ME 562 Summer 2010

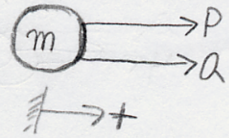
Homework 1

(6/9/10)

$\boxed{Q1}$ Problem 1-1 (P and Q have constant magnitudes)

<u>Step 1</u> $a_{max} = 3a_{min}$ — (1)

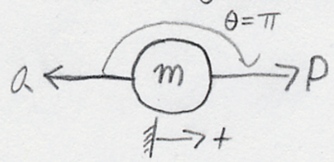
① $a = a_{max} \Rightarrow$ P and Q are in line and pointing in the same direction ($\theta = 0°$)



$$m a_{max} = P + Q$$

$$a_{max} = \frac{P+Q}{m} \quad (2)$$

② $a = a_{min} \Rightarrow$ P and Q are in line and pointing opposite of each other ($\theta = \pi$)



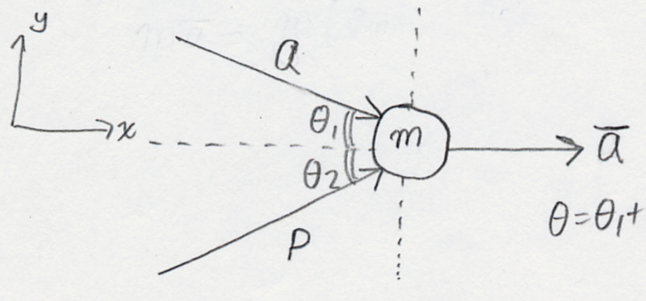
$$m a_{min} = P - Q$$

$$a_{min} = \frac{P-Q}{m} \quad (3)$$

Plug (2) & (3) into (1)

$$P + Q = 3(P - Q) \Rightarrow P = 2Q \quad (4)$$

<u>Step 2</u> $\bar{a} = \dfrac{a_{min} + a_{max}}{2} \Rightarrow m\bar{a} = \dfrac{m(a_{min} + a_{max})}{2}$



$\theta = \theta_1 + \theta_2$

<u>x:</u> $P\cos\theta_2 + Q\cos\theta_1 = m\bar{a} = \dfrac{(P+Q) + (P-Q)}{2}$

$$P\cos\theta_2 + Q\cos\theta_1 = P \quad (5)$$

Plug (4) into (5)

$$2\cos\theta_2 + \cos\theta_1 = 2 \quad (6)$$

<u>y:</u> $P\sin\theta_2 = Q\sin\theta_1$

$$2Q\sin\theta_2 = Q\sin\theta_1$$

$$2\sin\theta_2 = \sin\theta_1 \quad (7)$$

$$4\sin^2\theta_2 = \sin^2\theta_1$$

$$4(1-\cos^2\theta_2) = 1-\cos^2\theta_1$$

$$4\cos^2\theta_2 = 3+\cos^2\theta_1$$

$$\cos\theta_1 = \sqrt{4\cos^2\theta_2 - 3} \quad (8)$$

Plug (8) into (6)

$$2\cos\theta_2 + \sqrt{4\cos^2\theta_2-3} = 2$$

$$\sqrt{4\cos^2\theta_2-3} = 2-2\cos\theta_2$$

$$4\cos^2\theta_2 - 3 = 4 - 8\cos\theta_2 + 4\cos^2\theta_2$$

$$-3 = 4-8\cos\theta_2 \Rightarrow \cos\theta_2 = 7/8$$

$$\underline{\underline{\theta_2 = 28.955°}} \quad (9)$$

Using (7),

$$2\sin\theta_2 = \sin\theta_1$$

$$\sin\theta_1 = 2\sin(28.955) = 0.9682$$

$$\underline{\underline{\theta_1 = 75.522°}}$$

$$\theta = \theta_1+\theta_2 = 28.955° + 75.522° = \boxed{104.48°}$$

Q2 Problem 1-3

$\underline{e}_1 = l_1 \underline{i} + l_2 \underline{j} + l_3 \underline{k}$

$\underline{e}_2 = m_1 \underline{i} + m_2 \underline{j} + m_3 \underline{k}$

$\underline{e}_3 = n_1 \underline{i} + n_2 \underline{j} + n_3 \underline{k}$

a) $e_1, e_2,$ & e_3 are unit vectors

$$\left.\begin{array}{l} l_1^2 + l_2^2 + l_3^2 = 1 \\ m_1^2 + m_2^2 + m_3^2 = 1 \\ n_1^2 + n_2^2 + n_3^2 = 1 \end{array}\right]$$

b) e_1, e_2, e_3 are coplanar. Thus volume of parallelopiped is 0 and scalar triple product is 0.

