CHAPTER 1

Introductory Concepts

- Elements of Vector Analysis
- Newton's Laws
- Units
- The basis of Newtonian Mechanics
- D'Alembert's Principle

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Science of Mechanics: It is concerned with the motion of material bodies.

- Bodies have different scales: Microscropic, macroscopic and astronomic scales.

  In mechanics - mostly macroscopic bodies are considered.
- Speed of motion - serves as another important variable - small and high (approaching speed of light).

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- In Newtonian mechanics - study motion of bodies much bigger than particles at atomic scale, and moving at relative motions (speeds) much smaller than the speed of light.
- Two general approaches:
  - Vectorial dynamics: uses Newton's laws to write the equations of motion of a system, motion is described in physical coordinates and their derivatives;
  - Analytical dynamics: uses energy like quantities to define the equations of motion, uses the generalized coordinates to describe motion.

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1.1 Vector Analysis:

- Scalars, vectors, tensors:

– Scalar: It is a quantity expressible by a single real number.
Examples include: mass, time, temperature, energy, etc.

– Vector: It is a quantity which needs both direction and magnitude for complete specification.

– Actually (mathematically), it must also have certain transformation properties.

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These properties are: vector magnitude remains unchanged under rotation of axes.
ex: force, moment of a force, velocity, acceleration, etc.

– geometrically, vectors are shown or depicted as directed line segments of proper magnitude and direction.

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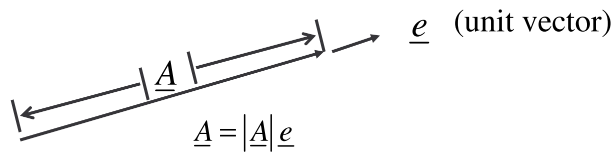
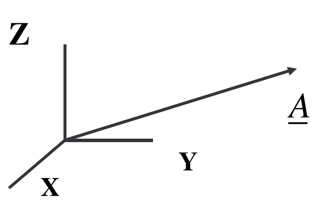


– if we use a coordinate system, we define a basis set $(\hat{i}, \hat{j}, \hat{k})$: we can write

$$\underline{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$



or, we can also use the three components and define

$$\{A\} = \{A_x, A_y, A_z\}^T$$

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– The three components A_x , A_y , A_z can be used as 3-dimensional vector elements to specify the vector.

– Then, laws of vector-matrix algebra apply.

– Tensors:

scalar - an array of zero dimension

vector - an array of one dimension

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– quantities which need arrays of two or higher dimension to specify them completely - called tensors of appropriate rank.

Again - to be a tensor, the object must also satisfy certain transformation properties of rotation and translation.

Exs: Second-order tensors: stress at a point in deformable body - stress tensor has nine components (a 3x3 matrix in a representation when the basis is defined), inertia tensor (again, a 3x3 matrix in usual notation) expressing mass distribution in a rigid body.

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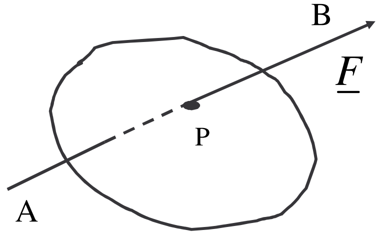

- **TYPES OF VECTORS: Consider a force $\underline{F}$ acting on a body at point P. The force has a line of action AB. This force can lead to translation of the rigid body, rotation of the rigid body about some point, as well as deformation of the body.**

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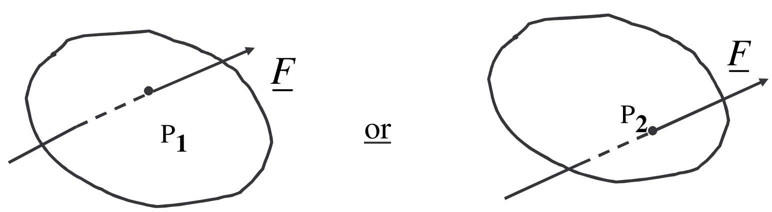

The same force $\underline{F}$ is now acting at two different points P_1 , P_2 of the body, i.e., the lines of action are distinct.

