FROM LAST TIME...

System Interfacing (DC Motors)

- DC motor theory
- DC motor nomenclature
- Torque/speed tradeoff
- DC output power
- H-bridge operation

\[ \omega = 0.5 \omega_{\text{NL}} \]

\[ T = 0.5 T_{\text{STALL}} \]

\[ V_{\text{IN}} \text{ increase} \]
UNIT 9: DC MOTOR GEARING AND CONTROL
SPUR GEARS

Images: http://science.howstuffworks.com/transport/engines-equipment/gear2.htm
http://robotshop.com
PLANETARY GEARS

Images: http://www.carbibles.com/transmission_bible_pg2.html
http://robotshop.com
SPUR AND PLANETARY GEARHEADS HAVE RELATIVE ADVANTAGES

<table>
<thead>
<tr>
<th>Factor</th>
<th>SPUR</th>
<th>PLANETARY</th>
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</thead>
<tbody>
<tr>
<td>Size</td>
<td>Larger</td>
<td>Smaller</td>
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<tr>
<td>Cost</td>
<td>Lower</td>
<td>Higher</td>
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<tr>
<td>Load Capacity</td>
<td>Lower</td>
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<tr>
<td>Operating Speed</td>
<td>Lower</td>
<td>Higher</td>
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<tr>
<td>Backlash</td>
<td>Higher</td>
<td>Lower</td>
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<tr>
<td>Efficiency</td>
<td>Lower</td>
<td>Higher</td>
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<tr>
<td>Noise</td>
<td>Lower</td>
<td>Higher</td>
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<tr>
<td>Centerline Offset</td>
<td>Offset</td>
<td>Inline</td>
</tr>
</tbody>
</table>

- Gears may be plastic, ceramic, or metal
  - There's a reason why plastic is cheaper!
**MOTOR & LOAD CHARACTERISTICS ARE MATCHED WITH GEAR TRAINS**

Gear ratio (or speed ratio):

\[ r = \frac{\omega_M}{\omega_L} = \frac{N_L}{N_M} \]

Torque ratio*:

\[ r = \frac{T_L}{T_M} \]

* assuming perfect efficiency
Precision Gearmotor - 90 RPM (6-12V)

**ROB - 12497**

**Description:** These precision gearmotors are incredibly tough and feature full metal gears to help you drive wheels, gears, or almost anything else that needs to turn. They have a gear ratio of 50:1 and operate up to 12 volts and deliver a stall torque of 138.8 oz-in. and a max speed of 90 RPM. Each precision gearmotor sports a 6mm diameter D-shaft that protrudes from them.

**Features:**

- Voltage: 6 - 12 Volts
- Gear Ratio: 50:1
- Stall Torque: 138.8 oz-in. (@ 12V)
- Speed: 90 RPM (@ 12V)
- No Load Current: 120 mA (@ 12V)
- Stall Current: 1A (@ 12V)
- Insulation Resistance: 20 MΩhm
- Dielectric Strength: 250VDC
- High Torque Construction
- DC Reversible
- Shaft Size: 6mm Diameter x 0.715" Length
- Weight: 8.05 oz.

**Documents:**

- Dimensional Drawing
Standard Gearmotor - 81 RPM (3-12V)
ROB - 12310

Description: These standard gearmotors are incredibly tough and feature full metal gears to help you drive wheels, gears, or almost anything else that needs to turn. They have a gear ratio of 50:1 and operate up to 12 volts and deliver a stall torque of 57 oz-in. and a max speed of 81 RPM. Each standard gearmotor sports a 6mm diameter D-shaft.

Features:
- Voltage: 3 - 12 Volts
- Gear Ratio: 50:1
- Stall Torque: 57 oz-in. (@ 12V)
- Speed: 81 RPM (@ 12V)
- No Load Current: 195 mA (@ 12V)
- Stall Current: 0.5A (@ 12V)
- Insulation Resistance: 10 MOhm
- Dielectric Strength: 300VDC
- DC Reversible
- Shaft Size: 6mm Diameter x 0.715" Length
- Weight: 4.2 oz.

