO}}}{k_B T} = \frac{P_{\text{NO}}}{k_B} = \frac{0.10 \times 10^3 \, J / m^3}{(1.3806 \times 10^{-23} \, J / K)(700 \, K)} = 1.035 \times 10^{22} \, m^{-3} \]

Find the number density of NO molecules in the \( J''=1.5, v''=0 \) level (level 1):

\[ \frac{n_1}{n_{\text{NO}}} = g_1 \exp \left[ -\frac{\epsilon_1}{\hbar c} \left( \frac{\hbar c}{k_B} \right) / T \right] \]

For level 1,

\[ g_1 = 2J''+1 = 4, \quad \frac{\epsilon_1}{\hbar c} = F(J'') + G(v'') = B_0 J''(J''+1) + \omega_c (v''+\frac{1}{2}) \]

\[ B_0 = 1.704 \, \text{cm}^{-1} \quad \Rightarrow \quad \frac{\epsilon_1}{\hbar c} = 952 \, \text{cm}^{-1} \]

The partition functions for NO are given by:

\[ Z_{\text{elec}} = g_{0,\text{elec}} = 4, \quad \theta_{\text{vib}} = \frac{\hbar c \omega_c}{k_B} = 2740 \, K, \quad Z_{\text{vib}} = \frac{\exp(-\theta_{\text{vib}} / 2T)}{1 - \exp(-\theta_{\text{vib}} / T)} = 0.144 \]

\[ \theta_{\text{rot}} = B_0 \left( \frac{\hbar c}{k_B} \right) = 2.39 \, K, \quad \frac{\hbar c}{k_B} = 1.439 \, K / \text{cm}^{-1} \]

\[ Z_{\text{rot}} = \frac{T}{\theta_{\text{rot}}} = 293 \]

\[ \frac{n_1}{n_{\text{NO}}} = 4 \exp \left[ -\left( 952 \, \text{cm}^{-1}\right) \left( 1.439 \, K / \text{cm}^{-1}\right) / (700 \, K) \right] \]

\[ = \frac{3.35 \times 10^{-3}}{(4)(0.144)(293)} = 3.35 \times 10^{-3} \]

\[ n_1 = 3.35 \times 10^{-3} \cdot n_{\text{NO}} = 3.47 \times 10^{19} \, m^{-3} \quad \quad g_2 = 2J'+1 = 2 \]

Now find \( K_{21} \):

\[ K_{21} = \frac{\left( \frac{g_2}{g_1} n_1 - n_2 \right) A_{21}^2}{4} \]

\[ = \frac{\left[ \frac{2}{4} \left( 3.47 \times 10^{19} \right) - 0 \, m^{-3} \right] \left( 226.9 \times 10^{-9} \, m \right)^2 \left( 4.852 \times 10^5 \, s^{-1} \right)}{4} = 1.084 \times 10^{11} \, m^{-1} s^{-1} \]
At this point we have all the information we need to calculate $\chi'_0$:

$$\Delta \omega_c = 2\pi c \Delta \tilde{\nu}_c = 2\pi \left(2.998 \times 10^{10} \text{ cm/s}\right) \left(0.165 \text{ cm}^{-1}\right) = 3.11 \times 10^{10} \text{ s}^{-1}$$

$$\chi''_0 = \left(\frac{g_2 n_1 - n_2}{g_1}\right) \lambda^2_{21} A_{21} = \left[\frac{2}{4} \left(3.47 \times 10^9\right) - 0 \text{ m}^{-3}\right] \left(226.9 \times 10^{-9} \text{ m}\right) \left(4.852 \times 10^5 \text{ s}^{-1}\right)$$

$$= 8.01 \times 10^{-8}$$

Now we need to find the Doppler width:

$$\Delta \tilde{\nu}_D = \tilde{\nu}_{a0} \sqrt{\frac{8 \ln 2}{m_a c^2}} \frac{k_b T}{m_a}$$

$$m_a = m_{NO} = (30 \text{ amu}) \left(1.661 \times 10^{-27} \frac{\text{kg}}{\text{amu}}\right) = 4.98 \times 10^{-26} \text{ kg}$$

$$\Delta \tilde{\nu}_D = (44,074 \text{ cm}^{-1}) \sqrt{\frac{8 \ln 2 \left(1.381 \times 10^{-23} \text{ J/K}\right) (700 \text{ K})}{(4.98 \times 10^{-26} \text{ kg}) (2.998 \times 10^4 \text{ m/s})^2}}$$

$$= (44,074 \text{ cm}^{-1}) \left(3.46 \times 10^{-6}\right) = 0.153 \text{ cm}^{-1}$$

$$\Delta \tilde{\nu}_C = 0.165 \text{ cm}^{-1}$$

$$a = \frac{\sqrt{\ln 2} \Delta \tilde{\nu}_C}{\Delta \tilde{\nu}_D} = \frac{0.165}{0.153} = 0.90$$

