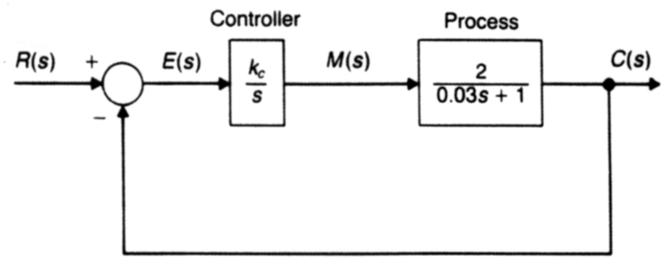
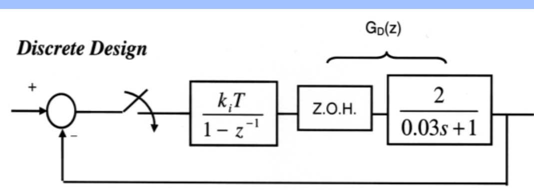
Frequency Domain Design

Continuous Design



Frequency Domain Design

Discrete Design



T = 0.03

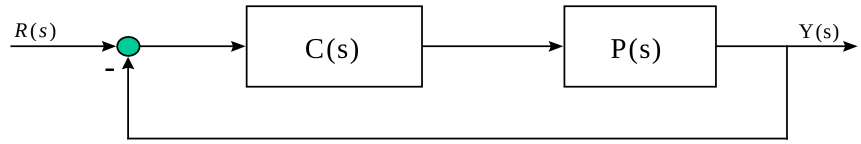
$$G_D(z) = \frac{1.264z^{-1}}{1 - 0.368z^{-1}} \ .$$

<u>*O.L.T.F.*</u>

$$D(z) \cdot G_D(z) = \frac{1.264 \ k_i T \cdot z^{-1}}{(1 - z^{-1})(1 - 0.368z^{-1})}$$

$$= \frac{1.264 k_i T \cdot z}{(z-1)(z-0.368)} \ .$$

Relationship between Second-order System Response & OLTP



Open-Loop Frequency Response:

$$G(s) = C(s)P(s) \quad \Rightarrow \quad G(j\omega) = C(j\omega)P(j\omega)$$

Assume

$$G(s) = \frac{\omega_n^{\ 2}}{s(s + 2\zeta\omega_n)} \quad \Rightarrow T(s) = \frac{\omega_n^{\ 2}}{s^2 + 2\zeta\omega_n s + \omega_n^{\ 2}}$$

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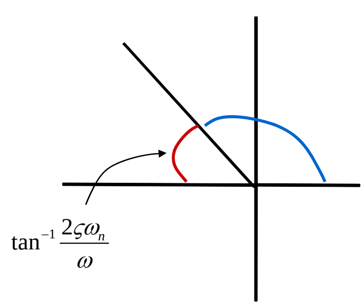
C.E.: $s^2 + 2\varsigma\omega_n s + \omega_n^{\ 2} = 0$

$$G(j\omega) = \frac{\omega_n^2}{-\omega^2 + 2j\varsigma\omega\omega_n}$$

$$|G(j\omega)| = \frac{\omega_n^2}{\sqrt{(-\omega^2)^2 + (2\varsigma\omega\omega_n)^2}}$$

$$\angle G(j\omega) = -\angle\left(-\omega^2 + 2j\varsigma\omega\omega_n\right)$$

$$= -\left(180^\circ - \tan^{-1}\frac{2\varsigma\omega_n}{\omega}\right)$$



Frequency Domain Design

Phase Margin (when $\left|G(j\omega)\right|_{\omega=\omega_{gc}}=1$)

$$\phi_m = 180^\circ + \angle G(j\omega_{gc})$$
$$= 180^\circ - \left(180^\circ - \tan^{-1}\frac{2\varsigma\omega_n}{\omega}\right)_{\omega=\omega_{gc}}$$
$$= \tan^{-1}\frac{2\varsigma\omega_n}{\omega_{gc}}$$

$$\left|G(j\omega_{gc})\right| = 1 = \frac{\omega_n^2}{\sqrt{\omega_{gc}^4 + \left(2\varsigma\omega_{gc}\omega_n\right)^2}}$$

$$\omega_{gc}^4 + (2\varsigma\omega_n)^2\omega_{gc}^{\ 2} - \omega_n^4 = 0$$

$$\left(\frac{\omega_{gc}}{\omega_n}\right)^2 = -2\varsigma^2 + \sqrt{4\varsigma^4+1}$$

$$\left(\frac{\omega_{gc}}{\omega_n}\right) = \sqrt{-2\varsigma^2 + \sqrt{4\varsigma^4+1}}$$

