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ME 562

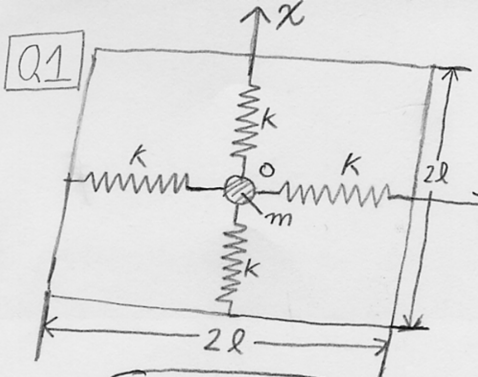
Advanced Dynamics

Summer 2010

Homework #3

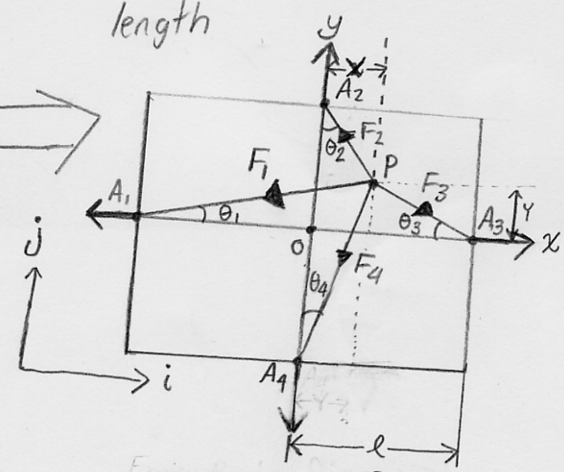
Q1

Assumption: 1) Springs never get compressed to zero length

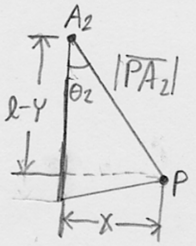


Initial Position

$\Longrightarrow$



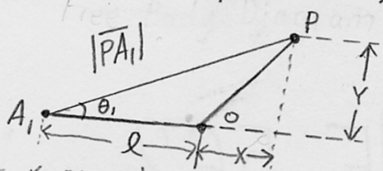
Final Position



$\cos\theta_2 = \dfrac{\ell - Y}{|PA_2|}$

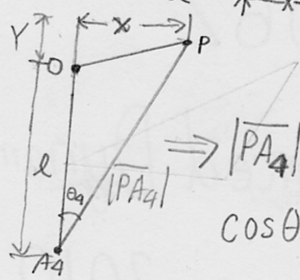
$\sin\theta_2 = \dfrac{X}{|PA_2|}$

$\Rightarrow |PA_2| = \sqrt{(\ell-Y)^2 + X^2}$



$\Longrightarrow |PA_1| = \sqrt{(\ell+X)^2 + Y^2}$

$\cos\theta_1 = \dfrac{\ell+X}{|PA_1|}$; $\sin\theta_1 = \dfrac{Y}{|PA_1|}$

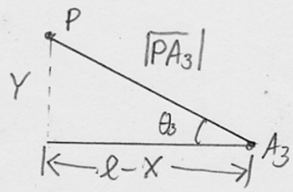


$\Longrightarrow |PA_4| = \sqrt{X^2 + (\ell+Y)^2}$

$\cos\theta_4 = \dfrac{\ell+y}{|PA_4|}$; $\sin\theta_4 = \dfrac{X}{|PA_4|}$

$\cos\theta_3 = \dfrac{\ell - X}{|PA_3|}$

$\sin\theta_3 = \dfrac{Y}{|PA_3|}$



$\Rightarrow |PA_3| = \sqrt{(\ell-X)^2 + Y^2}$

$$\text{Force}\begin{cases} \underline{F}_1 = k\cdot(|PA_1| - \ell)\cdot(-\cos\theta_1\,\underline{i} - \sin\theta_1\,\underline{j}) \\ \underline{F}_2 = k\cdot(|PA_2| - \ell)\cdot(-\sin\theta_2\,\underline{i} + \cos\theta_2\,\underline{j}) \\ \underline{F}_3 = k\cdot(|PA_3| - \ell)\cdot(\cos\theta_3\,\underline{i} - \sin\theta_3\,\underline{j}) \\ \underline{F}_4 = k\cdot(|PA_4| - \ell)\cdot(\sin\theta_4\, i - \cos\theta_4\,\underline{j}) \end{cases}$$