– same translational effect

– the translational effect depends only on magnitude and direction of the force, not on its point of application or the line of action- free vectors

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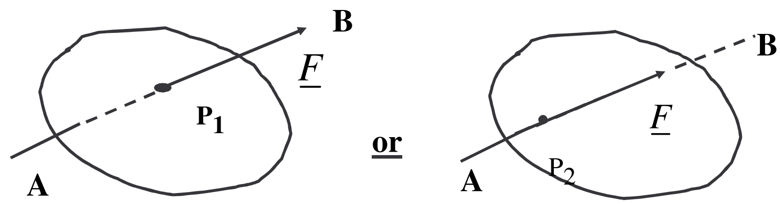

The force F has the **same line of action** **AB in the two cases. The points of application (P_1 and P_2) are different but moment about every point is the same → same rotational effect (as well as translational effect): effect of vector** ***F*** **depends on magnitude, direction as well as line of action -** **sliding vectors**

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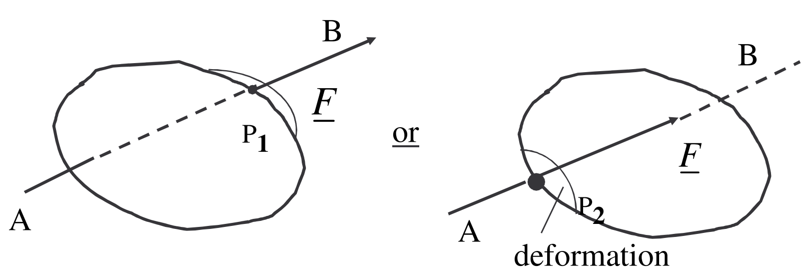
If the **body is deformable**, **the effect of force is different depending its point of application; whether the force acts at point P_1 or P_2. Thus, in such a case, the point of application is also crucial -** **bound vectors**.



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- **Equality of vectors:**

For free vectors $\underline{A}$ and $\underline{B}$, $\underline{A} = \underline{B}$ if and only if $\underline{A}$ and $\underline{B}$ have the same magnitude and the same direction.

- **Unit vectors:**

If $\underline{A}$ is a vector with magnitude A, $\underline{A}/A$ is a vector along $\underline{A}$ with unit length $\rightarrow$ $\underline{e}_A = \underline{A}/A$ or $\underline{A} = A\underline{e}_A$.

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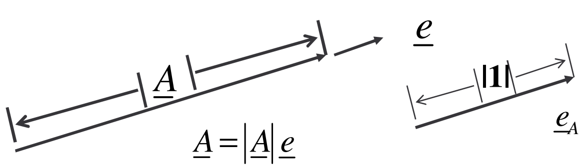

unit vector in the direction of $\underline{A}$

- **Addition of vectors**: Consider two vectors $\underline{A}$ *and* $\underline{B}$. Their addition is a vector $\underline{C}$ given by $\underline{C} = \underline{A} + \underline{B}$. Also $\underline{C} = \underline{B} + \underline{A}$ (addition is commutative). The result is also a vector.

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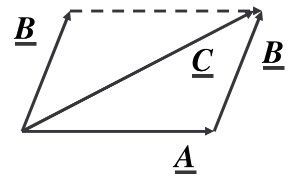
$$\underline{C} = \underline{A} + \underline{B}$$

Graphically, one can use the parallelogram rule of vector addition.



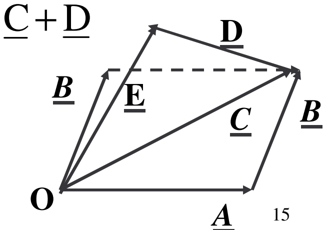
For more than two vectors, one can add sequentially - polygon of vectors. Consider The addition of vectors $\underline{A}$, $\underline{B}$, and $\underline{D}$.

$\underline{C} = \underline{A} + \underline{B}$, $\underline{E} = (\underline{A} + \underline{B}) + \underline{D} = \underline{C} + \underline{D}$

or

$\underline{E} = (\underline{A} + \underline{D}) + \underline{B}$

$= \underline{A} + (\underline{B} + \underline{D}).$



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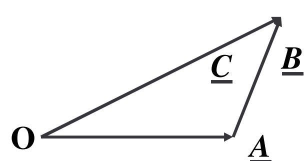
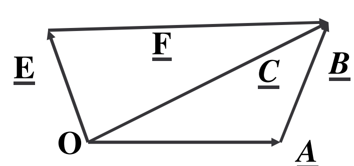
- **COMPONENTS OF A VECTOR:**

Consider the vector addition for $\underline{A}$ and $\underline{B}$: graphically:



$\underline{C} = \underline{A} + \underline{B}$

We can interpret $\underline{A}$ and $\underline{B}$ to be components of the vector $\underline{C}$. Clearly, the components of $\underline{C}$ are non-unique. As an example, $\underline{E}$ and $\underline{F}$ are also components.



