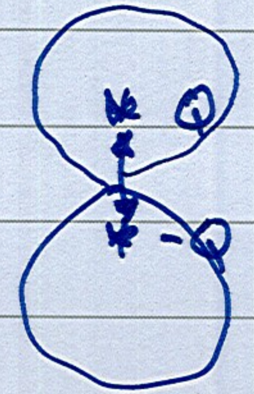
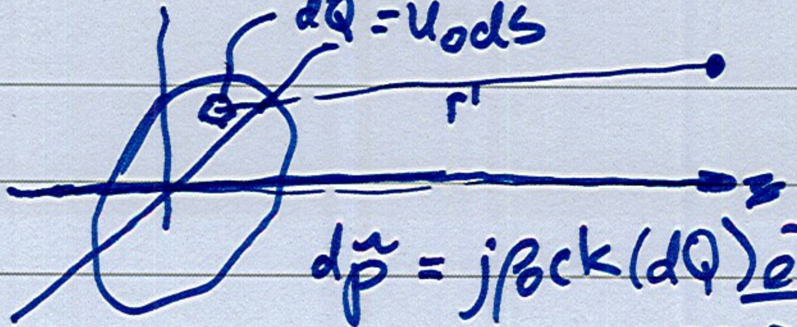
ME 513 Session 38 11/18/19

1

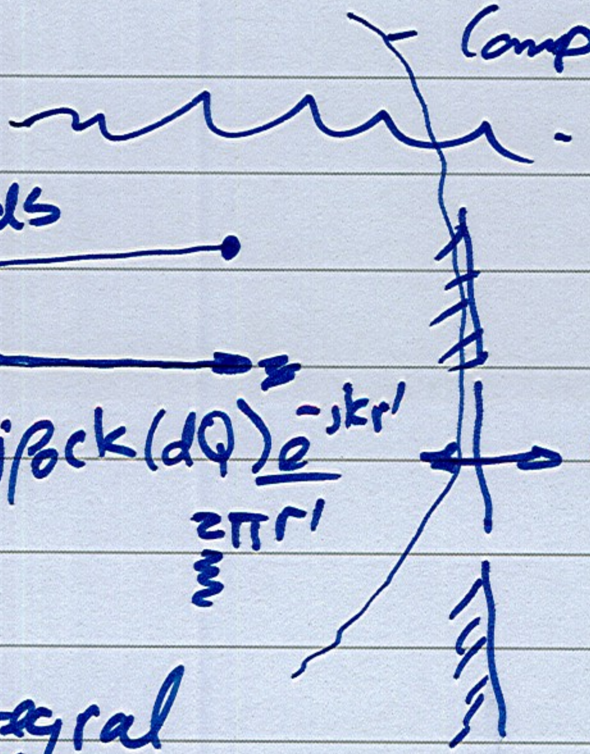
- dipole



Multipoles



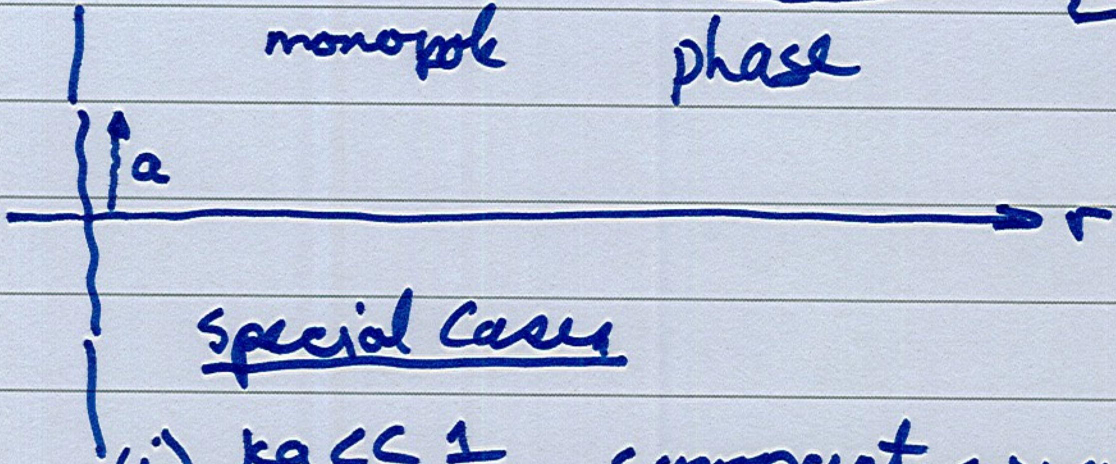
$d\tilde{p} = j\rho_o c k (dQ) \frac{e^{-jkr'}}{2\pi r'}$