$$\therefore \left[\begin{array}{l} \underline{F}_1 = k\left[(\ell+x)\left(\dfrac{\ell}{\sqrt{(\ell+x)^2+Y^2}} - 1\right)\underline{i} + y\left(\dfrac{\ell}{\sqrt{(\ell+x)^2+Y^2}} - 1\right)\underline{j}\right] \\ \underline{F}_2 = k\left[X\left(\dfrac{\ell}{\sqrt{x^2+(\ell-Y)^2}} - 1\right)\underline{i} - (\ell-Y)\left(\dfrac{\ell}{\sqrt{X^2+(\ell-Y)^2}} - 1\right)\underline{j}\right] \\ \underline{F}_3 = k\left[(X-\ell)\left(\dfrac{\ell}{\sqrt{(\ell-X)^2+Y^2}} - 1\right)\underline{i} + Y\left(\dfrac{\ell}{\sqrt{(\ell-X)^2+Y^2}} - 1\right)\underline{j}\right] \\ \underline{F}_4 = k\left[X\left(\dfrac{\ell}{\sqrt{X^2+(\ell+Y)^2}} - 1\right)\underline{i} + (\ell+Y)\left(\dfrac{\ell}{\sqrt{X^2+(Y+\ell)^2}} - 1\right)\underline{j}\right] \end{array}\right.$$

Equations of motion

$$\begin{cases} m\ddot{x} = \sum F_x \\ m\ddot{y} = \sum F_y \end{cases}$$

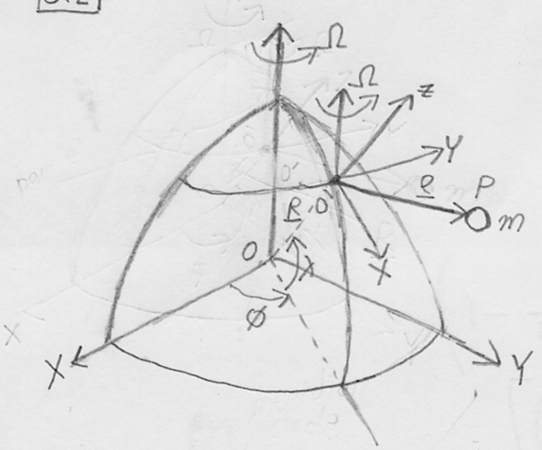
$$\therefore \ddot{x} = \frac{1}{m}\Bigg[k(\ell+X)\left(\frac{\ell}{\sqrt{(\ell+X)^2+Y^2}}-1\right)+kX\left(\frac{\ell}{\sqrt{X^2+(\ell-Y)^2}}-1\right)$$

$$+k(X-\ell)\left(\frac{\ell}{\sqrt{(\ell-X)^2+Y^2}}-1\right)+kX\left(\frac{\ell}{\sqrt{X^2+(\ell+Y)^2}}-1\right)\Bigg]$$

$$\ddot{y} = \frac{1}{m}\Bigg[kY\left(\frac{\ell}{\sqrt{(\ell+X)^2+Y^2}}-1\right)+k(Y-\ell)\left(\frac{\ell}{\sqrt{X^2+(\ell+Y)^2}}-1\right)\Bigg]$$

$$+kY\left(\frac{\ell}{\sqrt{X^2+(\ell-Y)^2}}-1\right)+k(\ell+Y)\left(\frac{\ell}{\sqrt{X^2+(Y+\ell^2)}}-1\right)\Bigg]$$