Documents:
- Dimensional Drawing
OBTAIN DESIRED SYSTEM PERFORMANCE WITH GEAR TRAINS

- Gear ratio (or speed ratio): \( r = \frac{N_L}{N_M} = \frac{\omega_M}{\omega_L} \)
- Mechanical advantage: \( r \cdot \eta = \frac{T_L}{T_M} \)
- Trade off speed for torque, or torque for speed
- Many motor and gearbox pairs are possible
INERTIAS DETERMINE MOTOR RESPONSIVENESS

\[ \eta \theta_L, \omega_L \]

\[ r = \frac{N_L}{N_M} \]

\[ J_T = J_M + \frac{J_L}{r^2} \]
INERTIAS DETERMINE MOTOR RESPONSIVENESS

Total inertia seen by the motor is:

\[ J_T = J_M + \frac{J_L}{r^2} \]

where

\( J_T \): Total inertia seen by motor
\( J_M \): Motor rotor inertia
\( J_L \): Load inertia
\( J_G \): Gear train inertia
\( r \): Gear ratio

"reflected inertia"

Motors inertia is normally small, at least in comparison with common load inertias. At a high enough gear ratio, however, motor inertia can become the dominant inertial effect!
Sometimes it takes a bit of work to convert inertia values into the proper units...

<table>
<thead>
<tr>
<th>Motor Data</th>
<th>118749</th>
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<th>118751</th>
<th>118752</th>
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<tbody>
<tr>
<td>1. Assigned power rating</td>
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<td>2. Nominal voltage</td>
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<td>3. No load speed</td>
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<td>4. Stall torque</td>
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<td>5. Speed / torque gradient</td>
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<td>6. No load current</td>
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<td>7. Starting current</td>
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<td>8. Terminal resistance</td>
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<td>9. Max. permissible speed</td>
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<td>10. Max. continuous current</td>
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<td>11. Max. continuous torque</td>
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<td>12. Max. power output at nominal voltage</td>
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<td>13. Max. efficiency</td>
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<td>14. Torque constant</td>
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<td>15. Speed constant</td>
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<td>16. Mechanical time constant</td>
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<td>17. Rotor inertia</td>
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<td>18. Terminal inductance</td>
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<td>19. Thermal resistance housing-ambient</td>
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<td>20. Thermal resistance rotor-housing</td>
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<td>21. Thermal time constant winding</td>
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</table>
INERTIAL RATIOS ESTABLISH SYSTEM PERFORMANCE

- Minimizing motor inertia \( (J_M) \) allows motor torque to be more effective in accelerating the load (as it does not “waste” energy accelerating its own inertia). Remember, however, that we often want to maximize power transfer, not load acceleration!

- If the motor inertia is greater than the reflected load inertia, the majority of the electrical power is going into rotating the motor, not the load. This is inefficient.

- If the reflected inertia is much larger than the motor inertia, the motor is limited in its ability to quickly start and stop. This results in poor dynamic response.
INERTIA MATCHING MAXIMIZES POWER TRANSFER (FOR FIXED TORQUE)

Kinetic energy of the load is:

\[ KE_L = \frac{1}{2} J_L \omega_L^2 \]

Power at the load is the time differentiation of kinetic energy:

\[ P = \frac{d}{dt} KE_L = \frac{d}{dt} \left( \frac{1}{2} J_L \omega_L^2 \right) = J_L \omega_L \cdot \frac{d}{dt} \omega_L \]

Power reflected at the motor end is:

\[ P_L = \frac{J_L}{r^2} \omega_M \cdot \frac{d}{dt} \omega_M = \frac{J_L}{r^2} \omega_M \cdot \alpha_M \]

Acceleration is proportional to applied torque:

\[ \alpha_M = \frac{T_M}{J_T} = \frac{T_M r^2}{J_M r^2 + J_L} \quad \text{and} \quad \omega_M = \alpha_M \cdot t \]

Since acceleration is constant, the power to the load is:

\[ P_L = J_L r^2 t \cdot \frac{T_M^2}{(J_M r^2 + J_L)^2} \]

Maximize \( P_L \) by differentiating \( P_L \) with respect to \( r \):

\[ J_M = \frac{1}{r^2} J_L \quad \text{or} \quad r = \sqrt{\frac{J_L}{J_M}} \]
TRADEOFF BETWEEN RESPONSE AND EFFICIENCY IS COMMON

- While 1:1 matching of reflected and motor inertias minimizes starting and stopping times, you may find need for other ratios. Keep in mind that a system's dynamic characteristics are governed by the larger inertia.