We can make it more systematic.

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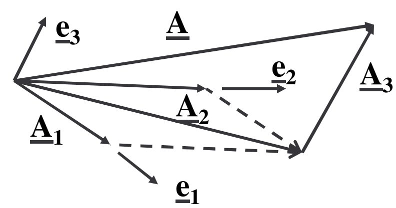
• Let $\underline{e}_1$, $\underline{e}_2$, $\underline{e}_3$ - three linearly independent unit vectors (not necessarily orthogonal) and let $\underline{A}$ be a vector.

We can write $\underline{A} = \underline{A}_1 + \underline{A}_2 + \underline{A}_3$ where $\underline{A}_i$, $i = 1, 2, 3$ are components of $\underline{A}$ along the directions specified by unit vectors $\underline{e}_i$, i = 1, 2, 3.

Then :

$$\underline{A} = A_1 \underline{e}_1 + A_2 \underline{e}_2 + A_3 \underline{e}_3$$



A_i, i = 1, 2, 3 are unique scalar components

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Let $\underline{B} = B_1 \underline{e}_1 + B_2 \underline{e}_2 + B_3 \underline{e}_3$ be another vector, with components expressed in same unit vectors. We can then write the sum as

$$\underline{C} = \underline{A} + \underline{B} = (A_1 + B_1)\underline{e}_1 + (A_2 + B_2)\underline{e}_2$$
$$+ \quad (A_3 + B_3)\underline{e}_3$$

The components of the vector $\underline{C}$ are then

$\rightarrow$ $C_1 = A_1 + B_1$, $C_2 = A_2 + B_2$

and

$C_3 = A_3 + B_3$.

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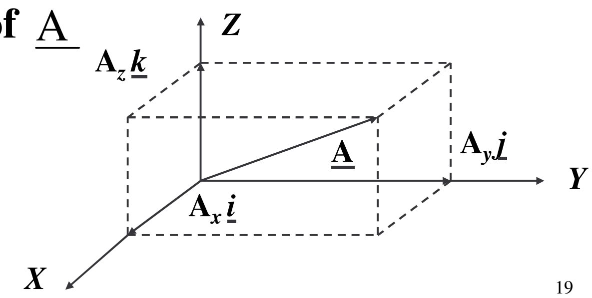
- **The more familiar case of unit vectors is the** Cartesian coordinate system **- (x, y, z)**

Let $\underline{i}$, $\underline{j}$, $\underline{k}$ **- unit vectors along x, y and z directions. Then**

$\underline{A}=A_x\underline{i}+A_y\underline{j}+A_2\underline{k}$ where A_x , A_y , A_z **are components of** $\underline{A}$ **along axes.**



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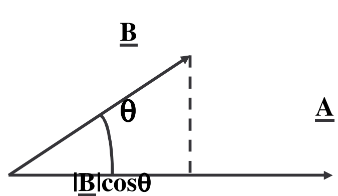
•SCALAR PRODUCT:

Definition: (DOT) For two vectors A and B, the dot product is defines as $\underline{A}\cdot\underline{B}=|\underline{A}|\,|\underline{B}|\cos\theta$

Dot product is Commutative**, i.e.,** $\underline{A}\cdot\underline{B}=\underline{B}\cdot\underline{A}$



If $\underline{A}=\sum_{i=1}^{3}A_i\underline{e}_i$, $\underline{B}=\sum_{i=1}^{3}B_i\underline{e}_i$,

then

$$\underline{A}\cdot\underline{B}=A_1B_1+A_2B_2+A_3B_3$$

provided the unit vectors are an orthogonal set, i.e.,

$$\underline{e}_1\cdot\underline{e}_2=\underline{e}_2\cdot\underline{e}_3=\underline{e}_3\cdot\underline{e}_1=0.$$

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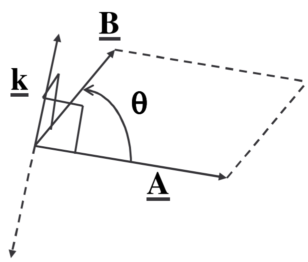
•**VECTOR PRODUCT**: **Let $\underline{A}$, $\underline{B}$ be two vectors that make an angle θ with respect to each other.**

Then, the vector or cross product is defined as a vector $\underline{C}$ with magnitude $|\underline{C}| = |\underline{A} \times \underline{B}| = |\underline{A}||\underline{B}|\sin\theta$.



