

1..

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$$G_p(z) = (1-z^{-1})Z\left[\frac{1}{s^2(s+5)}\right]$$

$$= \frac{1-z^{-1}}{5} Z\left[\frac{5}{s^2(s+5)}\right]$$

$$= \frac{1-z^{-1}}{5} z \left[\frac{(5T-1+e^{-5T})z + (1-e^{-5T}-5Te^{-5T})}{5(z-1)^2(z-e^{-5T})} \right]$$

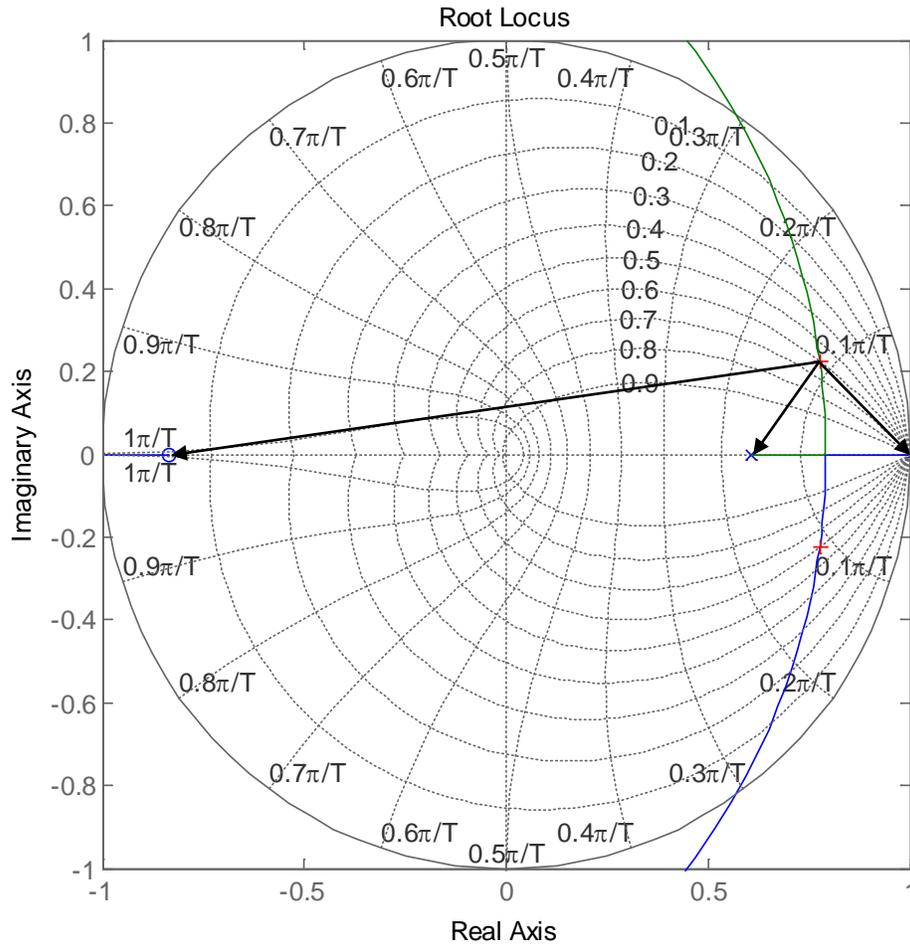
$$= \frac{(5T-1+e^{-5T})z + (1-e^{-5T}-5Te^{-5T})}{25(z-1)(z-e^{-5T})}$$

$$e^{-5T} = e^{-5 \times 0.1} = 0.607$$

$$\therefore G_p(z) = \frac{0.107z + 0.0895}{25(z-1)(z-0.607)} = \frac{4.28 \times 10^{-3}(z+0.836)}{(z-1)(z-0.607)}$$

$$= \frac{4.28 \times 10^{-3}z + 3.58 \times 10^{-3}}{z^2 - 1.607z + 0.607}$$

$$G(z) = K \times G_p(z)$$



Closed system pole location = $0.7819 + j0.2256$

$$r_{p_1} = 0.314 \quad r_{p_2} = 0.285 \quad r_{z_1} = 1.634$$

$$0.00428K_p = \frac{r_{p_1} \cdot r_{p_2}}{r_{z_1}} = \frac{0.314 \cdot 0.285}{1.634}$$

$$K_p = 12.7$$

From Matlab $K = 12.81$

2.

