ME 687 Lects 3, 4, and 5
Characteristics of Laser Radiation and Laser Systems

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August 24, 27, and 29, 2012
Outline of the Lecture

• Stimulated Emission and the Four-Level Laser

• Survey of Lasers Used for Spectroscopic Diagnostics

• Properties of Laser Radiation: Polarization of the Electric Field, Polarization Control of Laser Beams

• Properties of Laser Radiation: Gaussian Beam Propagation

• Frequency Conversion Techniques: Sum-Frequency Mixing and Difference-Frequency Mixing in Nonlinear Crystals
The concept of stimulated emission was first developed by Albert Einstein from thermodynamic considerations. Consider a system comprised of a two-level atom and a blackbody radiation field, both at temperature T.

\[ [\rho_v] = J - s / m^3 \]
Interaction of Radiation with Atoms and Molecules: The Two-Level System

From thermodynamic equilibrium

\[ N_2 A_{21} + N_2 B_{21} \rho_v = N_1 B_{12} \rho_v \quad [\rho_v] = J - s / m^3 \]

\[ \rho_v = \frac{8 \pi h \nu^3}{c^3} \frac{1}{(e^{h\nu/kT} - 1)} \quad ; \quad \frac{N_2}{N_1} = \frac{g_2}{g_1} \exp \left( - \frac{\varepsilon_2 - \varepsilon_1}{kT} \right) \Rightarrow \]

\[ A_{21} = \frac{8 \pi h \nu^3}{c^3} B_{21} \quad ; \quad g_2 B_{21} = g_1 B_{12} \]

Units of B must be consistent with units of \( \rho_v \), units of A are sec\(^{-1}\).
Further developments in this course will be in terms of A to avoid confusion on units.
The four-level laser system is an approximate description of the lasing process for almost every laser.

Population inversion between levels 2 and 1 maintained by the pumping process, fast relaxation rates $\gamma_{32}$ and $\gamma_{10}$. 

Types of Lasers Used for Diagnostics of Flames, Plasmas

HeNe, Argon-Ion Lasers
- Gas lasers, population inversion pumped by electric discharge
- Continuous as opposed to pulsed (lower peak powers)
- Fixed frequency as opposed to tunable (can't be tuned to species resonances): 633 nm for HeNe, 486 or 514 nm for Ar⁺
Nd:YAG Lasers
• Q-switched Nd:YAG laser is the workhorse laser for combustion diagnostics

• Q-switching gives 10 nsec pulse, > 1 J/pulse at 1064 nm, 100 MW peak power

• Typically YAG crystal is flashlamp-pumped, diode-laser-pumped models are now commercially available and capable of high-repetition-rate operation (10 kHz instead of 10 Hz)

• Fundamental output at 1064 nm rarely used for diagnostics, frequency-doubled output at 532, tripled at 355 nm, quadrupled at 266 nm more commonly used

• 532 nm output: dye laser pump, CARS pump beam, Mie and Rayleigh scattering, laser-induced incandescence (LII)
Q-Switched Nd:YAG Laser

Energy level diagram showing dominant pump bands and the 1064-nm laser transition for the Nd$^{3+}$ ion in the crystal host yttrium aluminum garnet. Diode pumping performed on the 790 nm transition.
Q-Switched Nd:YAG Laser

POWERLITE™ PRECISION II 9000 OPTICAL LAYOUT

1. Rear Mirror
2. Pockels Cell
3. 1/4 Wave Plate
4. Dielectric Polarizer
5. Oscillator Head
6. Output Coupler
7. IR Mirror
8. 1/2 Wave Plate
9. Amplifier Head
10. Rotator
11. Dichroics, 532 nm
12. Dichroics, 355 or 266 nm
Q-Switched Nd:YAG Laser

Flashlamp-pumped Nd:YAG rod in pump chamber

The pump chamber: a perfect balance between efficiency and mode quality
Q-Switched Nd:YAG Laser: Flashlamp-Pumped vs. Diode-Pumped

Flashlamp-pumped Q-switched Nd:YAG lasers

- Repetition rate: 10-30 Hz
- Pulse energy (532 nm): 1 J
- Pulse length: 2-10 ns
- Frequency width (532 nm): 0.001 cm$^{-1}$