Let $\underline{k}$ be the unit vector normal to the plane formed by vectors $\underline{A}$ and $\underline{B}$. It is fixed by the right hand screw rule. Then

$$\underline{A} \times \underline{B} = |\underline{A}||\underline{B}|\sin\theta \underline{k}$$

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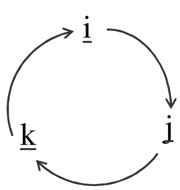
Some properties of cross product are :

$$\underline{A}\times\underline{B}=-\underline{B}\times\underline{A}$$

Consider unit vectors for the Cartesian coordinate system (x,y,z), ($\underline{i}$, $\underline{j}$, $\underline{k}$): Then



right-hand rule

$$|\underline{i} \times \underline{j}| = |\underline{j} \times \underline{k}| = |\underline{k} \times \underline{i}| = 1$$

$$\underline{i} \times \underline{i} = \underline{j} \times \underline{j} = \underline{k} \times \underline{k} = 0$$

$$\underline{i} \times \underline{j} = \underline{k} \quad ; \quad \underline{j} \times \underline{k} = \underline{i}$$

$$\underline{k} \times \underline{i} = \underline{j}$$

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Now, consider cross-product again. When vectors **A** and **B** are expressed in component form: $\underline{A}=A_x\underline{i}+A_y\underline{j}+A_z\underline{k}$ $\underline{B}=B_x\underline{i}+B_y\underline{j}+B_z\underline{k}$,

The cross product is evaluated by the operation

$$\underline{A}\times\underline{B}=\begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

$$\equiv(A_yB_z-A_zB_y)\underline{i}+(A_zB_x-A_xB_z)\underline{j}$$
$$+(A_xB_y-A_yB_x)\underline{k}$$

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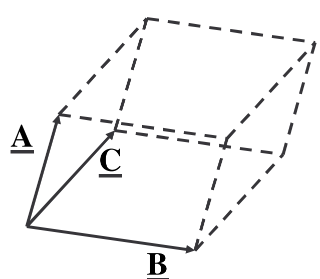
- **SCALAR TRIPLE PRODUCT**: Consider three vectors **A**, **B**, and **C**. The scalar triple product is given by $R=\underline{A}\bullet(\underline{B}\times\underline{C})=(\underline{A}\times\underline{B})\bullet\underline{C}$



Note that the result is the same scalar quantity. It can be interpreted as the volume of the parallelepiped having the vectors $\underline{A}$, $\underline{B}$ and $\underline{C}$ as the edges. The sign can be +ve or -ve.

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• **VECTOR TRIPLE PRODUCT**:

Consider vectors $\underline{A}$, $\underline{B}$, and $\underline{C}$. Then, vector triple product is defined as a vector $\underline{D}$, given by $\underline{D}=\underline{A}\times(\underline{B}\times\underline{C})$ **Note that** $\underline{A}\times(\underline{B}\times\underline{C}) \neq (\underline{A}\times\underline{B})\times\underline{C}$

One can show that

$$\underline{D}=\underline{A}\times(\underline{B}\times\underline{C})=(\underline{A}\bullet\underline{C})\underline{B}-(\underline{A}\bullet\underline{B})\underline{C}$$

• **DERIVATIVE OF A VECTOR**:

Suppose that a vector $\underline{A}$ **is a function of a scalar** ***u*****, i.e.,** $\underline{A}=\underline{A}(u)$ **. We can then consider change in vector** $\underline{A}$ **associated with change in the scalar** ***u*****.**

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Let $\underline{A}=\underline{A}(u)$ **and** $\underline{A}(u+\Delta u) \equiv \underline{A}(u)+\Delta\underline{A}$

Then $$\frac{d\underline{A}}{du} = \lim_{\Delta u \to 0} \frac{\underline{A}(u)+\Delta\underline{A} - \underline{A}(u)}{\Delta u}$$

or $$\frac{d\underline{A}}{du} = \lim_{\Delta u \to 0} \frac{\Delta\underline{A}}{\Delta u}$$

