Introduction to Bifurcation and Chaos

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Class Notes
Course grade will be based on:
(i) some home assignments;
(ii) seminar presentations prepared on the basis of assigned topics from literature;
(iii) project.
Introduction

Examples of nonlinear behavior in structural, fluid-mechanical and chemical/biological systems.
1. Onset of wavy vortices in the finite-length Couette-Taylor problem


Consider two concentric cylinders forming an annulus – There is fluid in the gap between the two cylinders. The inner cylinder rotates at a constant rate $\Omega$. At slow rates, the fluid undergoes steady shearing motion.
Onset of wavy vortices

At some threshold value of $\Omega$, uniform pattern becomes unstable and gives way to flow in the form of cells along the axial direction. The cell size (axial wave number) is $\sim$ the gap width. These are called Taylor cells.
1. Onset of wavy vortices........

The streamlines of low Taylor number T-C flow are circular. This means that if you were to insert a needle with dye into the flow, the dye would trace out a circular path. If we view this flow from a radial viewpoint, we see the picture here.

Ref: http://www.princeton.edu/~gasdyn/Research/T-C_Research_Folder/Intro_to_T-C_Flows.html
Onset of wavy vortices………

Once the flow becomes unstable the picture radically changes. The flow becomes dominated by toroidal Taylor vortices. A cross sectional view of the flow field looks like:
The figure illustrates $m=5$ ‘wavy’ flow near its onset – the axisymmetric (Taylor) flow with $n=9$ vortices (cells) in the axial direction becomes unstable.

End effect is also seen in the figure.
2. Faraday Waves

http://www.physics.utoronto.ca/nonlinear/

Applying a vertical sinusoidal oscillation to a dish of fluid effectively modulates the acceleration of gravity seen by the fluid. If this modulation exceeds a critical value, the normally flat state of the free surface becomes unstable to the formation of surface waves. These waves have a frequency which is half that of the driving oscillations (the first sub-harmonic). This effect was first reported by Michael Faraday in 1831.
In ordinary Newtonian fluids (those that do not exhibit shear thickening or shear thinning) the wave patterns include ones with 1-fold symmetry (stripes), 2-fold symmetry (squares), 3-fold symmetry (hexagons) as well as higher orders of symmetry.
2. Faraday Waves………

patterns on the surface

Stripes, as viewed on surface

Squares
2. Faraday Waves........

Hexagons

Spirals (metastable)
In a horizontal fluid layer maintained at temperature difference across it by heating from below and cooling from above, the fluid flows in a pattern of convection cells if the temperature difference exceeds a threshold value. The fluid motion transports additional heat over and above that carried by thermal conduction through the fluid. Many variations on this process have been studied since Bénard's first experiments around the turn of the 20th century.
The fluid is confined between rigid plates held at constant temperature. Lord Rayleigh showed (1916) that an initially motionless fluid layer becomes *unstable* to small flow perturbations when the temperature difference is sufficiently large. The flow pattern consists of *rolls* or *hexagonal cells*, depending on the temperature dependence of the fluid. The flow pattern may be stable, undergo secondary instabilities or become chaotic.
4. Hysteresis and Loop Formation

Mike Thompson and Group, UCL, Center for Nonlinear Dynamics

straight untwisted rod

input twist $(R)$

input slack $(D)$

smooth input of $D$

jump into a loop, "hockling"

"snarling"
5. Flow induced instabilities

Post-divergence shape of the rubber shell for mode $n=3$.

a. Computed shape for flow velocity 30 m/s;
b. Experimental shape, from Ref. 7

6. Buckling of rings under axial moments

Ref: F. Pai et al. 2001/U Missouri, Columbia

Fig. 4: Large static deformation of a circular ring subjected to a pair of twisting moments.
6. Buckling of rings under axial moments......

Fig. 5: Large deformation test and total-Lagrangian finite element analysis of a circular ring subjected to a pair of twisting moments.
7. Break-up of a liquid jet

A series of photographs showing different stages of fission that occur when a pendant water drop falls from a circular plate.

Viscous journal bearing flow: the flow between two periodically rotated eccentric cylinders, rotated in opposite directions.

9. Patterns of complexity-2

Rectangular cavity flow with periodically moving boundaries: in a period, the upper plate is moved to the right, and then the bottom plate is moved to the left.