$$\begin{aligned}
 G_p(z) &= (1 - z^{-1})Z\left[\frac{1}{s^3}\right] \\
 &= (1 - z^{-1})\frac{T^2(1 + z^{-1})z^{-1}}{2(1 + z^{-1})^3} \\
 &= \frac{T^2(1 + z^{-1})z^{-1}}{2(1 - z^{-1})^2} = \frac{T^2(z + 1)}{2(z - 1)^2}
 \end{aligned}$$

$$G_c(s) = K_p + K_d s$$

$$\begin{aligned}
 D(z) &= K_p + K_d \frac{z - 1}{Tz} = \frac{(K_p T + K_d)z - K_d}{Tz} \\
 &= \frac{K(z - \alpha)}{Tz}
 \end{aligned}$$

$$\text{where } K = K_p T + K_d \quad \alpha = \frac{K_d}{K_p T + K_d}$$

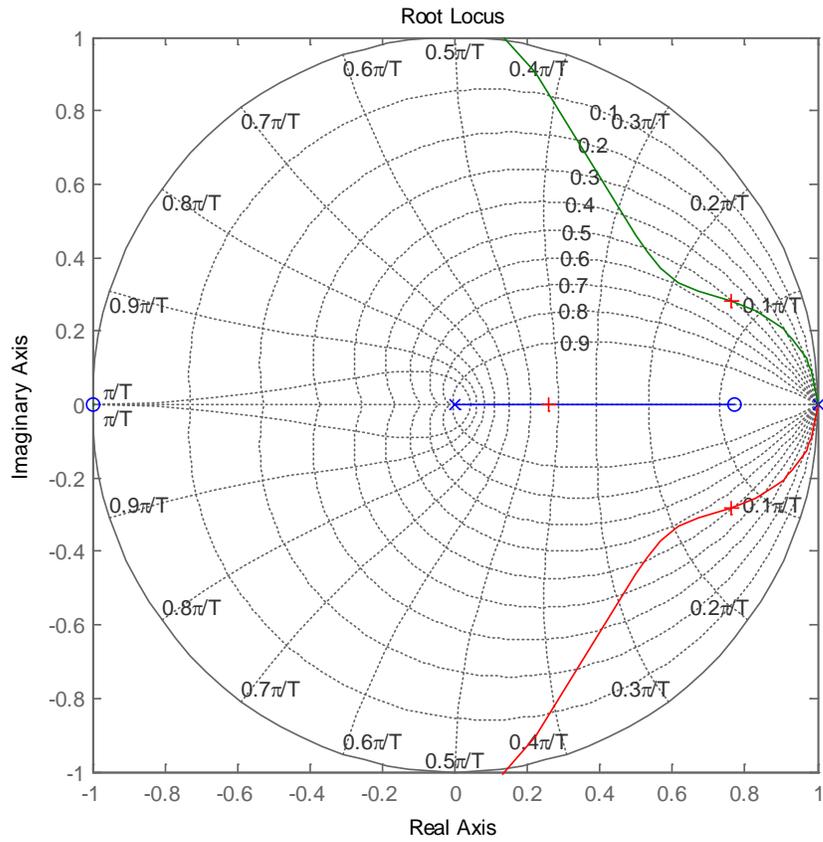
$$G(z) = D(z) \times G_p(z) = \frac{0.1K(z - \alpha)(z + 1)}{2 \times z(z - 1)^2}$$

Draw the root-locus of

$$\frac{K(z + 1)}{z(z - 1)^2} \quad \pi/10T = \frac{\pi}{1} = 3.14$$

desired pole location

$$\begin{aligned}
 z_{1,2} &= e^{(-\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2})T} \quad (\zeta\omega_n = 0.5 \times 4 = 2, \omega_n\sqrt{1-\zeta^2} = 4\sqrt{1-0.5^2} = 3.464) \\
 &= e^{-0.2} e^{+j0.3464} = 0.8187(\cos 0.3464 \pm j \sin 0.3464) \\
 &= 0.77 \pm j0.278
 \end{aligned}$$



defect angle

$$\phi_{p_{1,2}} = 129.6^\circ$$

$$\phi_{p_3} = 19.9^\circ$$

$$\phi_z = 8.9^\circ$$

$$-129.6^\circ - 129.6^\circ - 19.9^\circ + 1.9^\circ + \phi_d = -180^\circ$$

$$\therefore \phi_d = 90.2^\circ$$

$$\therefore \alpha = 0.77$$

$$r_{p_1} = r_{p_2} = 0.36$$

$$r_{p_3} = 0.82$$

$$r_{z_1} = 1.79$$

$$r_{z_2} = 0.278$$

$$\frac{0.1 K}{2} = \frac{(0.36)^2 \times 0.82}{1.79 \times 0.278} = 0.214$$

$$\therefore K = 4.28$$

$$K_d = \alpha \times K = 0.77 \times 4.28 = 3.296$$

$$K_p = \frac{K - K_d}{T} = \frac{4.28 - 3.296}{0.1} = 9.84$$

3.