This is the derivative of $\underline{A}$ **with respect to** ***u*****.**

<u>Ex:</u> The position vector $\underline{r}$**(t) for a particle moving depends on time. We define the velocity to be** $\underline{v}(t) \equiv \frac{d\underline{r}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta\underline{r}}{\Delta t}$

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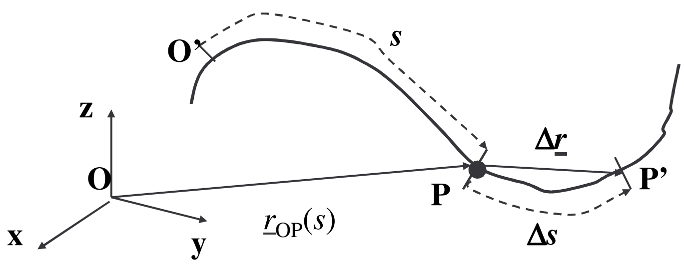
ex:



Consider a particle P moving along a curved path. Its position depends on distance from some landmark, O', i.e. $\underline{r}_{OP} \equiv \underline{r}_{OP}(s)$ **where 's' is the distance along the curve.**

We shall consider $\dfrac{d\underline{r}_{OP}(s)}{ds}$ **later in the next chapter.**

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Some useful properties and rules of differentiation are: $\dfrac{d}{du}(\underline{A}+\underline{B}) = \dfrac{d\underline{A}}{du} + \dfrac{d\underline{B}}{du}$

$$\frac{d}{du}(g(u)\underline{A}(u)) = g(u)\frac{d\underline{A}}{du} + \frac{dg(u)}{du}\underline{A}(u)$$

$$\frac{d}{du}(\underline{A}(u)\bullet\underline{B}(u)) = \underline{A}(u)\bullet\frac{d\underline{B}(u)}{du} + \frac{d\underline{A}(u)}{du}\bullet\underline{B}(u)$$

$$\frac{d}{du}(\underline{A}(u)\times\underline{B}(u)) = \underline{A}(u)\times\frac{d\underline{B}(u)}{du} + \frac{d\underline{A}(u)}{du}\times\underline{B}(u)$$

Finally, if $\underline{A} = A_1\underline{e}_1 + A_2\underline{e}_2 + A_3\underline{e}_3 = \sum A_i\underline{e}_i$,

then $\dfrac{d\underline{A}}{du} = \sum(\dfrac{dA_i}{du})\underline{e}_i + \sum A_i(\dfrac{d\underline{e}_i}{du})$

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Some more useful properties: Concept of a *Dyad* and *Dyadic*: Consider two vectors $\underline{a}$ and $\underline{b}$

Dyad: It consists of a pair of vectors $\underline{a}\underline{b}$ for two vectors $\underline{a}$ and $\underline{b}$. $\underline{a}$ - called an ***antecedent,*** $\underline{b}$ **-** called a ***consequent.***

Dyadic: It is a sum of dyads. Suppose that the vectors $\underline{a}$ and $\underline{b}$ are expressed in terms of a set of ***unit basis vectors*** $\underline{e}_1$, $\underline{e}_2$, $\underline{e}_3$, so that

$$\underline{a} = a_1\,\underline{e}_1 + a_2\,\underline{e}_2 + a_3\,\underline{e}_3 \text{ and } \underline{b} = b_1\,\underline{e}_1 + b_2\,\underline{e}_2 + b_3\,\underline{e}_3$$

Then, $\underline{A} \equiv \underline{a}\,\underline{b}$

$$= (a_1\,\underline{e}_1 + a_2\,\underline{e}_2 + a_3\,\underline{e}_3)(b_1\,\underline{e}_1 + b_2\,\underline{e}_2 + b_3\,\underline{e}_3)$$

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and, $\underline{A} \equiv \underline{a}\,\underline{b} = \sum_{i=1}^{3}\sum_{j=1}^{3} a_i b_j\,\underline{e}_i\,\underline{e}_j = \sum_{i=1}^{3}\sum_{j=1}^{3} A_{ij}\,\underline{e}_i\,\underline{e}_j$

In the Text of Greenwood, the unit vectors $\underline{e}_1$, $\underline{e}_2$, $\underline{e}_3$ are mostly limited to the ***Cartesian basis*** $(\underline{i}, \underline{j}, \underline{k})$

