Homework #8 ME 576

1.

(a)

$$1\ \text{kW}=1.34\ \text{HP}$$

$$\therefore\ 2\,\text{HP}=\frac{2}{1.34}=1.492\,\text{kW}$$

Back induced e.m.f. during idling

$$V_i = k_b n = 1\ (V\cdot s)(600\,rpm)\left(\frac{1}{60}\frac{\text{min}}{\text{sec}}\right)\left(2\pi\, {rad}/_{\text{sec}}\right)$$

$$= 20\pi\cdot(V)$$

Power consumption during idling

$$P = V_A \cdot I_A$$

$$= \left(R_A\cdot I_A + V_i\right) I_A = 1{,}492$$

$$R_A I_A{}^2 + (20\pi) I_A - 1{,}492 = 0$$

$$\therefore I_A = \frac{-10\pi \pm \sqrt{(10\pi)^2 + 10\times 1492}}{10} = 9.47(\text{Amp}).$$

For speed control, I_A is maintained at a constant level and so is T_{max}.

$$T_{\max} = K_T\cdot I_A = 1\left({kg\cdot m}/_{A}\right)\cdot 9.47(A)$$

$$= 9.47 kg\cdot m = 92.8\,N\cdot m.$$

Maximum acceleration is attained when the motor starts running

$$T_M = J_M\frac{d\omega(t)}{dt} + \underbrace{B\omega(t)}_{0} + \underbrace{\cancel{T_L}}_{0}$$

$$
\begin{aligned}
\dot{\omega}_{\max} &= \frac{T_{\max}}{J_m} \\
&= \frac{9.47(kg \cdot m)}{0.1(kg \cdot m^2)} = \frac{9.47 \times 9.8\left(kgm \cdot m^2 / \sec^2\right)}{0.1\left(kgm \cdot m^2\right)} \\
&= 928\left(rad / \sec^2\right) = 147.7\left(rev / \sec^2\right)
\end{aligned}
$$

(b). During idling, the power generated by the motor is consumed to overcome this dissipative energy due to damping

$$
\begin{aligned}
P &= T_m \cdot \omega \\
&= \left(J_m \frac{d\overset{0}{\cancel{\omega}}(t)}{dt} + B \cdot \omega(t) + \overset{0}{\cancel{T}}_L \right)\omega(t) \\
B &= \frac{P}{\omega(t)^2} = \frac{1,492(W)}{\left(20\pi\right)^2} = 0.38 \, N \cdot m / rad / \sec.
\end{aligned}
$$

(c).

$$
\begin{aligned}
V_A &= R_A I_A + V_i \\
&= R_A I_A + K_b \omega(t) \\
&\therefore \omega(t) = \frac{V_A - R_A I_A}{K_b} \\
\omega_{\max} &= \frac{250 - 10 \times 9.47}{1} = 155.3 \, rad / \sec = 1,483 \; rpm \\
P_{\max} &= V_{A\max} \cdot I_{A\max} \\
&= 250 \times 9.47 = 2,368W \\
&= 3.17 \; HP
\end{aligned}
$$

2. A brush type DC motor is used to drive an NC table. The torque constant K_T=2 kg·m/A and the voltage constant K_b=1 V·s. The electric resistance of the rotor circuit is 4 ohms and the total equivalent moment of inertia of the shaft-table is 6 kg-m^2. Ignore the effect of inductance in the armature circuit.

(a) Determine the maximum torque available from this motor with the rotor input voltage of 100V.

$$V_A = R_A \cdot (I_A)_{\max}$$
$$(I_A)_{\max} = \frac{V_A}{R_A} = \frac{100}{4} = 25(Amp)$$
$$T_{\max} = K_T \cdot (I_A)_{\max} = 2 \times 25 = 50\ kg \cdot m = 490\ N \cdot m$$

(b) The motor is required to deliver a constant torque up to 2,000 rpm by armature control. If the armature current is maintained at a constant level calculated in (a), determine the rotor voltage increase needed to increase the speed from 0 to 2,000 rpm.

$$\Delta V_A = \Delta V_i$$
$$= K_b \times n = 1 \times \frac{2000x2\pi}{60} = 209\,V$$

(c) Calculate the maximum acceleration achievable by this motor-table unit under no external load.

$$\left.\frac{dn(t)}{dt}\right|_{\max} = \frac{T_{\max}}{J} = \frac{50 \times (9.8)}{6\left(kg \cdot m^2\right)} = 81.6\ {}^{rad}\!/_{\sec^2}$$
$$= 12.998\ {}^{rev}\!/_{\sec^2}$$

(d) What percentage of stator current increase or reduction is needed to further increase the speed of the motor to 3000 rpm by field control? Determine the <u>torque</u> at this speed.

For field control

$V_i = K\phi n$: constant

$$K\phi_1 n_1 = K\phi_2 n_2$$

$\frac{\phi_1}{\phi_2} = \frac{2000}{3000}$ reduction of 33.3%

$$T_{n=3000} = T_{\max} \times \frac{2}{3} = 327\ N \cdot m$$

3.

$$R_r = 10\,\Omega \quad L_r = 0.06\,H \quad V_f = 220V \quad n=1,\ p=3$$

$$\omega_p = 60 \times 2\pi \, {}^{\text{rad}}\!/_{\text{sec}} = \left(376 \, {}^{\text{rad}}\!/_{\text{sec}}\right)$$

$$\omega_f = \frac{\omega_p}{n} = 60 \times 2\pi \, {}^{\text{rad}}\!/_{\text{sec}}$$

$$T_m = \frac{p \cdot nV^2 sR_r}{\omega_p\left(R_r^{\,2} + s^2\omega_p^{\,2}L_r^{\,2}\right)}$$

$$= \frac{3 \times 1 \times (220)^2 \cdot (10) \cdot s}{(60 \times 2\pi)\left[10^2 + s^2 \cdot (60 \times 2\pi)^2 \times (0.06)^2\right]}$$

$$= \frac{1{,}452{,}000 \cdot s}{37{,}699 + 192{,}884\ s^2} = \frac{38.5s}{1 + 511\ s^2}$$

$$s = 1 - \frac{\omega_m}{\omega_p} \cdot n = 1 - \frac{\omega_m}{\omega_p}$$

or $\quad \omega_m = (1-s)\omega_p$