

Homework #6

1. (a)

$$G_p(z) = G_p(s) = \frac{5}{s(s+5)}$$

$$G_p(z) = (1-z^{-1})Z\left[\frac{5}{s^2(s+5)}\right] \quad e^{-Ts} = e^{-0.1 \times 5} = e^{-0.5} = 0.6065$$

$$= \frac{0.0214z^{-1} + 0.018z^{-2}}{(1-z^{-1})(1-0.6065z^{-1})}$$

Dead beat controller: $T_d = z^{-1}$

$$D(z) = \frac{1}{G_p(z)} \frac{T_d(z)}{1-T_d(z)} = \frac{(z-1)(z-0.607)}{0.0214z+0.018} \frac{z^{-1}}{1-z^{-1}} = \frac{46.7(z-0.607)}{z+0.841}$$

(b)

$$T_d(z) = a_1 z^{-1} + a_2 z^{-2}$$

Since the input is a step input, for zero steady state error

$$1 - T(z) = (1 - z^{-1})N(z)$$

$$U(z) = \frac{Y(z)}{R(z)} \cdot \frac{R(z)}{G(z)}$$

$$= T(z) \cdot \frac{1}{1-z^{-1}} \cdot \frac{(1-z^{-1})(1-0.607z^{-1})}{0.0214(1+0.841z^{-1})z^{-1}}$$

$$T(z) = a_1 z^{-1} + a_2 z^{-2} = (1+0.841z^{-1})z^{-1} T_1$$

$$1 - T(z) = 1 - T_1 z^{-1} - 0.841 T_1 z^{-2} = (1 - z^{-1})(1 + n_1 z^{-1})$$

$$= 1 + (n_1 - 1)z^{-1} - n_1 z^{-2}$$

$$T_1 = 1 - n_1 \quad \Rightarrow \quad T_1 = 0.543$$

$$0.841 T_1 = n_1 \quad \Rightarrow \quad n_1 = 0.457$$

$$\therefore a_1 = 0.543 \quad a_2 = 0.458$$

$$T(z) = 0.543z^{-1} + 0.458z^{-2}$$

$$D(z) = \frac{1}{G(z)} \cdot \frac{T(z)}{1-T(z)} = \frac{1}{G(z)} \frac{T(z)}{(1-z^{-1})N(z)}$$

$$= \frac{\cancel{(1-z^{-1})}(1-0.607z^{-1})}{0.0214 \cancel{(1+0.841z^{-1})z^{-1}}} \frac{\cancel{(1+0.841z^{-1})} z^{-1} \cdot 0.543}{\cancel{(1-z^{-1})}(1+0.457z^{-1})}$$

$$= \frac{25.4(1-0.607z^{-1})}{1+0.457z^{-1}}$$

2.

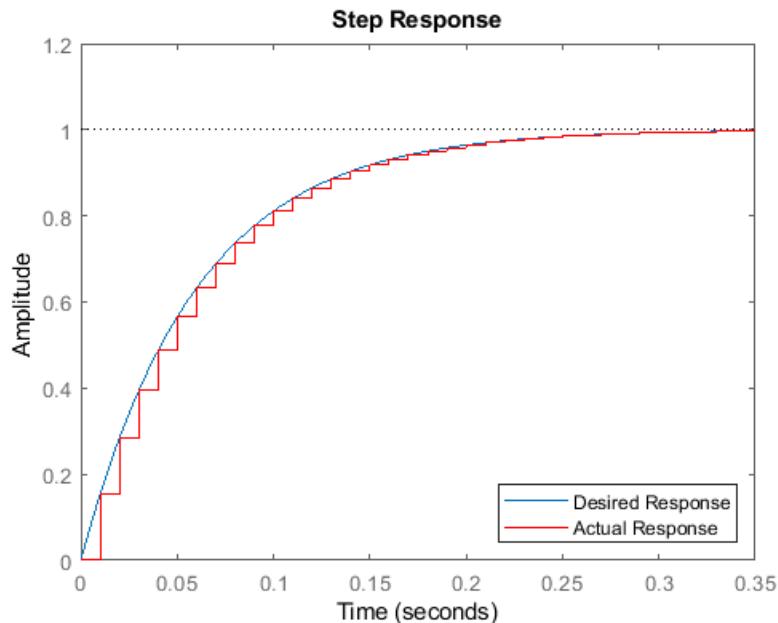
$$\begin{aligned}
 \text{a) } G_p(s) &= G_p(s) = \frac{2}{s(s+2)} \quad T = 0.01 \\
 G_p(z) &= (1 - z^{-1}) Z \left[\frac{G_p(s)}{s} \right] \\
 &= \frac{9.934 \times 10^{-5} z + 9.868 \times 10^{-5}}{(z-1)(z-0.9802)} = \frac{9.934 \times 10^{-5}(z+0.993)}{(z-1)(z-0.9802)}
 \end{aligned}$$

For first order response

$$\begin{aligned}
 T_d &= \frac{K(1-\delta)z^{-1}}{1-\delta z^{-1}} \quad \delta = e^{-T/\tau} = e^{-0.01/0.06} = 0.846 \\
 &= \frac{0.154 z^{-1}}{1 - 0.846 z^{-1}}
 \end{aligned}$$

$$\begin{aligned}
 D(z) &= \frac{(1 - z^{-1})(1 - 0.9802 z^{-1})}{(9.934 \times 10^{-5} + 9.868 \times 10^{-5} z^{-1}) z^{-1}} \quad \cancel{\frac{0.154 z^{-1}}{(1 - z^{-1})}} \\
 &= 1550 \frac{1 - 0.9802 z^{-1}}{(1 + 0.9934 z^{-1})} \\
 \left(\because D(z) = \frac{1}{G_p(z)} \cdot \frac{T(z)}{1 - T(z)} \right)
 \end{aligned}$$

b)



3.

a) Since the plant has one integrator, we can use I or PI controller.