A ***Conjugate Dyadic*** for the dyadic $\underline{A}$ is obtained by interchanging the order of vectors $\underline{a}$ and $\underline{b}$ and is denoted by $\underline{A}^T$. Thus,

$$\underline{A}^T \equiv \underline{b}\,\underline{a} = \sum_{i=1}^{3}\sum_{j=1}^{3} b_i a_j\,\underline{e}_i\,\underline{e}_j = \sum_{i=1}^{3}\sum_{j=1}^{3} A_{ji}\,\underline{e}_i\,\underline{e}_j$$

A dyadic is ***symmetric*** if $\underline{A} = \underline{A}^T$, that is $A_{ij} = A_{ji}$

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An example of a symmetric dyadic is the inertia dyadic:

$$\underline{I} \equiv \sum_{i=1}^{3}\sum_{j=1}^{3} I_{ij}\,\underline{e}_i\,\underline{e}_j = I_{xx}\underline{i}\,\underline{i} + I_{xy}\underline{i}\,\underline{j} + I_{xz}\underline{i}\,\underline{k}$$

$$+ I_{yx}\underline{j}\,\underline{i} + I_{yy}\underline{j}\,\underline{j} + I_{yz}\underline{j}\,\underline{k}$$

$$+ I_{zx}\underline{k}\,\underline{i} + I_{zy}\underline{k}\,\underline{j} + I_{zz}\underline{k}\,\underline{k}$$

A dyadic is ***skew-symmetric*** if $\underline{A} = -\underline{A}^T$, that is, it is negative of its conjugate. Note that symmetry property of a dyadic is independent of the unit vector basis or its orthogonality.

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Some operational properties:

1. The ***sum of two dyadics*** is a dyadic obtained by adding the corresponding elements in the same basis:

   $\underline{C} = \underline{A} + \underline{B}$ if and only if $C_{ij} = A_{ij} + B_{ij}$

2. The ***dot product*** of a dyadic with a vector is a vector. Consider vectors $\underline{a}$ and $\underline{b}$, and the derived dyadic $\underline{A} = \underline{a}\underline{b}$. The dot product with the vector $\underline{c}$ is the vector $\underline{d}$ given by

   $$\underline{d} \equiv \underline{A}\cdot\underline{c} = (\underline{a}\underline{b})\cdot\underline{c} = \underline{a}(\underline{b}\cdot\underline{c})$$

   which is a vector in the direction of vector $\underline{a}$.
   Note that $\underline{e} \equiv \underline{c}\cdot\underline{A} = \underline{c}\cdot(\underline{a}\underline{b}) = (\underline{c}\cdot\underline{b})\underline{c} \neq \underline{d}$

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⇒ In general, pre-multiplying a vector by a dyadic ≠ post-multiplying the vector by the same dyadic.

For a *symmetric* dyadic, the order does not matter. Consider the product of Inertia dyadic with the angular velocity vector:

$$\underline{I}\cdot\underline{\omega} = (I_{xx}\underline{i}\,\underline{i} + I_{xy}\underline{i}\,\underline{j} + I_{xz}\underline{i}\,\underline{k} + I_{yx}\underline{j}\,\underline{i} + I_{yy}\underline{j}\,\underline{j} + I_{yz}\underline{j}\,\underline{k}$$

$$+ I_{zx}\underline{k}\,\underline{i} + I_{zy}\underline{k}\,\underline{j} + I_{zz}\underline{k}\,\underline{k})\cdot\underline{\omega} = \underline{I}\cdot(\omega_x\underline{i} + \omega_y\underline{j} + \omega_k\underline{k})$$

Now, note that $I_{xx}\underline{i}\,\underline{i}\cdot(\omega_x\underline{i} + \omega_y\underline{j} + \omega_k\underline{k})$

$= I_{xx}\underline{i}(\omega_x\underline{i}\cdot\underline{i} + \omega_y\underline{i}\cdot\underline{j} + \omega_k\underline{i}\cdot\underline{k}) = I_{xx}\omega_x\underline{i}$ etc.