For $x = 0$, $V(a, x) = V(0.90, 0) = 0.457$

$$g_v(\omega) = \sqrt{\frac{\ln 2}{\pi}} \frac{2}{\Delta \omega_D} V(a, x)$$

$$\Delta \omega_D = \Delta \tilde{\nu}_D \left(2\pi c\right) = (0.153 \text{ cm}^{-1}) \left(2\pi\right) \left(2.998 \times 10^{10} \text{ cm/s}\right) = 2.88 \times 10^{10} \text{ s}^{-1}$$

$$g_v(\omega) = \sqrt{\frac{\ln 2}{\pi}} \frac{2}{2.88 \times 10^{10} \text{ s}^{-1}} = 0.457 = 1.49 \times 10^{-11} \text{ s}$$

$$A(\omega) = 1 - \exp\left[-K_{21} g_v(\omega) L\right]$$

$$= 1 - \exp\left[-\left(1.084 \times 10^{11} \text{ m}^{-1}\text{s}^{-1}\right) \left(1.49 \times 10^{-11} \text{ s}\right) \left(0.2 \text{ m}\right)\right] = 0.276$$

For $\Delta x = x = 0.1$, $\tilde{\nu} - \tilde{\nu}_{a0} = x \frac{\Delta \tilde{\nu}_D}{2 \sqrt{\ln 2}} = 0.1 \left(0.153 \text{ cm}^{-1}\right) = 0.00919 \text{ cm}^{-1}$

$$V(a, x) = V(0.90, 0.1) = 0.455$$
\[ g_v(\omega) = \sqrt{\frac{\ln 2}{\pi}} \frac{2}{2.88 \times 10^{10} \text{ s}^{-1}} 0.455 = 1.48 \times 10^{-11} \text{ s} \]

\[ A(\omega) = 1 - \exp \left[ -\left(1.084 \times 10^{11} \text{ m}^{-1} \text{s}^{-1}\right) \left(1.48 \times 10^{-11} \text{ s}\right)(0.2 \text{ m}) \right] = 0.274 \]

For a He pressure of 400 kPa,

\[ \Delta \tilde{v}_c = \frac{400}{50} \left( 0.165 \text{ cm}^{-1} \right) = 1.32 \text{ cm}^{-1} \]

\[ a = \sqrt{\ln 2} \frac{\Delta \tilde{v}_c}{\Delta \nu_p} = \sqrt{\ln 2} \frac{1.32}{0.153} = 7.18 \]

For \( a > 3 \) we are off the Voigt profile charts so we will assume pure collisional broadening. For this case then

\[ g(\omega) = g_c(\omega) = \frac{2}{\pi \Delta \omega_c} \frac{1}{1 + (\Delta x)^2} \quad \Delta x = 2 \frac{\omega - \omega_0}{\Delta \omega_c} \]

\[ \Delta \omega_c = 2\pi c \Delta \tilde{v}_c = (2\pi) \left( 2.998 \times 10^5 \text{ cm/s} \right) (1.32 \text{ cm}^{-1}) = 2.49 \times 10^{11} \text{ s}^{-1} \]

For part (b) the value of the line strength is unchanged, \( K_{21} = 1.084 \times 10^{11} \text{ m}^{-1} \text{s}^{-1} \).

At line center,

\[ \Delta x = 0, \quad g_c(\omega) = \frac{2}{\pi \Delta \omega_c} = \frac{2}{\pi \left( 2.49 \times 10^{11} \text{ s}^{-1} \right)} = 2.56 \times 10^{-12} \text{ s} \]

\[ A(\omega) = 1 - \exp \left[ -K_{21} g_c(\omega) L \right] \]

\[ = 1 - \exp \left[ -\left(1.084 \times 10^{11} \text{ m}^{-1} \text{s}^{-1}\right) \left(2.56 \times 10^{-12} \text{ s}\right)(0.2 \text{ m}) \right] = 0.054 \]

For \( \Delta x = 0.1 \)

\[ \Delta x = 0, \quad g_c(\omega) = \frac{2}{\pi \Delta \omega_c} \frac{1}{1 + (\Delta x)^2} = \frac{2}{\pi \left( 2.49 \times 10^{11} \text{ s}^{-1} \right)} 0.99 = 2.53 \times 10^{-12} \text{ s} \]

\[ A(\omega) = 1 - \exp \left[ -K_{21} g_v(\omega) L \right] \]

\[ = 1 - \exp \left[ -\left(1.084 \times 10^{11} \text{ m}^{-1} \text{s}^{-1}\right) \left(2.53 \times 10^{-12} \text{ s}\right)(0.2 \text{ m}) \right] = 0.053 \]
The absorption line shapes for the two cases are shown below:

**DataHW2P3aF2012**
- $P_{He} = 50$ kPa
- $a = 0.90$
- $L = 0.2$ m

**DataHW2P3bF2012**
- $P_{He} = 400$ kPa
- $a = 7.2$
- $L = 0.2$ m