Rayleigh Integral

On axis sound pressure

$$\tilde{p}(r,0) = \underbrace{2j\rho_0 c k \frac{U_0}{r} e^{-jkr}}_{\text{monopole}} \; \underbrace{e^{-j\frac{kr}{2}\left[\sqrt{1+\frac{a^2}{r^2}}-1\right]}}_{\text{phase}} \; \underbrace{\sin\left[\frac{kr}{2}\left(\sqrt{1+\frac{a^2}{r^2}}-1\right)\right]}_{\text{spatial variation}}$$



<u>Special cases</u>

(i) $ka \ll 1$ compact source

(ii) farfield $\frac{a}{r} \ll 1$

monopole on ~~an~~ a hard surface

farfield

$$\sqrt{1+\frac{a^2}{r^2}} \qquad \frac{a^2}{r^2} \ll 1 \qquad (1+x)^{1/2}$$

$$= 1 + \frac{x}{2} + \frac{x^3}{3} + \cdots$$

$$\simeq 1 + \frac{a^2}{2r^2}$$

$$\sin\frac{kr}{2}\left[1 + \frac{a^2}{2r^2} - 1\right]$$

$$= \sin\frac{kr}{4} \quad \sin\left(\frac{ka}{4}\left(\frac{a}{r}\right)\right) \qquad ka \ll 1$$

$$\frac{a}{r} \ll 1$$

$$\simeq \frac{ka}{4}\left(\frac{a}{r}\right)$$

sub into the complete soln

$$Q = u_0 \pi a^2$$

$$|\tilde{p}(r,0)| \approx \rho_0 c\, u_0 (ka) \frac{a}{r}$$

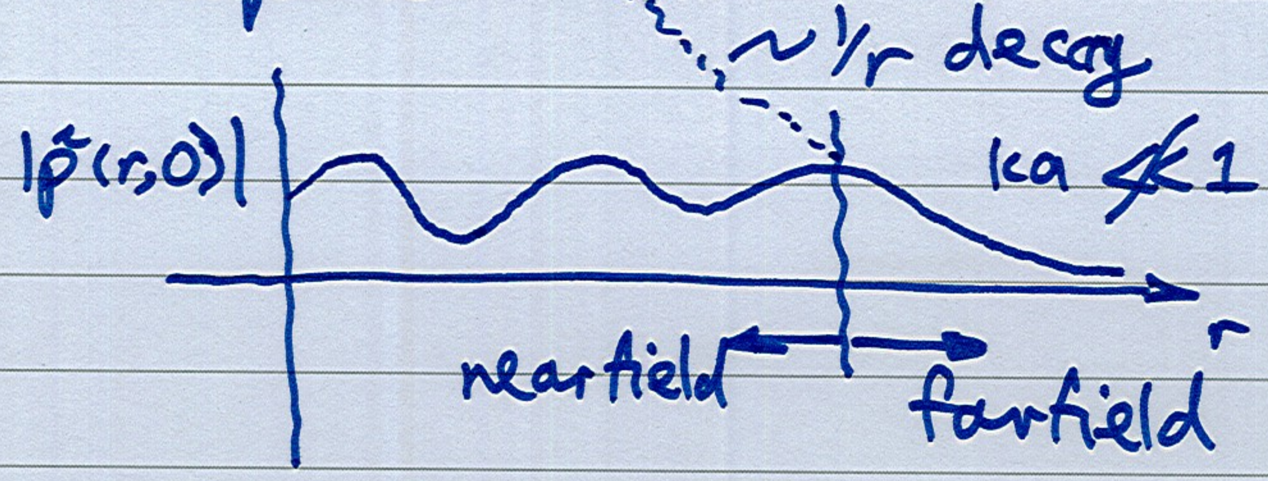
$$\approx \rho_0 c \frac{kQ}{2\pi r}$$

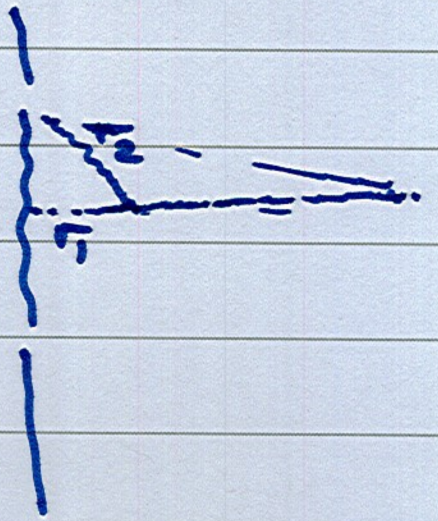
exactly the same as a monopole on a hard surface