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Thus, $\underline{I}\cdot\underline{\omega} = (I_{xx}\omega_x + I_{xy}\omega_y + I_{xz}\omega_z)\underline{i}$

$$+ (I_{yx}\omega_x + I_{yy}\omega_y + I_{yz}\omega_z)\underline{j}$$

$$+ (I_{zx}\omega_x + I_{zy}\omega_y + I_{zz}\omega_z)\underline{k} = \underline{\omega}\cdot\underline{I}$$

Since dot product of a dyadic and a vector is a new vector, ***dyadic*** is really an ***operator*** acting on a vector. A interesting symmetric dyadic is the ***unit dyadic:***

$$\underline{U} \equiv \underline{i}\,\underline{i} + \underline{j}\,\underline{j} + \underline{k}\,\underline{k}$$

For any vector $\underline{a}$, $\underline{U}\cdot\underline{a} \equiv \underline{i}(\underline{i}\cdot\underline{a}) + \underline{j}(\underline{j}\cdot\underline{a}) + \underline{k}(\underline{k}\cdot\underline{a})$

$= \underline{a}\cdot\underline{U} = \underline{a}$ So, it leaves every vector unchanged.

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Finally, consider the ***cross product*** of a dyadic with a vector:

$$\underline{c}\times\underline{A}\neq\underline{A}\times\underline{c}. \quad \text{If } \underline{A}=\sum_{i=1}^{3}\sum_{j=1}^{3}A_{ij}\,\underline{e}_i\,\underline{e}_j, \text{ then}$$

$$\underline{c}\times\sum_{i=1}^{3}\sum_{j=1}^{3}A_{ij}\,\underline{e}_i\,\underline{e}_j=\sum_{i=1}^{3}\sum_{j=1}^{3}A_{ij}(\underline{c}\times\underline{e}_i)\,\underline{e}_j$$

– another dyadic

See Section 7.5 for Greenwood.

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1.2 **Newton's Laws**: **There are three laws**

1. Every body continues in its state of rest, or of uniform motion in a straight line, unless compelled to change that state by forces acting upon it.

2. The time rate of change of linear momentum of a body is proportional to the force acting upon it and occurs in the direction in which the force acts.

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3. To every action there is an equal and opposite reaction.

- **Laws of motion for a particle:**

Let $\underline{F}$ **=** **applied force,**

m = mass of the particle, $\underline{v}$ **= velocity at an instant,**

$\underline{p} \equiv m\underline{v}$ **= linear momentum of the particle.**

Then, $\underline{F} = k\frac{d}{dt}(\underline{p}) = k\frac{d}{dt}(m\underline{v})$

$= km\underline{a}$ **(for a body with constant mass)**

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Here, k > 0 constant; it is chosen such that k = 1 depending on the choice of units.

$$\rightarrow \underline{F} = \frac{d}{dt}(\underline{p}) = \frac{d}{dt}(m\underline{v})$$

$= m\underline{a}$ **(for constant mass system)**

Since, $\underline{F}$ and $\underline{a}$ **are vectors, we can express them in the appropriate coordinate system.**

ex: in a Cartesian coordinate system (x,y,z):

$$\underline{F} = F_x\underline{i} + F_y\underline{j} + F_z\underline{k} = m(a_x\underline{i} + a_y\underline{j} + a_z\underline{k})$$

or $F_x = ma_x,\ F_y = ma_y,\ F_z = ma_z$

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<u>Imp</u>: Newton's laws are applicable in a special reference frame - called the inertial reference frame.

Note that in practice, any fixed reference frame or rigid body will suffice.

<u>READING ASSIGNMENT #1: The discussion in text</u>.

1.3 UNITS:

One first introduces dimensions associated with each quantity.

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Note that the quantities related by Newton's Laws are: F - force, M - mass, L - length, T - time.

Since there is one relation among the four quantities (second law), three of the units are independent, and the fourth fixed by the requirement of <u>principle of dimensional homogeneity</u>.

- **Absolute system - mass, length, and time are fundamental quantities, where as force is considered a derived quantity.**

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In the absolute system, the units of various quantities are: Mass - kilogram (kg); time - second (sec.); length - meter (m)

Now, consider the dimensional relation:

$F = ML/T^2 = kg.m/s^2$

This unit is called a Newton: it is the force needed to give an object of mass 1 kg an acceleration of 1 m/s^2.

- Gravitational system - In this case, length, time, and force are assumed fundamental, where as mass is derived. 1 slug = $lb.s^2/ft$

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READING ASSIGNMENT #2:

1.4. The Basics of Newtonian Mechanics.

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1.5 D'ALEMBERT'S PRINCIPLE:

Consider Newton's second law: $\underline{F}=m\underline{a}$

If we write $(\underline{F}-m\underline{a})=0$,

one can imagine (-m$\underline{a}$) to represent another force, the so called inertia force. Then, we just have summation of forces = 0, that is, an equivalent statics problem. We will see that this principle has profound significance when considering derivation of Lagrange's equations.