Q2



$$\underline{v}_p = \dot{\underline{R}} + (\dot{\underline{\rho}})_{rel} + \underline{\omega} \times \underline{\rho}$$

$$\underline{a}_p = \ddot{\underline{R}} + (\ddot{\underline{\rho}})_{rel} + 2\underline{\omega} \times (\dot{\underline{\rho}})_{rel} + \dot{\underline{\omega}} \times \underline{\rho} + \underline{\omega} \times (\underline{\omega} \times \underline{\rho})$$

$$\underline{\rho} = x\,\underline{i} + y\,\underline{j} + z\,\underline{k}$$

$$\therefore (\dot{\underline{\rho}})_{rel} = \dot{x}\,\underline{i} + \dot{y}\,\underline{j} + \dot{z}\,\underline{k}$$

$$(\ddot{\underline{\rho}})_{rel} = \ddot{x}\,\underline{i} + \ddot{y}\,\underline{j} + \ddot{z}\,\underline{k}$$

And $\underline{\omega} = \Omega \times \underline{k} = \Omega(-\cos\lambda\,\underline{i} + \sin\lambda\,\underline{k})$

$$\therefore \underline{\omega} \times \underline{\rho} = \Omega(-\cos\lambda\,\underline{i} + \sin\lambda\,\underline{k}) \times (x\,\underline{i} + y\,\underline{j} + z\,\underline{k})$$

$$= \Omega[-y\sin\lambda\,\underline{i} + (z\cos\lambda + x\sin\lambda)\,\underline{j} - y\cos\lambda\,\underline{k}]$$

$$\underline{\omega} \times (\underline{\omega} \times \underline{\rho}) = \Omega^2[-\sin\lambda(z\cos\lambda + x\sin\lambda)\,\underline{i} - y\,\underline{j} - \cos\lambda(z\cos\lambda + x\sin\lambda)\,\underline{k}]$$

$$2\underline{\omega} \times (\dot{\underline{\rho}})_{rel} = 2(-\Omega\cos\lambda\,\underline{i} + \Omega\sin\lambda\,\underline{k}) \times (\dot{x}\,\underline{i} + \dot{y}\,\underline{j} + \dot{z}\,\underline{k})$$

$$= 2\Omega[-\dot{y}\sin\lambda\,\underline{i} + (\dot{z}\cos\lambda + \dot{x}\sin\lambda)\,\underline{j} - \dot{y}\cos\lambda\,\underline{k}]$$

$\dot{\underline{\omega}} \times \underline{\rho} = 0$; $\underline{R} = R\,\underline{k}$ (constant w.r.t xyz)

$$\dot{\underline{R}} = \underline{\omega} \times \underline{R}$$

$$\ddot{\underline{R}} = \underline{\omega} \times (\underline{\omega} \times \underline{R}) = \Omega^2 R(-\cos\lambda\sin\lambda\,\underline{i} - \cos^2\lambda\,\underline{k})$$

$$\boxed{\begin{aligned} \underline{a}_p = {} & \Omega^2 R(-\cos\lambda\sin\lambda\,\underline{i} - \cos^2\lambda\,\underline{k}) + \ddot{x}\,\underline{i} + \ddot{y}\,\underline{j} + \ddot{z}\,\underline{k} \\ & + 2\Omega[-\dot{y}\sin\lambda\,\underline{i} + (\dot{z}\cos\lambda + \dot{x}\sin\lambda)\,\underline{j} - \dot{y}\cos\lambda\,\underline{k}] \\ & + \Omega^2[-\sin\lambda(z\cos\lambda + x\sin\lambda)\,\underline{i} - y\,\underline{j} - \cos\lambda(z\cos\lambda + x\sin\lambda)\,\underline{k}] \end{aligned}}$$