For an I-Controller,

$$D(B) = \frac{k_i T}{1 - B}$$

$$G(B) = D(B) \times G_p(B) = \frac{0.11k_i T^2 B}{(1 - B)^2}$$

$$e_n = \frac{1}{1 + G(B)} \times r_n$$

$$= \frac{(1 - B)^2}{(1 - B)^2 + 0.11k_i T^2 B} \times \frac{0.1TB}{(1 - B)^2}$$

$$\therefore e_{ss} = \lim_{B \rightarrow 1} (1 - B) \cdot e_n = 0$$

$$\text{b) } \frac{\Delta Z_n}{d_n} = \frac{\frac{0.11TB}{1-B}}{1 + (140) \frac{0.11TB}{1-B}} = \frac{0.11TB}{(1-B) + 15.4TB}$$

$$\Delta Z_{ss} = \lim_{B \rightarrow 1} \left[(1 - B) \frac{0.11TB}{(1 - B) + 15.4TB} \cdot \frac{5}{1 - B} \right]$$

$$= 0.0357$$

c) Add a feed forward controller or PI controller

4.

$$\begin{aligned}
 G_p(s) &= \frac{0.6K}{\left(\frac{s}{35} + 1\right)\left(\frac{s}{4} + 1\right)} \quad T=0.02 \quad K=40 \\
 &= \frac{84}{(s+35)(s+4)} \\
 G_p(z) &= (1-z^{-1})Z\left[\frac{84}{s(s+4)(s+35)}\right] \\
 &= \frac{0.524z^{-1} + 0.405z^{-2}}{1 - 1.423z^{-1} + 0.458z^{-2}} = \frac{0.524z + 0.405}{z^2 - 1.423z + 0.458}
 \end{aligned}$$

$$\begin{aligned}
 a) \quad G_f(z) &= \frac{1}{G_p(z)} \\
 &= \frac{1 - 1.423z^{-1} + 0.458z^{-2}}{0.524z^{-1} + 0.405z^{-2}}
 \end{aligned}$$

For realizable controller

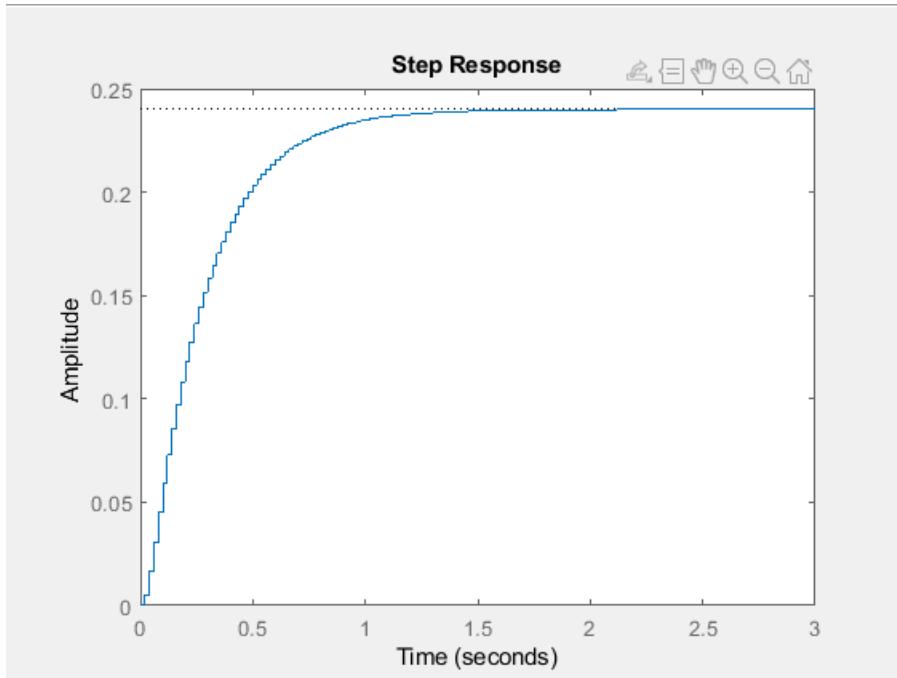
$$G_f(z) = \frac{1 - 1.423z^{-1} + 0.458z^{-2}}{0.524 + 0.405z^{-1}}$$

$$b) \quad k_p = 0.01$$

$$\frac{Y(z)}{R(z)} = \frac{G_f(z)G_p(z) + k_pG_p(z)}{1 + k_pG_p(z)} \text{ with feed forward control}$$

$$\begin{aligned}
 \frac{Y(z)}{R(z)} &= \frac{k_pG_p(z)}{1 + k_pG_p(z)} \\
 &= \frac{0.01 \frac{0.524z^{-1} + 0.405z^{-2}}{1 - 1.423z^{-1} + 0.458z^{-2}}}{1 + 0.01 \frac{0.524z^{-1} + 0.405z^{-2}}{1 - 1.423z^{-1} + 0.458z^{-2}}} \\
 &= \frac{0.00524z^{-1} + 0.00405z^{-2}}{1 - 1.418z^{-1} + 0.462z^{-2}} \\
 y_{ss} &= \frac{0.00524 + 0.00405}{1 - 1.418 + 0.462} = \frac{0.00929}{0.044} = 0.211
 \end{aligned}$$

Without feedforward control



With feedforward control

