ME 562 Advanced Dynamics
Summer 2010
HOMEWORK # 6

Due: July 23, 2010

Q1. Two wheels, each of mass *m*, are connected by a massless axle of length *l*. Each wheel is considered to have its mass concentrated as a particle at its hub. The wheels roll without slipping on a horizontal plane. The hub of wheel A is attached by a spring of stiffness *k* and unstressed length *l* to a fixed point O. Use *r*, θ, and ϕ as generalized coordinates. Then, find: (a) the constraints satisfied by these three variables; (b) the relationships between the virtual displacements in the three variables; (c) if the constraints are holonomic or non-holonomic. **(see Problem 6-25 in the text for a figure).**

Q2. (see Problem 6-7 in the text for a figure). A double pendulum consists of two massless rods of length *l* and two particles of mass *m* which can move in the vertical plane. Assume frictionless joints and define the configuration of the system using the coordinates θ and ϕ. Recall that the system is in the vertical plane.

(i) Derive the generalized forces for the generalized coordinates θ and ϕ corresponding to the weights forces of the two particles.

(ii) Then, use Lagrange's equations for holonomic systems and derive the differential equations of motion for the system.

Q3. (see Problem 6-13 in the text for a figure). A smooth tube in the form of a circle of radius *r* is pinned at O and rotates in its vertical plane with a constant angular velocity ω. The position of a particle of mass *m* that slides inside the tube is given by the relative coordinate ϕ. ϕ is the angle that the line joining the center of the ring/tube (O') to the particle makes with OO'. Use Lagrange's equations for holonomic systems to derive the differential equation for ϕ, the only generalized coordinate. Note that $\dot{\theta} = \omega$ is constant and is specified, thus it is not a generalized coordinate.

Q4. (see Problem 6-22 in the text for a figure). A dumbbell is composed of two particles, each of mass *m*, connected by a massless rod of length *l*. One particle of the dumbbell is connected by a pin to the edge of a disk of radius *r*, which is massless except for a particle of mass *m* at its center. The disc can roll without slipping on a horizontal surface. Assume frictionless joints and define the configuration of the system using the coordinates θ and ϕ which are absolute rotation angles. The system is in the vertical plane. Then, use Lagrange's equations for holonomic systems and derive the differential equations of motion for the system.