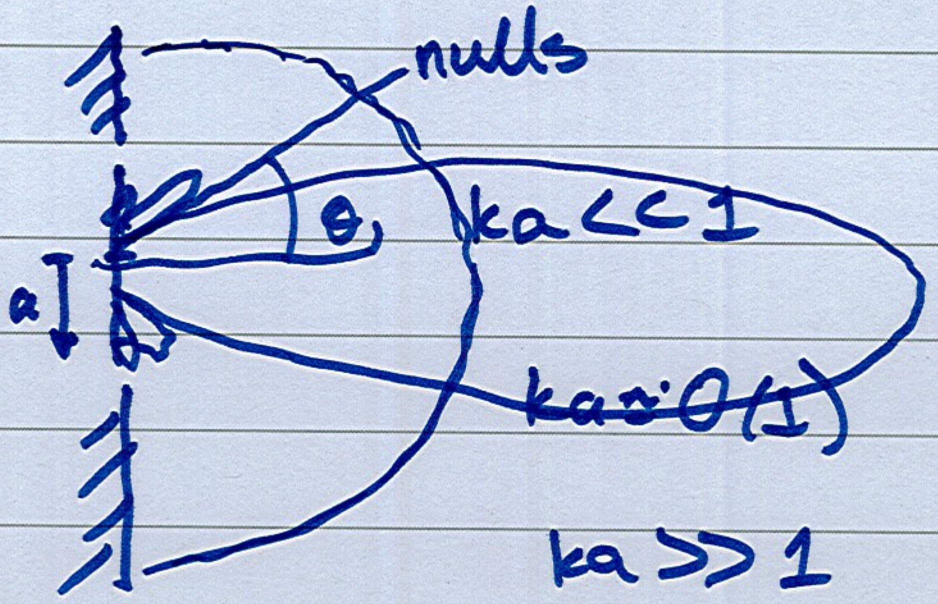
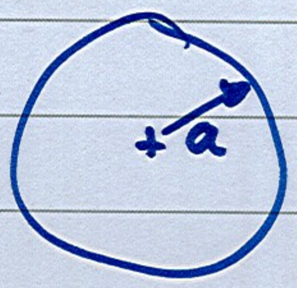




<u>Farfield</u>

Farfield Expression

$$\tilde{p}(r,\theta) = j\frac{\rho_0 c}{2} u_0 \left(\frac{a}{r}\right) e^{-jkr} \left[2\, \frac{J_1(ka\sin\theta)}{ka\sin\theta} \right]$$

Directivity Factor

<u>Angles of the nulls</u>

given by $ka\sin\theta_i = j_{1i}$

j_{1i} ith zero of the Bessel function Tabulated in A5

$J_1(j_{11}) = 0$ $\quad ka\sin\theta_1 = j_{11}$

solve for θ_1

$J_1(j_{12}) = 0$ $\quad ka \sin\theta_2 = j_{12}$

solve for θ_2

at high frequencies
$ka >> 1$

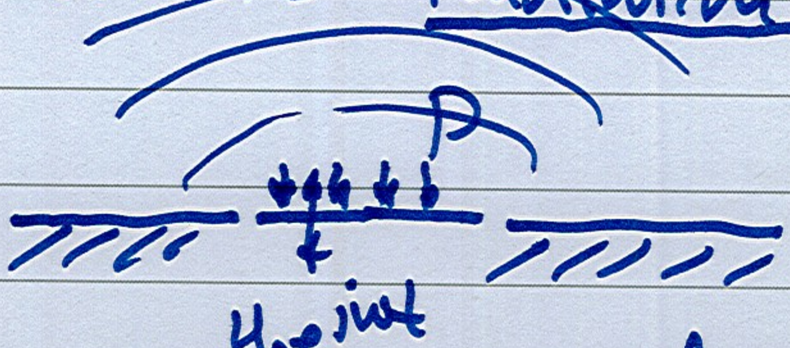
j_{11} is reached at progressively angles

"beamwidth" of the main lobe
becomes narrower

Public Address systems

- many small hf drivers pointing in various directions to insure uniform coverage
- relatively large lf drivers → omni