a)

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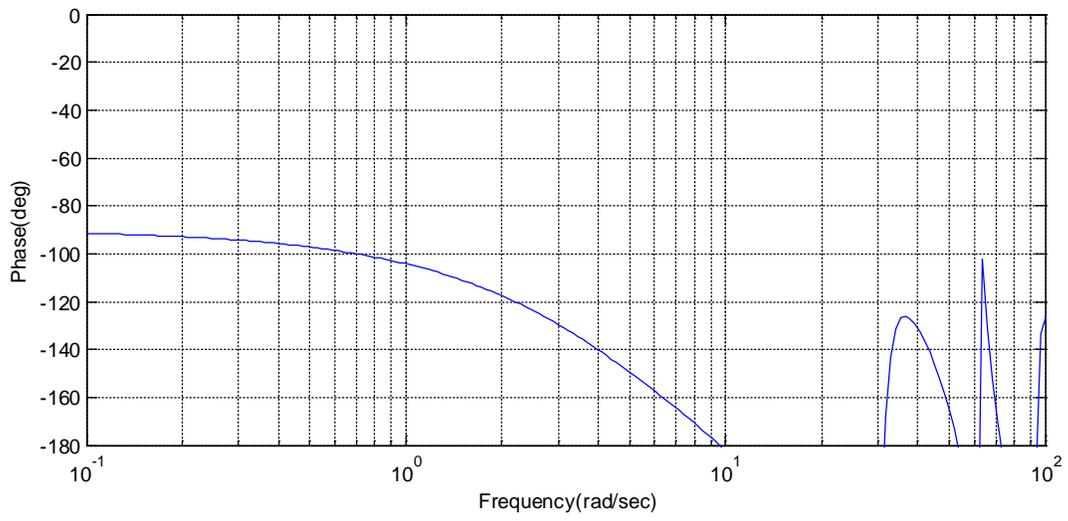
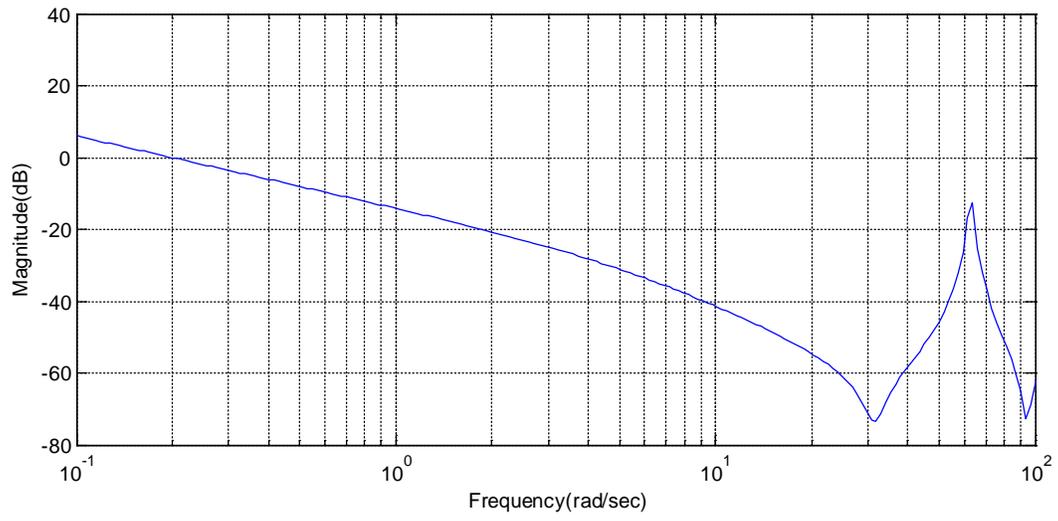