$$e_1 \cdot (e_2 \times e_3) = 0$$

$$= \begin{vmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{vmatrix} = l_1 \begin{vmatrix} m_2 & m_3 \\ n_2 & n_3 \end{vmatrix} - l_2 \begin{vmatrix} m_1 & m_3 \\ n_1 & n_3 \end{vmatrix} + l_3 \begin{vmatrix} m_1 & m_2 \\ n_1 & n_2 \end{vmatrix}$$

$$\underline{\underline{0 = l_1 (m_2 n_3 - n_3 n_2) - l_2 (m_1 n_3 - m_3 n_1) + l_3 (m_1 n_2 + m_2 n_1)}}$$

Leaving answer as $e_1 \cdot (e_2 \times e_3) = 0$ by itself is also fine.

c) e_1, e_2, e_3 are mutually orthogonal. Thus the dot product of two of three of these vectors is zero.

$\underline{e}_1 \cdot \underline{e}_2 = 0$

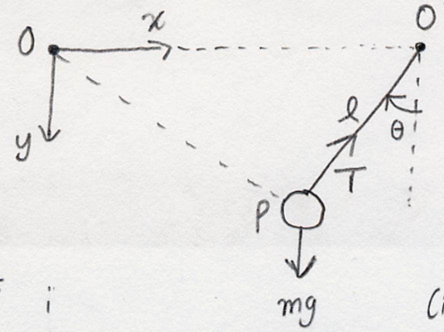
$\underline{e}_2 \cdot \underline{e}_3 = 0$

$\underline{e}_3 \cdot \underline{e}_1 = 0$

$$\left.\begin{array}{l} l_1 m_1 + l_2 m_2 + l_3 m_3 = 0 \\ m_1 n_1 + m_2 n_2 + m_3 n_3 = 0 \\ n_1 l_1 + n_2 l_2 + n_3 l_3 = 0 \end{array}\right\}$$

Q3

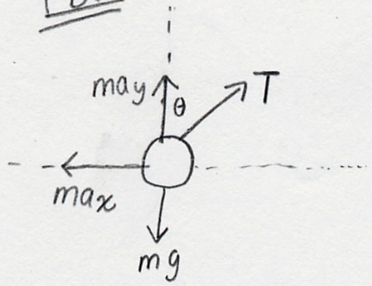
$\underline{r}_{OO'} = x_{OO'}\, i$



(i) Position of particle P w.r.t. inertial frame given as

$$\underline{r}_{OP} = \underline{r}_{OO'} + \ell(\cos\theta\, j - \sin\theta\, i)$$

FBD



(ii) Velocity

$$\dot{\underline{r}}_{OP} = \dot{\underline{r}}_{OO'} + \ell\dot{\theta}(-\sin\theta\, \underline{j} - \cos\theta\, \underline{i})$$

where $\dot{\underline{r}}_{OO'} = \dot{x}_{OO'}\, i$

(iii) Acceleration

$$\ddot{\underline{r}}_{OP} = \ddot{\underline{r}}_{OO'} - \ell\ddot{\theta}(\sin\theta\, \underline{j} + \cos\theta\, \underline{i}) - \ell\dot{\theta}^2(\cos\theta\, \underline{j} - \sin\theta\, \underline{i})$$

$$= \ddot{x}_{OO'}\, i - \ell\ddot{\theta}(\sin\theta\, \underline{j} + \cos\theta\, \underline{i}) - \ell\dot{\theta}^2(\cos\theta\, \underline{j} - \sin\theta\, \underline{i}) \checkmark$$

Equations of motion

$\boxed{1}$ $\Sigma F_x = ma_x$

$$T\sin\theta = ma_x = m(\ddot{x}_{OO'} - \ell\ddot{\theta}\cos\theta + \ell\dot{\theta}^2\sin\theta) \checkmark$$

$\boxed{2}$ $\Sigma F_y = ma_y$

$$T\cos\theta - mg = ma_y = m[\ell\ddot{\theta}\sin\theta + \ell\dot{\theta}^2\cos\theta] \checkmark$$

Apply steady state conditions ($\dot{\theta} = \ddot{\theta} = 0$)

$$\left.\begin{aligned} T\sin\theta &= m\ddot{x}_{OO'} \\ T\cos\theta - mg &= 0 \end{aligned}\right\} \text{Equations of motion in steady-state}$$