Diode-pumped Q-switched Nd:YAG lasers

- Repetition rate: 1-30 kHz
- Pulse energy (532 nm): 10 mJ
- Pulse length: 2-100 ns
- Frequency width (532 nm): 1.0 cm$^{-1}$
High-Repetition-Rate Laser System

Edgewave Diode-Pumped Solid State Nd:YAG Laser: 5 kHz Rep Rate, Dual-Head, 6 mJ/Pulse at 532 nm, 7 nsec Pulses

Sirah Credo Dye Laser: 5 kHz Rep Rate, 500 \( \mu \)J/Pulse at 283 nm (2.5 W average power in UV)
Dye Laser – Tunable Frequency Output

- Fundamental output is tunable.
- Changing the angle of grating changes laser frequency.
- Different wavelength regions accessed with different dyes (400 nm – 1000 nm, further extended with frequency conversion techniques like frequency doubling).
- Frequency bandwidths can be very narrow (< 0.01 cm\(^{-1}\)).
- Dye laser pulses are typically the same length temporally as the pump source, dye has very short fluorescence lifetime.
Continuum Dye Laser – Nd:YAG-Pumped
Excimer Laser - Pulsed, Ultraviolet

- Gas laser, upper laser level is electronic level of molecules like KrF\(^*\) (248 nm), XeCl\(^*\) (308 nm), and ArF\(^*\) (193 nm); these molecules are stable only in the excited state.
- Pulse lengths of 30 nsec, pulse energies of up to a few hundred mJ, rep rates of more than 1 kHz.
- Tunable narrowband excimer lasers have been developed.
- Application of these lasers for UV Raman has increased because of high Raman cross sections in UV (cross section prop. to \(\nu^4\)), narrowband excimer can be tuned away from interfering LIF lines.
Ultrafast (Femtosecond) Laser Systems

- Based on oscillation and amplification in Titanium:Sapphire crystals (Ti:S)

- Pulse lengths of a few 10’s of fsec to a few psec with the same system architecture, output powers of ~10 W, rep rates of more than 1 kHz (many mJ per pulse at 1 kHz rep rate)

- Tunable radiation produced using computer controlled optical parametric amplifiers

Spectra-Physics Ultrafast Laser System
Coherent Ultrafast (Femtosecond) Laser System

**Mantis Mode-Locked Ti:S Laser with Integrated OPSL Pump Laser**
- 800 nm

**Silhouette 128-Pixel MIIPS Pulse Shaper**

**Evolution-HE Nd:YLF Pump Laser**
- 527 nm
- 90 W at 10 kHz

**Legend Elite Ti:S Regenerative Amplifier and Single-Pass Amplifier**
- 800 nm
- 13 W at 5 kHz
- 10.5 W at 10 kHz

**OPerA Solo Optical Parametric Amplifier + Nonlinear Crystals**
- 200 nm - 10 μm

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Coherent Ultrafast Laser System
Ultrafast (Femtosecond) Laser Systems

Coherent Ultrafast Optical Parametric Amplifier

Typical OPerA Solo Tuning Curve
Legend Elite USP-1K-HE Pump (3.3 mJ)
Semiconductor or Diode Laser

• Very small and efficient, emission at wavelengths above about 350 nm

• Single frequency mode output using external cavity (ECDL) or distributed feedback (DFB) architecture to write grating on diode itself

• Frequency can be swept at very high rates

• Further development of these lasers is active area of research (higher power, lower and higher wavelengths)

Yariv, Introduction to Quantum Electronics, 1976
Electromagnetic Properties of Laser Radiation

- Infinite Plane Wave Approximation:
  Infinite plane wave with electric field polarized in x-direction, propagates in z-direction at speed c, properties uniform in x-y plane

\[
\vec{E}(\vec{r}, t) = \hat{x} \left\{ \frac{1}{2} E_0 \exp \left[ -i (k z - \omega t) \right] + \frac{1}{2} E_0^* \exp \left[ i (k z - \omega t) \right] \right\} \\
= \hat{x} \frac{1}{2} E_0 \exp \left[ i (k z - \omega t) \right] + c.c.
\]

\[\omega = \text{angular frequency} = 2\pi v = \frac{2\pi c}{\lambda} \quad \quad k = \frac{2\pi}{\lambda}\]
Electromagnetic Properties of Laser Radiation