Since close to earth's surface, $\Omega^2 x << \Omega^2 R$

$\Rightarrow \Omega = 7.29 \times 10^{-5}$ rad/sec

Thus Ω^2 is neglected

$$\boxed{\underline{a}_p \approx \underbrace{(\ddot{x} - 2\Omega\dot{y}\sin\lambda)}_{a_x}\,\underline{i} + \underbrace{(\ddot{y} + 2\Omega(\dot{x}\sin\lambda + \dot{z}\cos\lambda))}_{a_y}\,\underline{j} + \underbrace{(\ddot{z} - 2\Omega\dot{y}\cos\lambda)}_{a_z}\,\underline{k}}$$

(i) particle with a force F and pull of gravity

$$F_x = ma_x \Rightarrow F_x - m(\ddot{x} - 2\Omega\dot{y}\sin\lambda) = 0$$

$$\boxed{\frac{F_x}{m} = \ddot{x} - 2\Omega\dot{y}\sin\lambda}$$

$$F_y = ma_y \Rightarrow F_y - m(\ddot{y} + 2\Omega(\dot{x}\sin\lambda + \dot{z}\cos\lambda)) = 0$$

$$\boxed{\frac{F_y}{m} = \ddot{y} + 2\Omega(\dot{x}\sin\lambda + \dot{z}\cos\lambda)}$$

$$F_z - mg = ma_z \Rightarrow F_z - m(\ddot{z} - 2\Omega\dot{y}\cos\lambda) - mg = 0$$

$$\boxed{\frac{F_z}{m} = \ddot{z} - 2\Omega\dot{y}\cos\lambda + g}$$

(ii) particle is in free flight & acted upon only by gravity & air drag

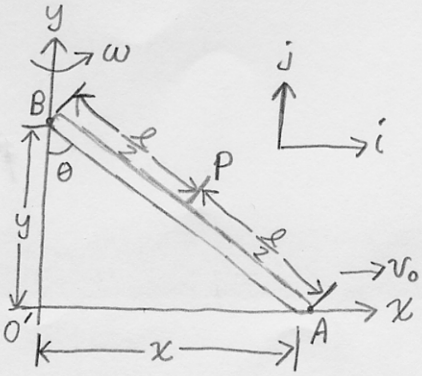
$[F = -(c\cdot m)V]$

$$F_x = -(c\cdot m)\dot{x} \Rightarrow \boxed{\ddot{x} - 2\Omega\dot{y}\sin\lambda + c\dot{x} = 0}$$

$$F_y = -(c\cdot m)\dot{y} \Rightarrow \boxed{\ddot{y} + 2\Omega(\dot{x}\sin\lambda + \dot{z}\cos\lambda + c\dot{y} = 0}$$

$$F_z = -(c\cdot m)\dot{z} \Rightarrow \boxed{\ddot{z} - 2\Omega\dot{y}\cos\lambda + c\dot{z} + g = 0}$$

Q3



$$x = l\sin\theta$$

$$v_0 = \dot{x} = l\dot{\theta}\cos\theta$$

$$\therefore \dot{\theta} = \frac{v_0}{l\cos\theta}$$

$$\underline{\omega} = \omega\,\underline{j}$$

$$\underline{v}_p = \underline{\dot{R}} + (\underline{\dot{\rho}})_r + \underline{\omega}\times\underline{\rho}$$

$$\underline{\dot{R}} = 0, \quad \underline{\rho} = \left(\frac{l}{2}\sin\theta\right)\underline{i} + \left(\frac{l}{2}\cos\theta\right)\underline{j}$$

$$(\underline{\dot{\rho}})_r = \left(\frac{l}{2}\dot{\theta}\cos\theta\right)\underline{i} - \left(\frac{l}{2}\dot{\theta}\sin\theta\right)\underline{j}$$

$$\underline{\omega}\times\underline{\rho} = -\frac{\omega l}{2}\sin\theta\,\underline{k}$$

$$\underline{v}_p = \frac{l}{2}\dot{\theta}\cos\theta\,\underline{i} - \frac{l}{2}\dot{\theta}\sin\theta\,\underline{j} - \frac{l}{2}\omega\sin\theta\,\underline{k}$$

