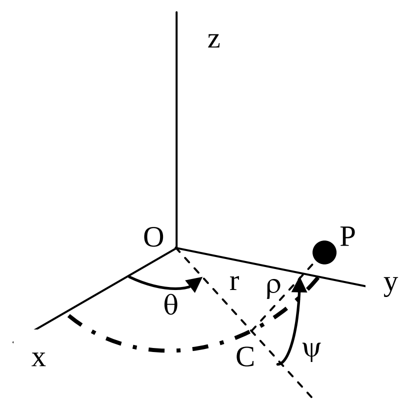
ME 562 Advanced Dynamics
Summer 2010
HOMEWORK # 2

Due: June 14, 2010

Q1. Follow the developments in class notes and derive the Frenet's formulas (summarized on page 19 of the Chapter 2 powerpoints) for a spatial curve. Recall that these formulas relate the rates of change of the unit vectors associated with the 'path variable' description of motion of an object to the properties (radius of curvature and twist) of the space curve. They are also shown here on the right.

$$\boxed{\begin{aligned} \frac{d\underline{e}_t}{ds} &= \frac{\underline{e}_n}{\rho} \equiv \kappa \underline{e}_n \\ \frac{d\underline{e}_b}{ds} &= -\frac{\underline{e}_n}{\tau} \\ \frac{d\underline{e}_n}{ds} &= -\frac{\underline{e}_t}{\rho} + \frac{\underline{e}_b}{\tau} \end{aligned}}$$

Q2. Toroidal coordinates (ρ, θ, ψ) are useful for magnetohydrodynamic studies in tokamaks. More specifically, consider a torus with radius of the centerline (a circle) to be *r*. This centerline is in the (x,y) plane. Now, consider a point P whose position we want to write with respect to the observer located at O. Let P' be the projection of the position of point P on to the (x,y) plane. The line joining this point P' with O intersects the centerline of the torus at C. We define the coordinates of the point P by using the radial line OC and the inclined line CP. Then, note from the figure that the Cartesian coordinates of the position P, in terms of the (x,y,z) components is given by $x = (r + \rho\cos\psi)\cos\theta$, $y = (r + \rho\cos\psi)\sin\theta$, $z = \rho\sin\psi$. So, θ is the angle the line OC makes with the x axis, and ψ is the angle that line CP makes with OC. Also, ρ is the distance along the line CP. See the diagram shown. Note that the three variables (ρ, θ, ψ) are sufficient to define the position of any point P when *r* is given. Derive expressions for the unit vectors in terms of the Cartesian basis ($\underline{i}, \underline{j}, \underline{k}$) for this coordinate system, and describe the derivatives of the unit vectors with respect to the toroidal coordinates. **Note** that *r* is a given fixed constant and (ρ, θ, ψ) are the toroidal variables. Thus, the unit vectors will be designated by $\underline{e}_\rho, \underline{e}_\theta$ and $\underline{e}_\psi$.



Q3. Obtain expressions for velocity of the point P in terms of the toroidal coordinates (ρ, θ, ψ) and the toroidal basis $\underline{e}_\rho, \underline{e}_\theta$ and $\underline{e}_\psi$ in Q2.

Q4. A particle moves in a uniform gravitational field with a constant downward acceleration g, that is, only the weight force acts on it. At a certain time, the particle velocity **v** is at an angle θ above the horizontal. At this time, find: (i) the normal component of acceleration a_n; (ii) the tangential component of acceleration a_t; (iii) the radius of curvature of the path at this instant ρ; (iv) the rate of change in velocity direction $\dot{\theta}$; (v) the rate of change in curvature $\dot{\rho}$; and (vi) $\ddot{\theta}$. Note that all the answers for these quantities must be in terms of the given (known) quantities **v**, g and θ.

Q5. A water particle *P* moves outward along the impeller vane of a centrifugal pump with a constant tangential velocity of 20 m/s relative to the impeller. The impeller is rotating at a uniform rate of 1200 rpm in the clockwise direction, as shown. The impeller vane is such that it is an arc of a circle with center located at O' on the periphery of the outer circle. Let $\underset{\sim}{e}_1$ and $\underset{\sim}{e}_2$ be unit vectors attached to the impeller (rotating with it) with directions as defined in the figure. Find (i) the velocity, and (ii) the acceleration of the fluid particle when it is about to leave the impeller at P. Use $\underset{\sim}{e}_1$ and $\underset{\sim}{e}_2$ to express the results.