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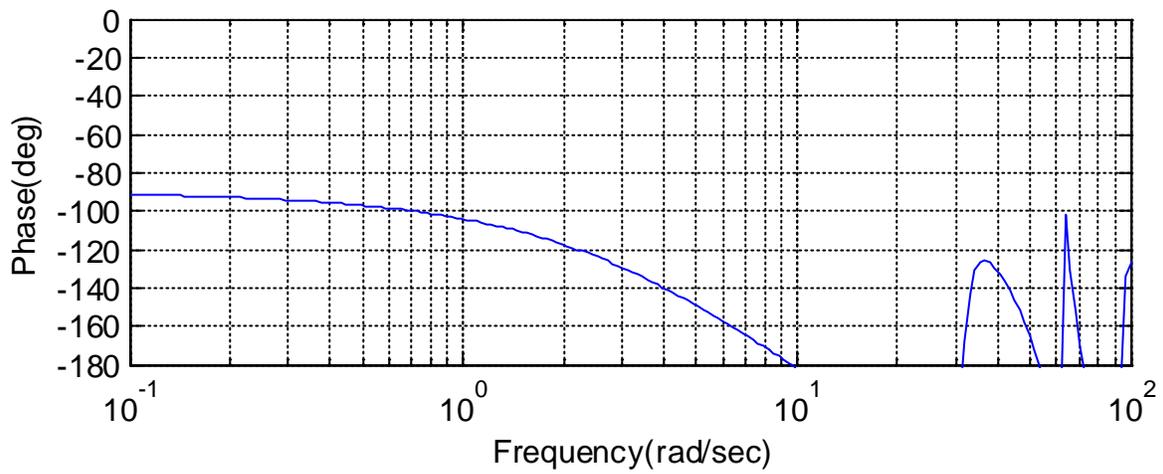
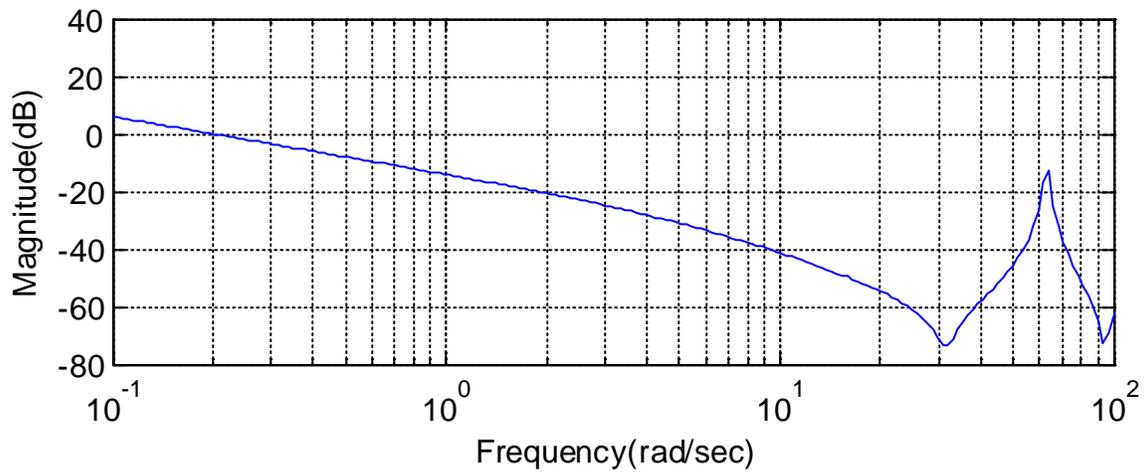
$$G(z) = K \times G_p(z)$$



