Laplace and z-Transforms

Modified from Table 2-1 in Ogata, Discrete-Time Systems

The sampling interval is Δ seconds. In the table below all signals are assumed to be 0 for $t < 0$ seconds, whereas in ME 579 we do have signals that are two sided and define: $X(z) = \sum_{n=-\infty}^{+\infty} x_n z^{-n}$. When we have a signal that is non-zero for negative time, we can split the signal into a positive time component, p_n, and a negative time component, q_n, and you can show that $X(z) = P(z) + Q(z^{-1})$. When you do this, you need to be careful with how you deal with the $t = 0, n = 0$ point and take into account any delays.

Laplace Transform $X(s)$	Continuous Signal $x(t)$	Sampled Signal $x(n\Delta) \equiv x_n$	z-Transform $X(z)$	Region of Convergence
-	-	Kronecker delta $\delta_n = 0, n \neq 0;$ $\delta_n = 1, n = 0.$	1	all z
-	-	Kronecker delta $\delta_{n-k} = 0, n \neq k;$ $\delta_{n-k} = 1, n = k.$	z^{-k}	$\lvert z\rvert > 0$
$\frac{1}{s}$	$= 1$ for $t \geq 0$	$=1$ for $n \geq 0$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$\frac{1}{s+a}$	e^{-at}	$e^{-a\Delta n}$	$\frac{1}{1-e^{-a\Delta}z^{-1}}$	$\lvert z\rvert > e^{-a\Delta}$
$\frac{1}{s^2}$	t	$n\Delta$	$\frac{\Delta z^{-1}}{(1-z^{-1})^2}$	$\lvert z\rvert > 1$
$\frac{2}{s^3}$	t^2	$(n\Delta)^2$	$\frac{\Delta^2 z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$	$\lvert z\rvert > 1$
$\frac{6}{s^4}$	t^3	$(n\Delta)^3$	$\frac{\Delta^3 z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$	$\lvert z\rvert > 1$

Continuing the table, and reminding ourselves that these time functions are all defined to be zero for $t = n\Delta < 0$ seconds.

Laplace Transform $X(s)$	Continuous Signal $x(t)$	Sampled Signal $x(n\Delta) \equiv x_n$	z-Transform $X(z)$	Region of Convergence
$\frac{1}{(s+a)^2}$	te^{-at}	$\Delta n e^{-a\Delta n}$	$\frac{\Delta e^{-a\Delta} z^{-1}}{(1-e^{-a\Delta}z^{-1})^2}$	$\|z\| > e^{-a\Delta}$
$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-a\Delta n)e^{-a\Delta n}$	$\frac{1-(1+a\Delta)e^{-a\Delta}z^{-1}}{(1-e^{-a\Delta}z^{-1})^2}$	$\|z\| > e^{-a\Delta}$
$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}sin(\omega t)$	$e^{-a\Delta n}sin(\omega\Delta n)$	$\frac{e^{-a\Delta}z^{-1}sin(\omega\Delta)}{1-2e^{-a\Delta}cos(\omega\Delta)z^{-1}+z^{-2}}$	$\|z\| > e^{-a\Delta}$
$\frac{(s+a)}{(s+a)^2+\omega^2}$	$e^{-at}cos(\omega t)$	$e^{-a\Delta n}cos(\omega\Delta n)$	$\frac{1-e^{-a\Delta}z^{-1}cos(\omega\Delta)}{1-2e^{-a\Delta}cos(\omega\Delta)z^{-1}+z^{-2}}$	$\|z\| > e^{-a\Delta}$
-	-	b^n	$\frac{1}{(1-bz^{-1})}$	$\|z\| > \|b\|$
-	-	$b^{(n-q)}, n \geq q;$ $0, n < q$	$\frac{z^{-q}}{(1-bz^{-1})}$	$\|z\| > \|b\|$
-	-	nb^{n-1}	$\frac{z^{-1}}{(1-bz^{-1})^2}$	$\|z\| > \|b\|$
-	-	n^2b^{n-1}	$\frac{z^{-1}(1+bz^{-1})}{(1-bz^{-1})^3}$	$\|z\| > \|b\|$