l/s flush mounted on wall can be modeled as monopole on a hard surface

$ka \ll 1$

Complete solution

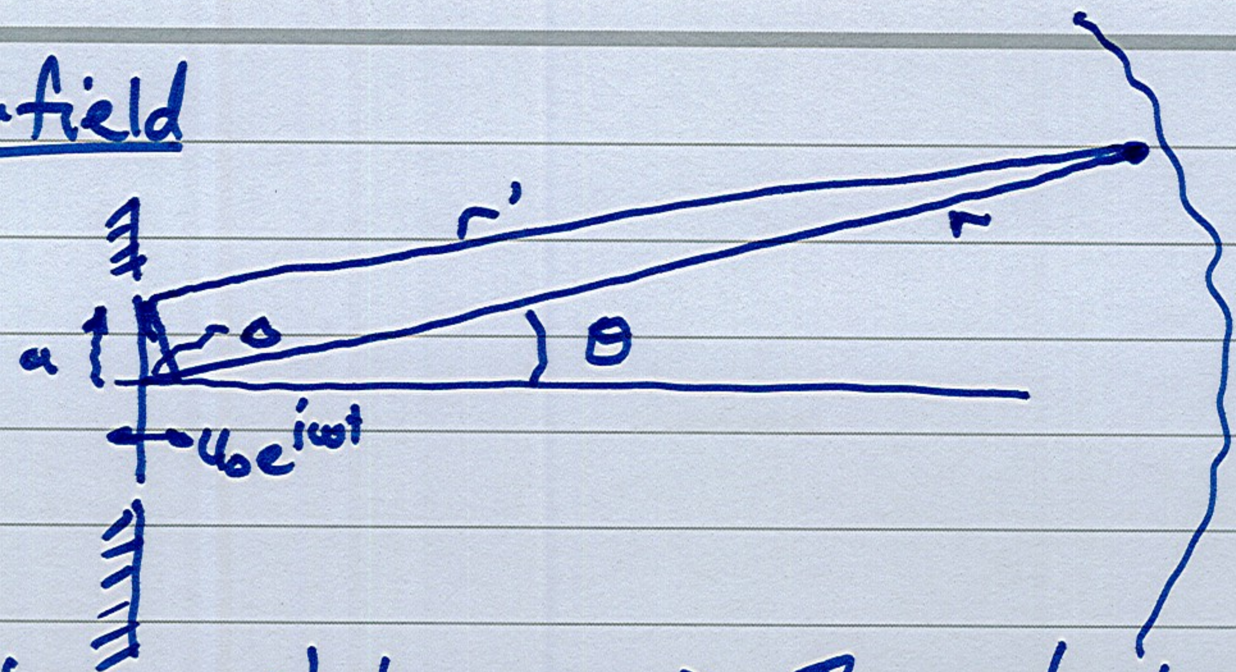




- oscillatory nearfield that can increase or decrease

- important when making measurements close to a large

farfield



$\frac{a}{r} \ll 1$
farfield

- replace r' by r in the spherical term
- replace r' by r+Δ in the phase term

$$\tilde{p}(r,\theta) = \underbrace{j\frac{\rho_0 c}{2} u_0 \left(\frac{a}{r}\right) e^{-jkr}}_{\text{monopole}} \underbrace{\left[2\,\frac{J_1(ka\sin\theta)}{ka\sin\theta}\right]}_{\text{Directivity factor}}$$