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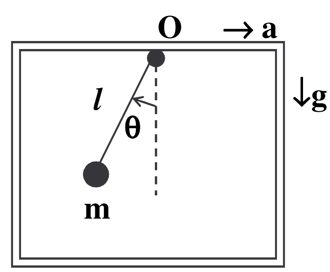
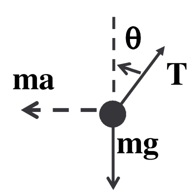
Ex 1.1: (Text)

Consider a massless rigid rod suspended from point O in a box which is accelerating to the right at a constant acceleration 'a'.



Find: The tension in the cable and the angle θ, when rod has reached a steady position relative to the box.

Free body diagram:



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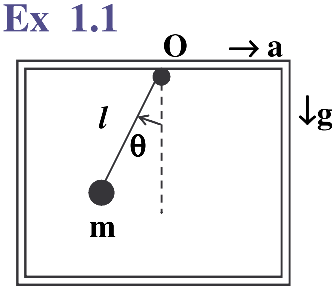
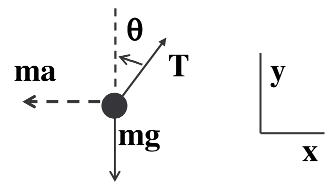
Ex 1.1



Free body diagram:



D'Alembert's principle:

$\sum F_x - ma_x = 0$: $T\sin\theta - ma = 0$;

$\sum F_y - ma_y = 0$: $T\cos\theta - mg = 0$;

$\Rightarrow \; T = m\sqrt{(a^2+g^2)}$, $\tan\theta = a/g$

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<u>Ex 1.2</u>: (to clearly point out the difference in components and orthogonal projections)

- **Consider the vector $\underline{A}$ with $|\underline{A}| = 5$, at an angle of 60° with the horizontal.**

- **Suppose that we want to express it in terms of unit vectors $\underline{e}_1$, $\underline{e}_2$ where**
  $\underline{e}_1 = \rightarrow(\underline{i})$, $\underline{e}_2 = |1| \measuredangle 135°$ $(=(-\underline{i}+\underline{j})/\sqrt{2})$.

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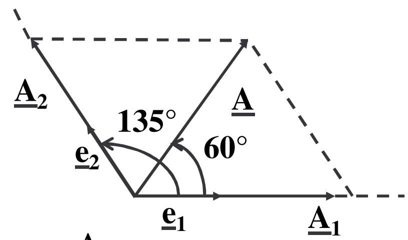
The vector sum can be represented as $\underline{A}=\underline{A}_1+\underline{A}_2$

or $\underline{A}=A_1\underline{e}_1+A_2\underline{e}_2$



Now

$\underline{A}=5(\cos 60\,\underline{i}+\sin 60\,\underline{j})=A_1\,\underline{e}_1+A_2\underline{e}_2$

$=A_1\,\underline{i}+A_2(-\underline{i}+\underline{j})/\sqrt{2}$

$\rightarrow (5\,\underline{i}+5\sqrt{3}\,\underline{j})/2=A_1\,\underline{i}+A_2(-\underline{i}+\underline{j})/\sqrt{2}$

or $\underline{\underline{i}}: 5/2=A_1-A_2/\sqrt{2}$; $\underline{\underline{j}}: A_2=5\sqrt{6}/2$

$\rightarrow$ $\boxed{\underline{A}=[5(1+\sqrt{3})\,\underline{e}_1+5\sqrt{6}\,\underline{e}_2]/2}$

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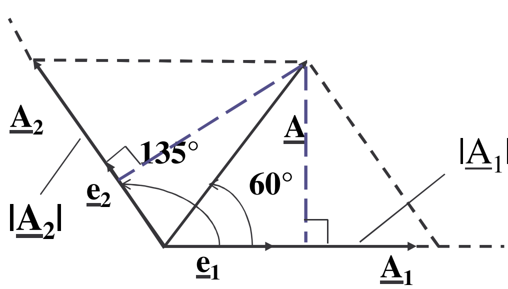
In the above,

component of $\underline{A}$ along $\underline{e}_1$ *is* $5(1+\sqrt{3})\,\underline{e}_1/2$;

component of $\underline{A}$ along $\underline{e}_2$ *is* $5\sqrt{6}\,\underline{e}_2/2$.

- **What about orthogonal projections of $\underline{A}$?**



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