- If dynamic response is important, try to keep the ratio less than 10:1. Bosch Rexroth recommends:
  - < 2:1 for quick positioning
  - < 5:1 for moderate positioning
  - <10:1 for quick velocity changes
WORM GEARS

Images: http://electrolift.com/the-worm-gear-advantage.php
http://www.longwaymotor.com/imagefile/pro/125013500416790.jpg
DYNAMIC RESPONSE

\[ V_{\text{IN}} \quad R_A \quad i_A \quad L_A \quad V_{\text{EMF}} \]

\[ + \quad V_M \quad - \]

\[ T_M \quad \rightarrow \ \theta, \ \omega \quad T_L \]

\[ i_A \quad K_T \quad T_M \]

\[ q, \ \omega \]
DYNAMIC RESPONSE

\[ V_M = \frac{1}{L_A s + R_A} i_A \]

\[ T_M = K_T i_A \]
DYNAMIC RESPONSE

\[ V_{IN} \rightarrow V_M \rightarrow \frac{1}{L_{AS} + R_A} \rightarrow i_A \rightarrow K_T \rightarrow T_M \]

\[-K_{EMF} \rightarrow \omega \rightarrow T_L \rightarrow \theta, \omega \]

\[ i_A = \frac{V_M}{L_{AS} + R_A} \]

\[ T_M = K_T i_A \]

\[ V_{EMF} = -K_{EMF} \omega \]

\[ J_M \]

\[ T_L \]

\[ T_M \]

\[ V_{IN} \]

\[ V_M \]

\[ i_A \]

\[ K_T \]

\[ \omega \]
DYNAMIC RESPONSE

\[ \begin{align*}
  V_{IN} & \rightarrow V_M \\
  & \rightarrow \frac{1}{L_{AS} + R_A} \\
  & \rightarrow i_A \\
  & \rightarrow K_T \\
  & \rightarrow T_M \\
  & \rightarrow T_{out} \\
  & \rightarrow -K_{EMF} \\
  & \rightarrow \omega
\end{align*} \]
DYNAMIC RESPONSE

\[ \tau = J \dot{\omega} + b \omega \]

\[ \Omega(s) = \frac{K_T}{L_A J_M s^2 + (b_M L_A + R_A J_M)s + (R_A b_M + K_{EMF} K_T)} \cdot V_{IN}(s) \]

Jeff Shelton – 3 March 2015
DYNAMIC RESPONSE

\[ \Omega(s) = \frac{K_T}{(R_A J_M) s + (R_A b_M + K_{EMF} K_T)} \cdot V_{IN}(s) \]
\[ \Omega(s) = \frac{K_T}{(R_A J_M)s + K_{EMF} K_T} \cdot V_{IN}(s) \]
DYNAMIC RESPONSE

Approximations:

- Neglecting both Armature Inductance \( L_A \) and Load Friction \( b_M \)

\[
\Omega(s) = \frac{K_T}{R_A J M s + K_{EMF} K_T} \cdot V_{IN}(s) \Rightarrow \frac{\Omega(s)}{V_{IN}(s)} = \frac{G(0)}{\tau_{MS} + 1}
\]

- Steady-state gain (speed/volt):

\[
G(0) = \frac{K_T}{K_{EMF} K_T} = \frac{1}{K_{EMF}}
\]

- Mechanical Time Constant:

\[
\tau_M = \frac{R_A J M}{K_{EMF} K_T}
\]
OPERATING ISSUES

Two basic modes of operation:

- Current (torque) mode – controlling current *through* winding
- Voltage (velocity) mode – controlling voltage *across* winding

Velocity mode is usually easier to implement (e.g., through PWM), but torque mode is certainly possible.
Voltage (Mode) Control Amplifier

- Power amplifier produces an output voltage in response to the input command.
- Motor will reach a steady-state speed for a given constant input voltage.
- Zero voltage (command) produces breaking (due to electrical dissipation)
**MOTOR DRIVERS (AMPLIFIERS)**

Voltage (Mode) Control Amplifier

Unipolar

\[
\frac{V_A}{V_M} = \frac{R_A}{R_A + R_B}
\]

\[
V_{CMD} = \frac{R_A}{R_A + R_B} V_M \quad \Rightarrow \quad V_M = \frac{R_A + R_B}{R_A} V_{CMD}
\]

