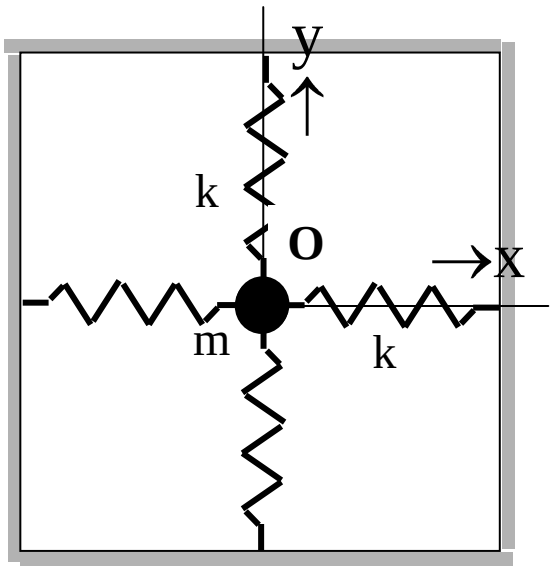
ME 562 Advanced Dynamics
Summer 2010
HOMEWORK # 3

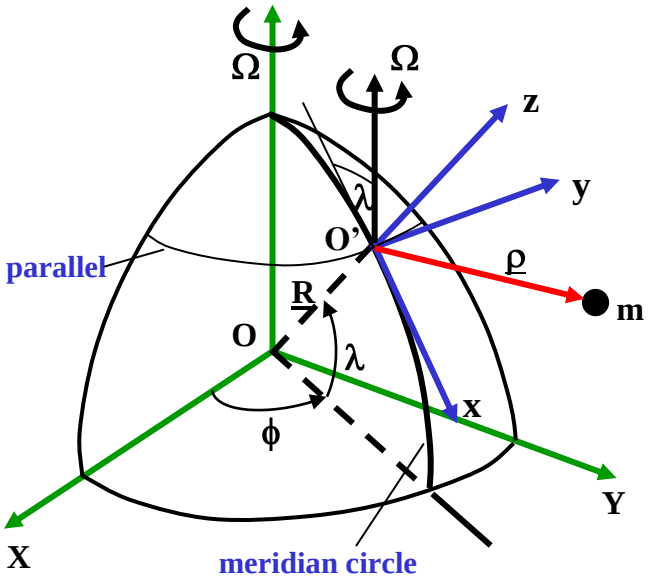
Due: June 21, 2010

Q1. Consider the particle of mass *m* located in the center of a box of sides *2l* and constrained to move in the horizontal plane by the four identical and linearly elastic springs with spring constant *k* each. Each of the springs has free length *l*. Let O be the origin of the coordinate system located at the center of the box. To derive the equation of motion of the particle, we displace the mass particle now to a position with coordinates (*x*, *y*). Then, derive (i) the forces in each of the spring as they act on the mass particle, (ii) the general equations for motion for the particle in the (*x*, *y*) coordinates. Note: you are not to make any assumptions regarding the size of displacement relative to the size of the box other than that the springs never get compressed to zero length.



Q2. Follow the developments in class notes and (i) re-derive (include all the details of derivation of expression for the inertial acceleration) the equations of motion for a particle with a force **F** and **the pull of gravity**, taking into account the rotation of the earth. (ii) Consider the case when the particle is in free flight and is acted upon only by gravity and the air drag (the drag force is modeled as proportional to the velocity of the particle relative to atmosphere and the atmosphere is rotating with the earth, (that is, **F** = -(c/m)**v**)). Then, show that the equations of motion are:

$$\ddot{x} - 2\Omega\dot{y}\sin\lambda + c\dot{x} = 0,$$
$$\ddot{y} + 2\Omega(\dot{x}\sin\lambda + \dot{z}\cos\lambda) + c\dot{y} = 0,$$
$$\ddot{z} - 2\Omega\dot{y}\cos\lambda + c\dot{z} + g = 0.$$



Q3. Consider the rod *AB* of length *l*, whose two ends *A* and *B* are pinned to the *x* and *y* axis, respectively. Thus, the end *A* only slides along *Ox* while the end *B* slides along *Oy*. The *Oxy* plane rotates about the vertical *OY* axis at a constant angular velocity ω. Suppose that the end *A* of the rod slides along the *x* axis with a constant speed v_0 relative to the *Oxy* plane. Find the

velocity and acceleration of the midpoint *P* of the rod, as seen by an observer in XYZ system. (**Problem 2-18 in the text).**

Q4. A smooth uniform sphere of mass *m* and radius *r* is squeezed between two massless levers, each of length *l*, which are inclined at an angle ϕ with the vertical. A force P is applied between the ends of the levers, as shown. The sphere is at rest before the force is applied. The levers make an angle of $\phi = 30°$. Find the upward acceleration of the sphere immediately after the force is applied. Comment on the changes in problem formulation that will be needed if the sphere was not smooth, that is, the sphere and the levers were to have friction forces between them. Don't solve this aspect of the problem, only comment on this situation. **(Problem 3-1 in the text).**