K=16 dB=6.3

$$G_p(s) = \frac{1}{s^2}$$

$$G_p(z) = (1 - z^{-1})Z\left[\frac{1}{s^3}\right]$$
$$= \frac{T^2(z+1)}{2(z-1)^2}$$



b)

$$G_p(s) = \frac{1}{s(s+5)} \quad T=0.02$$

$$G_p(z) = (1 - z^{-1})Z\left[\frac{1}{s^2(s+5)}\right]$$

$$= \frac{1 - z^{-1}}{5} Z\left[\frac{5}{s^2(s+5)}\right]$$

$$= \frac{1 - z^{-1}}{5} z \left[\frac{(5T - 1 + e^{-5T})z + (1 - e^{-5T} - 5Te^{-5T})}{5(z-1)^2(z - e^{-5T})} \right]$$

$$= \frac{(5T - 1 + e^{-5T})z + (1 - e^{-5T} - 5Te^{-5T})}{25(z-1)(z - e^{-5T})}$$

$$e^{-5T} = e^{-5 \times 0.02} = 0.905$$

$$\therefore G_p(z) = \frac{0.00484z + 0.00468}{25(z-1)(z-0.905)}$$

$$= \frac{1.936 \times 10^{-4}z + 1.872 \times 10^{-4}}{z^2 - 1.905z + 0.905}$$

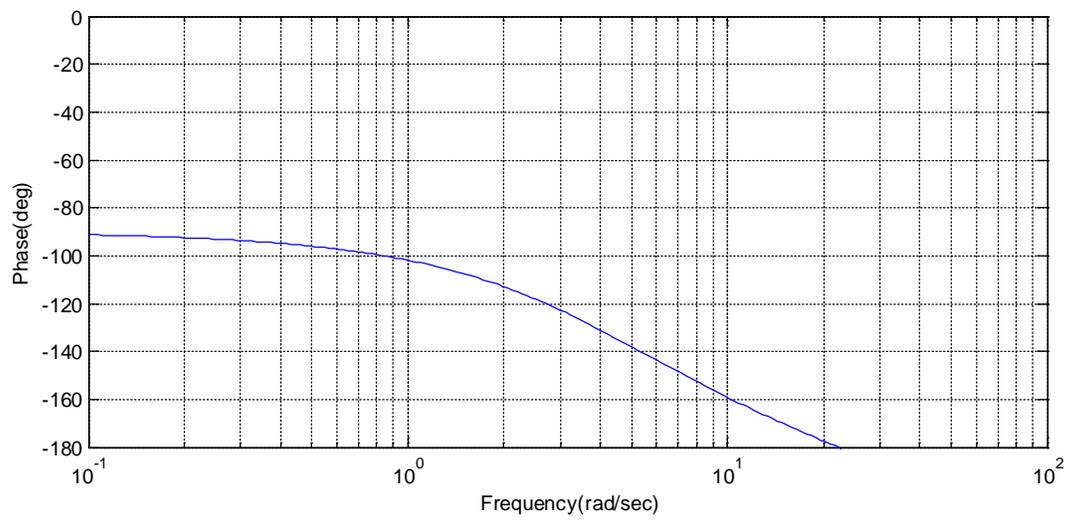
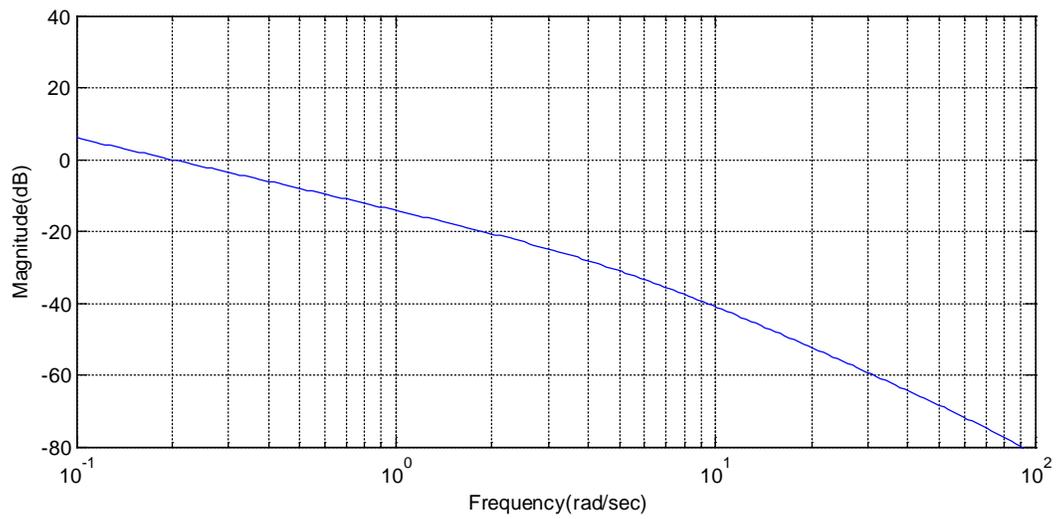
$$G(z) = K \cdot G_p(z)$$

Set $K=1$ and draw Bode plot using

$$z = e^{j\omega T}$$

For $PM=70^\circ$, $\omega_{gc} \cong 1.8$ rad/s

Increase gain by 20 dB $\Rightarrow K=10$



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$$= \frac{T^2(z+1)}{2(z-1)^2}$$