Bipolar
MOTOR DRIVERS (AMPLIFIERS)

Current (Mode) Control Amplifier

- Power amplifier produce an motor current in response to the input command.
- Normally uses feedback around the motor and amplifier. Feedback will compensate the back-EMF to the limit of the power supply.
- Improves dynamic performance – voltage rises sharply during initial transient.
MOTOR DRIVERS (AMPLIFIERS)

Current (Mode) Control Amplifier
MOTOR DRIVERS (AMPLIFIERS)

PWM Amplifier

- Darlington Connection
  - For low power applications (less than 0.5 Amp): can connect the digital part directly to analog transistors.
  - Darlington connection uses two stage amplification to bring the current capacity to about 1 Amp.

![Darlington Transistor Diagram](image-url)
ELECTRIC MOTORS

DC Motors
- Brushed
  - Permanent Magnet
  - Series Wound
  - Shunt Wound
  - Compounded
  - Separately Excited
  - Brushless
    - Stepper

AC Motors
- Synchronous
  - Reluctance
  - Hysteresis
  - Permanent Magnet
- Induction
  - Wound Rotor
  - Squirrel Cage
MAGNETIC FIELD SOURCE DEFINES BRUSHED DC MOTOR CLASSIFICATION

- Permanent-Magnet DC Motors (PMDC)
- Field Coil Induced Magnetic Field
ALTERNATE SOURCES OF MAGNETIC FIELD

Field Coil Induced Magnetic Field

- Series Wound DC Motor
  - High starting torque and no-load speed
  - Poor speed regulation
  - Good for getting heavy loads moving
- Shunt Wound DC Motor
  - Low starting torque and no-load speed
  - Poor torque regulation
  - Nearly constant speed, regardless of load
- Compound DC Motor
  - High starting torque
  - Good speed and torque regulation
  - Combines good features of series and shunt
- Separately Excited DC Motor
  - High torque capabilities at low speeds
LINEAR MOTORS

Rotary-to-linear converters, such as lead-screw and belt-and-pulley, have losses and dynamic effect that will need to address in addition to motor control.

Direct production of linear force/motion can be accomplished using linear motor technology (un-warp a rotary motor):
LINEAR MOTORS

- Can produce smoother motion than rotary motor plus converters.
- Motor needs to be as long as the motion path.
- No load inertia matching characteristics – no gearbox.
- Most of the motor does not participate in force generation.
- System stiffness needs to be generated through control.
BRUSHLESS DC (BLDC) MOTORS

Permanent magnet on rotor (usually the outer case) and three phase coil excitation on stator. In this case, the stator is the armature, as it is the portion through which current flows.

Use rotor angular position feedback to electronically commutate the coil (phase) currents.
ELECTRIC MOTORS

DC Motors
- Brushed
  - Permanent Magnet
  - Series Wound
  - Shunt Wound
  - Compounded
  - Separately Excited
- Brushless
  - Stepper

AC Motors
- Synchronous
  - Reluctance
  - Hysteresis
  - Permanent Magnet
- Induction
  - Wound Rotor
  - Squirrel Cage
BRUSHLESS DC (BLDC) MOTORS

BRUSHLESS DC (BLDC) MOTORS

[Image: http://www.rcuniverse.com/magazine/reviews/1344/BrushlessMotors7.jpg]
BRUSHLESS DC (BLDC) MOTORS

- Uses three-phase DC signals and requires three channels of power amplification.
- Excitation is a function of rotor position. On-off excitation switching needs discrete point measurement.
  - Hall effect sensors are generally used.
  - Non-excited coil back-emf can also be used.
BRUSHLESS DC (BLDC) MOTORS

Pros
- No Brushes
  - Less maintenance
  - Less electrical noise
  - Can use higher voltages
  - More efficient, due to friction reduction

Cons
- Require Additional Components
  - More electronics
  - Rotor position sensor
- Higher Torque Ripple
  - Can be reduced by using sinusoidal excitation
    - Requires linear or PWM amplifier with higher precision rotor position measurement.
  - Can be reduced by adding more commutation points
    - Not practical – needs too much more electronics.
COMING UP...

System Interfacing

- Stepper motor designs
- Stepper motor actuation
- Stepper motor characteristics