5.4.1.2 Radiation Impedance

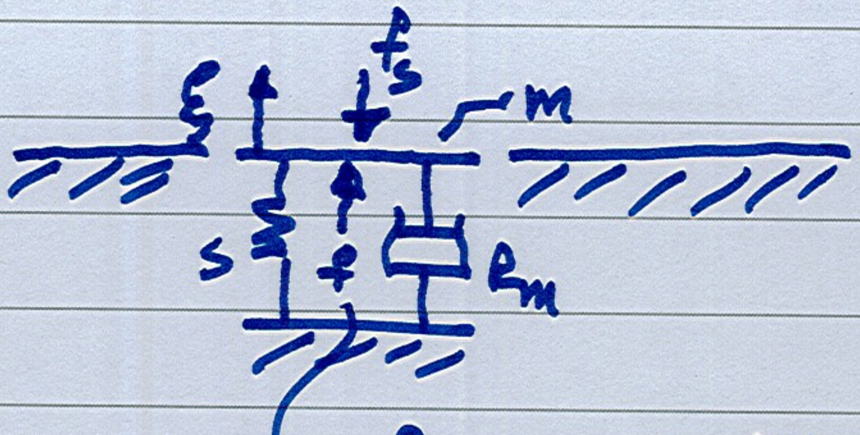


$$Z_r = \frac{\text{force exerted on the radiator by the sound field}}{\text{source velocity}}$$

$$Z_r = \frac{\int_S P_{surface}\, dS}{u_0}$$

u_0 ~ rigid piston velocity

~~[illegible]~~ loudspeakers - finite internal impedance



simple model of l/s diaphragm

force applied by voice coil

EOM

$$f - f_s = m\frac{d^2\xi}{dt^2} + R_m\frac{d\xi}{dt} + s\xi$$

Assume harmonic motion $e^{j\omega t}$

$u = j\omega\xi$ diaphragm velocity

$$f - f_s = j\omega m u + R_m u + \frac{s}{j\omega} u$$

$$= \left[R_m + j\left(\omega m - \frac{s}{\omega}\right)\right] u$$

in vacuo mechanical impedance of Ro l/s Z_M

$$f - f_s = Z_m u$$

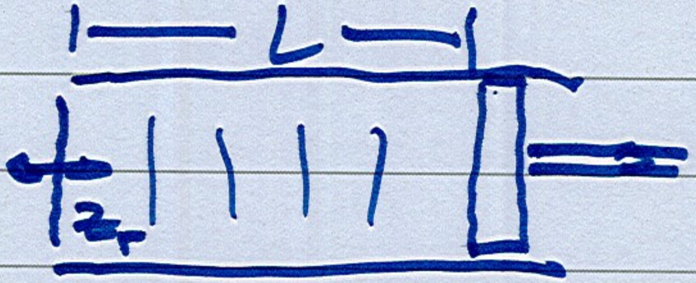
$$f = f_s + Z_m u \qquad Z_r = \frac{f_s}{u}$$

$$= (Z_r + Z_m)\, u$$

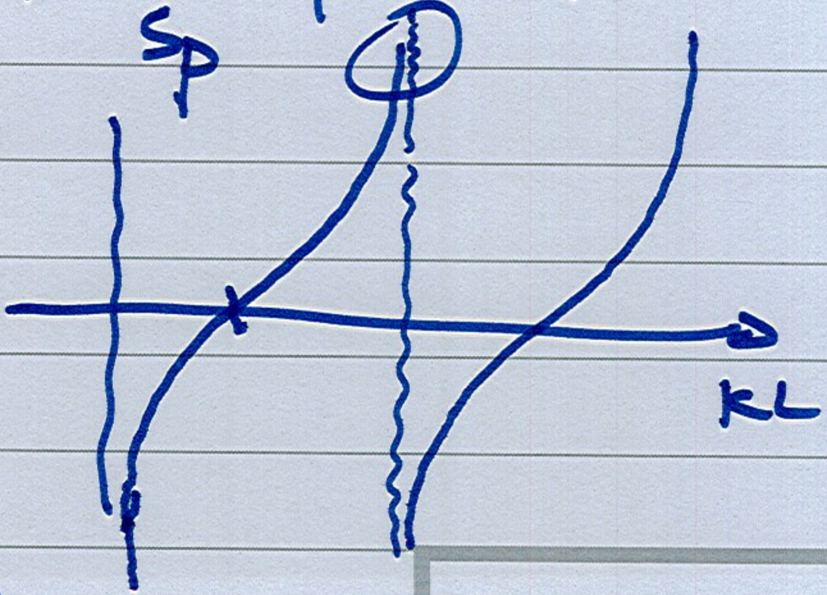
radiation impedance mechanical impedance

$$u = \frac{f}{Z_r + Z_m}$$

L/S response is determined by both the mechanical & radiation impedances



movable piston of area S_p



$$Z_r = -j S_p \rho_0 c \cot kL$$

$$u = \frac{f}{Z_r + Z_m}$$

$\uparrow$
∞ when $\overset{L}{\cancel{A}} = \lambda/2$

$u \to 0$

$$Z_r = \underset{\Sigma}{\overset{\text{resistance}}{R_r}} + j X_r \leftarrow \text{reactance}$$

