**ME 562 Advanced Dynamics**
**Summer 2010**
**HOMEWORK # 7**

**Due: August 2, 2010**

**Q1. (see Problem 6-24 in the text).** A disk of radius $R$ rotates about its fixed vertical axis of symmetry at a constant rate $\omega$. A simple pendulum of length $l$ and particle mass $m$ is attached at a point on the edge of the disk. As generalized coordinates, let $\theta$ be the angle of the pendulum from the downward vertical, and let $\phi$ be the angle between the vertical plane of the pendulum and the vertical plane of the radial line from the center of the disk to the attachment point, where positive $\dot{\phi}$ is in the same sense as $\omega$.

1. Find $T_2$, $T_1$, and $T_0$.
2. Obtain the differential equations of motion.
3. Assuming that $R = l$, $\omega^2 = g/2l$, and the initial conditions are $\theta(0) = 0$, $\dot{\theta}(0) = 0$, show that $\theta$ cannot exceed 72.93°.

**Q2. (see Problem 6-11 in the text for a figure).** Consider the system shown. It consists of particles $m_1$ and $m_2$ that are connected by a massless rod of length $l$. These particles move on a frictionless horizontal plane, the motion of $m_1$ being confined to a fixed frictionless circular track of radius $R$. Denote the generalized coordinates for describing the dynamics of the system by the angles θ and ϕ. Then, derive the following:

1. The expression for kinetic energy of the system.
2. The equations of motion of the system for the coordinates θ and ϕ.
3. Identify the ignorable coordinate in the system, and define the generalized momentum $p$ associated with it.
4. Define the Routhian function for the system and use it to derive the equation of motion for the reduced one degree-of-freedom system.

**Q3. (see Problem 7-2 the text).** Four particles, each of mass $m/4$, are located at $(x_0, 0, 0)$, $(-x_0/3, y_0, -z_0/2)$, $(-x_0/3, -y_0, -z_0/2)$, $(-x_0/3, 0, z_0)$, relative to the $xyz$ axes. The particles are connected by rigid massless rods. Solve for the values of $x_0$, $y_0$, and $z_0$ such that the system has principal moments of inertia $I_{xx}$, $I_{yy}$, and $I_{zz}$.

**Q4. (see Problem 7-6 in the text).** A certain rigid body has the following inertia matrix with respect to some reference point:

$$[I] = \begin{bmatrix} 450 & -60 & 100 \\ -60 & 500 & 7 \\ 100 & 7 & 550 \end{bmatrix} kg - m^2$$

a. Solve for the principal moments of inertia at the same reference point.
b. Find the rotation matrix such that one can transform (rotate) from the xyz system to the principal coordinate system.

**Q5. (see Problem 8-3 in the text).** A solid homogeneous sphere of mass $m_0$ and radius $r$ rolls without slipping on a triangular block of mass $m$ which can slide on a frictionless floor. Assume that the system is initially motionless.

a. Derive the equations of motion for the system.
b. Find the velocity of the block as a function of time.