Given time $t$

Given spatial location $z$

$E_x(z)$

$E_x(t)$

$\lambda$

$1/\nu$

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Linear Polarization, 45° to x-axis

\[ \vec{E}(z,t) = \hat{x} E_{0x} \cos(kz - \omega t) + \hat{y} E_{0y} \cos(kz - \omega t + \pi) \]
Time Dependence of Linear Polarization

Linear Polarization, 45° to x-axis, z = 0

\[ \vec{E}(z,t) = \hat{x} E_{0x} \cos(\omega t) + \hat{y} E_{0y} \cos(\omega t) \]

\[ t = 0 \]

\[ t = \pi/\omega \]

\[ t = \pi/2\omega \]

\[ t = 3\pi/2\omega \]
Polarization of Laser Radiation

Circular Polarization: electric field vector moves in a circle

\[ \vec{E}(z,t) = \hat{x} E_{0x} \cos(kz - \omega t) + \hat{y} E_{0y} \cos(kz - \omega t + \frac{\pi}{2}) \]
In a birefringent crystal like calcite, refractive index is different for light polarized parallel to (e-wave) and perpendicular to (o-wave) the optic axis.

In calcite $n_o > n_e$

In Glan polarizer, crystal cut at angle $\theta$ such that

$$n_e < \frac{1}{\sin \theta} < n_o$$

so that o-ray undergoes total internal reflection.

Hecht, *Optics*, 1987
Half-wave plate are used to rotate the polarization of linearly polarized light. Quarter-wave plates are used to convert circular to linear polarization and vice versa.
"Good" laser beams have a Gaussian intensity profile

\[ I(r) = \frac{2P}{\pi w^2} e^{-2r^2/w^2} \quad P = \text{power (W)}, \quad w = 1/e^2 \text{ radius} \]

Gaussian Beam Propagation

Gaussian beams propagate as Gaussians

\[ w(z) = w_0 \sqrt{1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2} = w_0 \sqrt{1 + \left( \frac{z}{z_R} \right)^2} \]

for \( z \gg z_R \), \( w(z) = \frac{\lambda z}{\pi w_0} \)

\( z_R = \text{Rayleigh range} = \frac{\pi w_0^2}{\lambda} \)


FIGURE 17.1
Notation for a lowest-order gaussian beam diverging away from its waist.
Gaussian Beam: Transmission Through Aperture

Power transmission through circular aperture.

Effect of transmission through circular aperture.

Gaussian Beam Focusing

Focused waist: \( w_0 \, w_{\text{lens}} = \frac{\lambda \, f}{\pi} \)

\( d_0 = 1/e^2 \) diameter of focus \( = 2 \, w_0 \)

\[ D = \frac{(\pi/2)(1/e^2 \text{ diameter on lens})}{\pi} = \pi \, w_{\text{lens}} \]

(99% energy contained)

\[ d_0 = \frac{2 \, f \, \lambda}{D} \]

Gaussian beams focus as Gaussians

Siegman, Lasers, 1986
Nonlinear crystals are used commonly to convert laser beam from one spectral region to another, e.g., frequency-doubling to obtain ultraviolet light from visible light.

The process shown on the left is Type I sum-frequency mixing (SFM) in beta barium borate (BBO) or potassium dihydrogen phosphate (KDP). When $\omega_1 = \omega_2$, the process is referred to as frequency doubling.
Second-Harmonic Generation – Sum-Frequency Mixing (SFM) Process

$\beta$-BBO Crystal

$\omega$ – Fundamental Beam

$2\omega$ – Second Harmonic Beam
Frequency Doubling in Nonlinear Crystals

Yariv, Quantum Electronics, 1975
Fourier analysis of polarization induced in the nonlinear crystal by light field at angular frequency $\omega$ reveals component at angular frequency $2\omega$. Polarization in the crystal at $2\omega$ serves as source term for development of light field at $2\omega$. Light field at $2\omega$ builds up to significant intensity in the crystal only if all the atoms in the crystal are oscillating with the correct phase – the phase-matching condition.

Yariv, *Quantum Electronics*, 1975
Difference-Frequency Mixing (DFM) or the Optical Parametric Process

\[ \omega_3 = \omega_2 + \omega_1 \]

- \( \omega_3 \) – Pump Beam
- \( \omega_1 \) – Idler Beam
- \( \omega_2 \) – Signal Beam

\( \beta \)-BBO Crystal