Power Radiated by The piston

$$\overline{\Pi} = \frac{1}{T}\int_0^T \mathrm{Re}\{f_s\}\,\mathrm{Re}\{u\}\,dt$$

harmonic case

$$\overline{\Pi} = \frac{1}{2}\mathrm{Re}\{f_s u^*\} \qquad z_r = \frac{f_s}{u}$$

$$= \frac{1}{2}\mathrm{Re}\{z_r u u^*\}$$

$$= \frac{|u|^2}{2}\mathrm{Re}\{z_r\}$$

$$= \frac{|u|^2}{2} R_r$$

mean square piston velocity

Circular rigid piston – in a baffle radiating into free space

$$R_r = \pi a^2 \rho_0 c \left(1 - \frac{2J_1(2ka)}{2ka} \right)$$

H_1 = Hankel function

$$X_r = 2\pi a^2 \rho_0 c \left(\frac{H_1(2ka)}{2ka} \right)$$

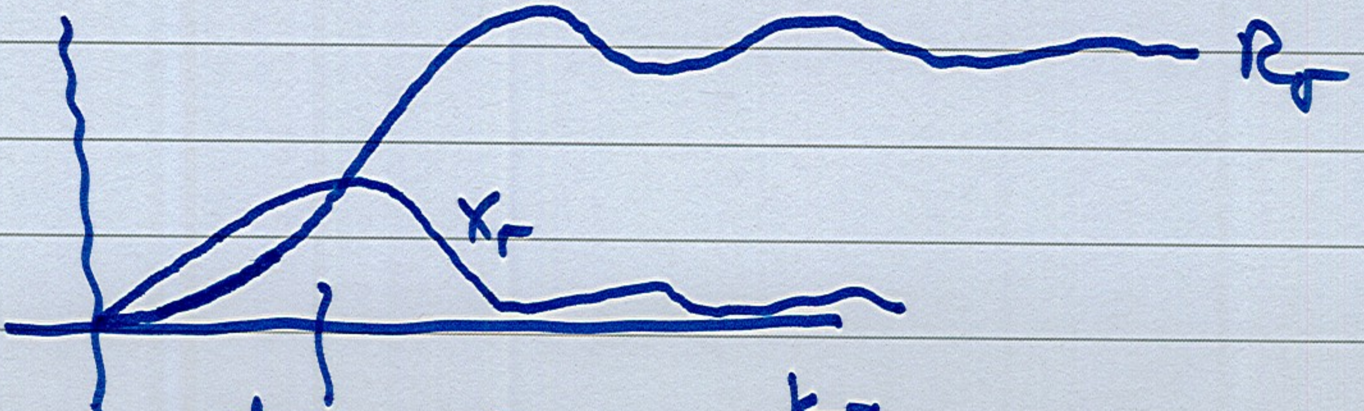
Struve function

when $ka \ll 1$ small source

$$R_r \simeq \frac{\pi a^2}{2} \rho_0 c (ka)^2$$

$$X_r \simeq \pi a^2 \rho_0 c (ka) \frac{8}{3\pi}$$

when $ka \ll 1$ X_r dominates mass-like

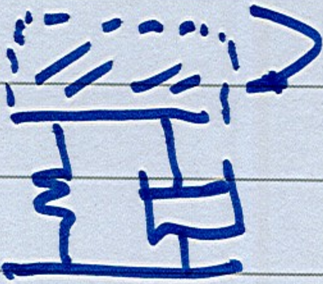


$\leftarrow$ $ka \ll 1$ $\rightarrow$ $ka \gg 1$

X_r $\qquad$ R_r

$$X_r = \omega m_r$$

$$m_r = \pi a^2 \rho_0 \left(\frac{8a}{3\pi}\right)$$ effective mass



$$\omega = \sqrt{\frac{s}{m + m_r}}$$

$ka \ll 1$

added mass has the effect of lowering the natural freq of the l/s