Frequency Domain Design

$$\left|G(j\omega_{gc})\right| = 1 = \frac{\omega_n^2}{\sqrt{\omega_{gc}^4 + \left(2\varsigma\omega_{gc}\omega_n\right)^2}}$$

$$\omega_{gc}^4 + (2\varsigma\omega_n)^2\omega_{gc}^{\ 2} - \omega_n^4 = 0$$

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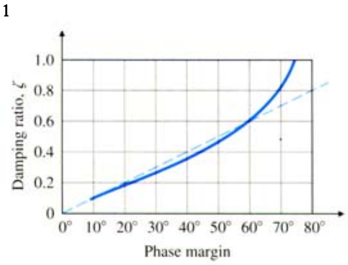
$$\left(\frac{\omega_{gc}}{\omega_n}\right) = \sqrt{-2\varsigma^2 + \sqrt{4\varsigma^4+1}}$$

$$\text{PM} = \tan^{-1}\frac{2\varsigma\omega_n}{\omega_{gc}}$$
$$= \tan^{-1}\frac{2\varsigma}{\sqrt{-2\varsigma^2 + \sqrt{4\varsigma^4+1}}}$$



Relation between Open Loop and Closed Loop Characteristics

- *Relationship Between PM of Open-loop TF G(s) and Damping Ration ζ of CLTF T(s): (second-order approximation)*

$$PM = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{4\zeta^4+1}}}$$

$$\zeta \approx \frac{PM}{100}\text{(good for } PM < 70^\circ\text{)}$$

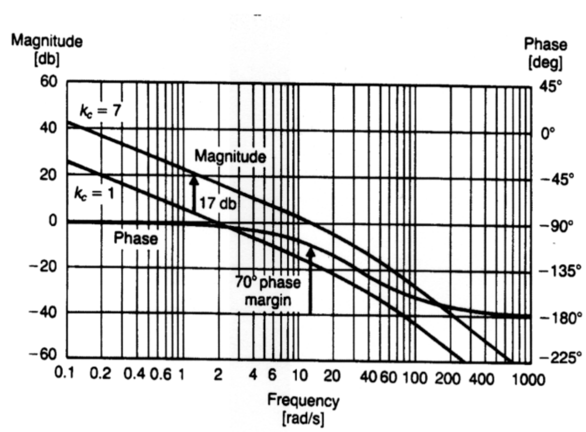
- M_r: maximum closed-loop magnitude

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

- ω_{BW}: *bandwidth, frequency where the closed-loop magnitude drops to -3 dB (or $1/\sqrt{2}$).*

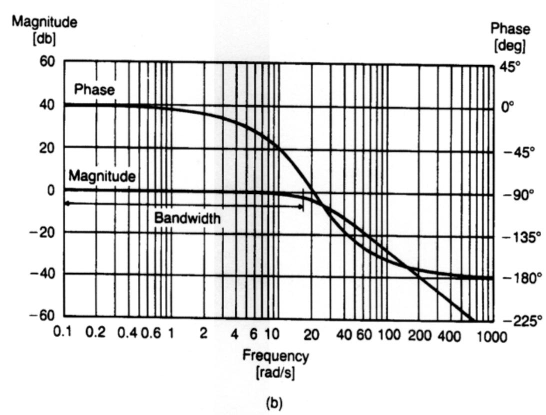
$$\omega_c \le \omega_{BW} \le 2\omega_c$$

Frequency Domain Design



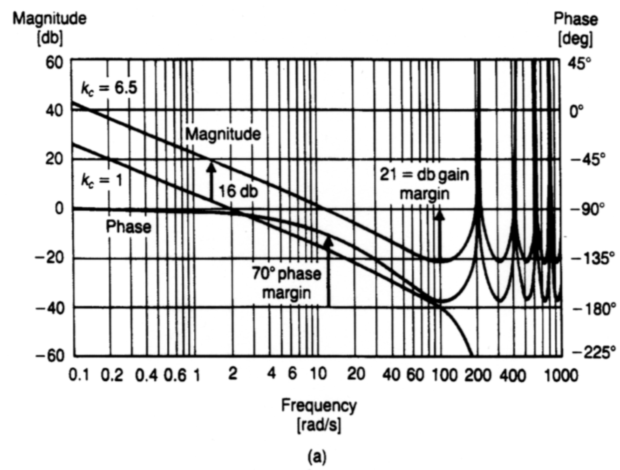
(a)

Frequency Domain Design

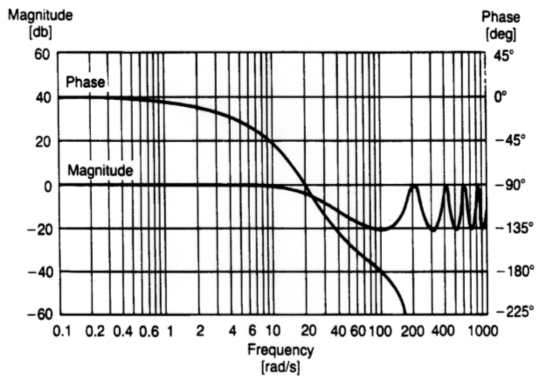


(b) closed-loop frequency response plot for Example 12.3 with $k_c = 7$.

Frequency Domain Design



Frequency Domain Design



(b)

(b) closed-loop frequency response plot for Example 12.4 with $k_c = 6.5$.