$$\boxed{\underline{v}_p = \frac{v_0}{2}\underline{i} - \frac{1}{2}v_0\tan\theta\,\underline{j} - \frac{1}{2}\omega l\sin\theta\,\underline{k}}$$

$$\underline{a}_p = \underline{\ddot{R}} + \underline{\dot{\omega}}\times\underline{\rho} + \underline{\omega}\times(\underline{\omega}\times\underline{\rho}) + (\underline{\ddot{\rho}})_r + 2\underline{\omega}\times(\underline{\dot{\rho}})_r$$

$$\underline{\ddot{R}} = 0, \quad \underline{\dot{\omega}}\times\underline{\rho} = 0$$

$$(\underline{\dot{\rho}})_r = \frac{v_0}{2}\underline{i} - \frac{v_0}{2}\tan\theta\,\underline{j}$$

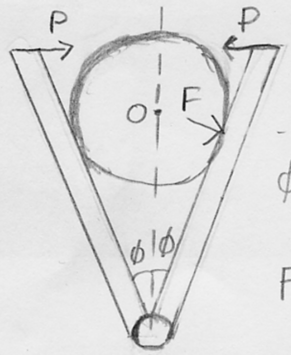
$$(\underline{\ddot{\rho}})_r = -\frac{v_0}{2}\dot{\theta}\sec^2\theta\,\underline{j} = -\frac{v_0^2}{2l\cos^3\theta}\underline{j}$$

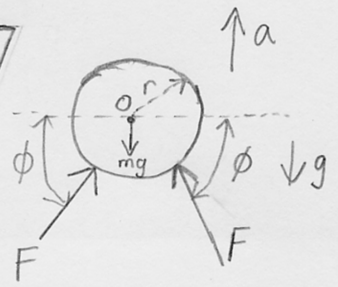
$$\underline{\omega}\times(\underline{\omega}\times\underline{\rho}) = -\frac{1}{2}l\omega^2\sin\theta\,\underline{i}$$

$$2\underline{\omega}\times(\underline{\dot{\rho}})_r = 2\omega\underline{j}\times\left(\frac{v_0}{2}\underline{i} - \frac{v_0}{2}\tan\theta\,\underline{j}\right) = -\omega v_0\,\underline{k}$$

$$\boxed{\underline{a}_p = -\frac{1}{2}l\omega^2\sin\theta\,\underline{i} - \frac{v_0^2}{2l\cos^3\theta}\underline{j} - \omega v_0\,\underline{k}}$$

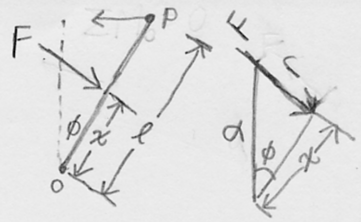
Q4





$$ma = 2F\sin\phi - mg$$

$$a = \frac{2F}{m}\sin\phi - g$$



$x = \alpha\cos\phi$ ①
$r = \alpha\sin\phi$ ②

② → ①

$$x = \left(\frac{r}{\sin\phi}\right)\cos\phi = r\cot\phi$$

$$\Sigma M_o = 0: P\ell\cos\phi - Fx = 0$$

$$P\ell\cos\phi - Fr\cot\phi = 0$$

$$\therefore F = \frac{P\ell}{r}\sin\phi$$

$$a = \frac{2P\ell}{mr}\sin^2\phi - g, \quad \phi = 30^\circ$$

$$\boxed{\therefore a = \frac{P\ell}{2mr} - g}$$

If sphere is not smooth, the sphere and the levers would have frictional forces between them. The friction force would be pointing in the opposite direction of the motion.

① $a = \frac{2P\ell}{mr}\sin^2\phi - g > 0$, the friction force would be added in the negative a direction & decrease the acceleration of moving upward.

② $a = \frac{2P\ell}{mr}\sin^2\phi - g < 0$, the friction force would point upward along the bar & decrease the acceleration.