Directivity factor $O(1)$

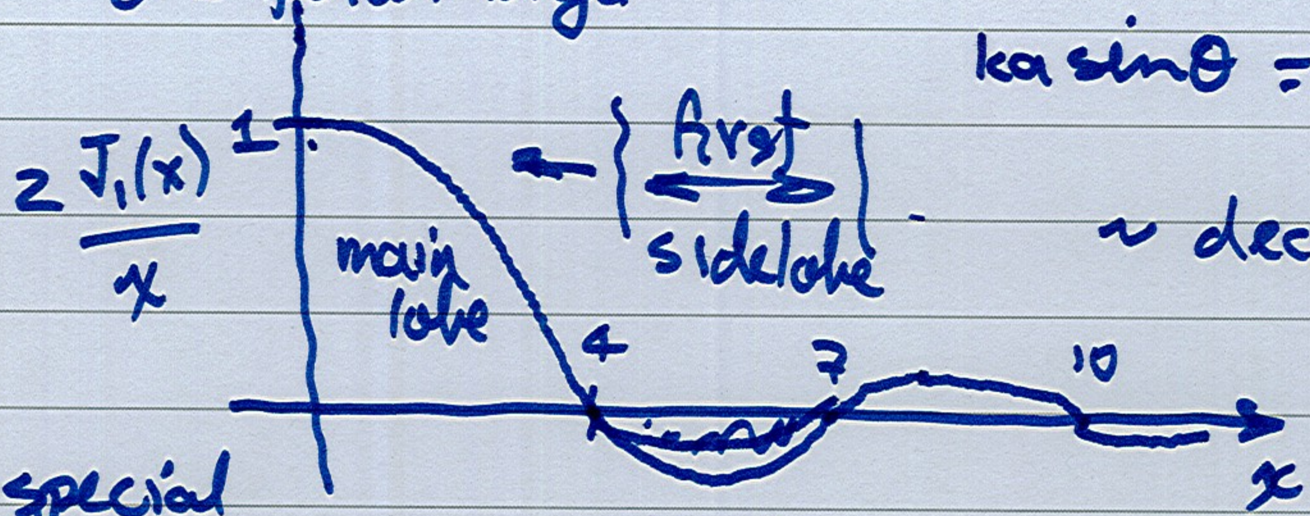
J_1 The Bessel function of the first kind a = piston radius

$k = \omega / c$

θ = polar angle

$$\left[2 \frac{J_1(ka \sin\theta)}{ka \sin\theta} \right]$$

$ka \sin\theta = x$



~ decaying cosine

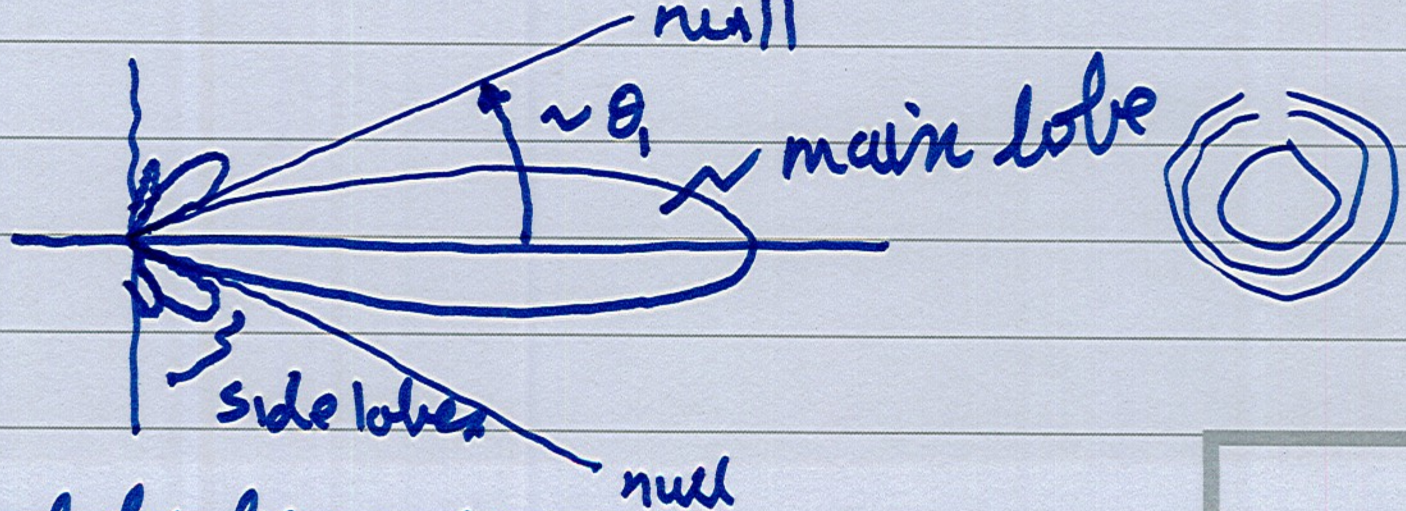
special

$ka \ll 1$ directivity $\simeq 1$ for all angles

- uniform radiation to all angles (omnidirectional)

$ka \gg 1$ high frequency

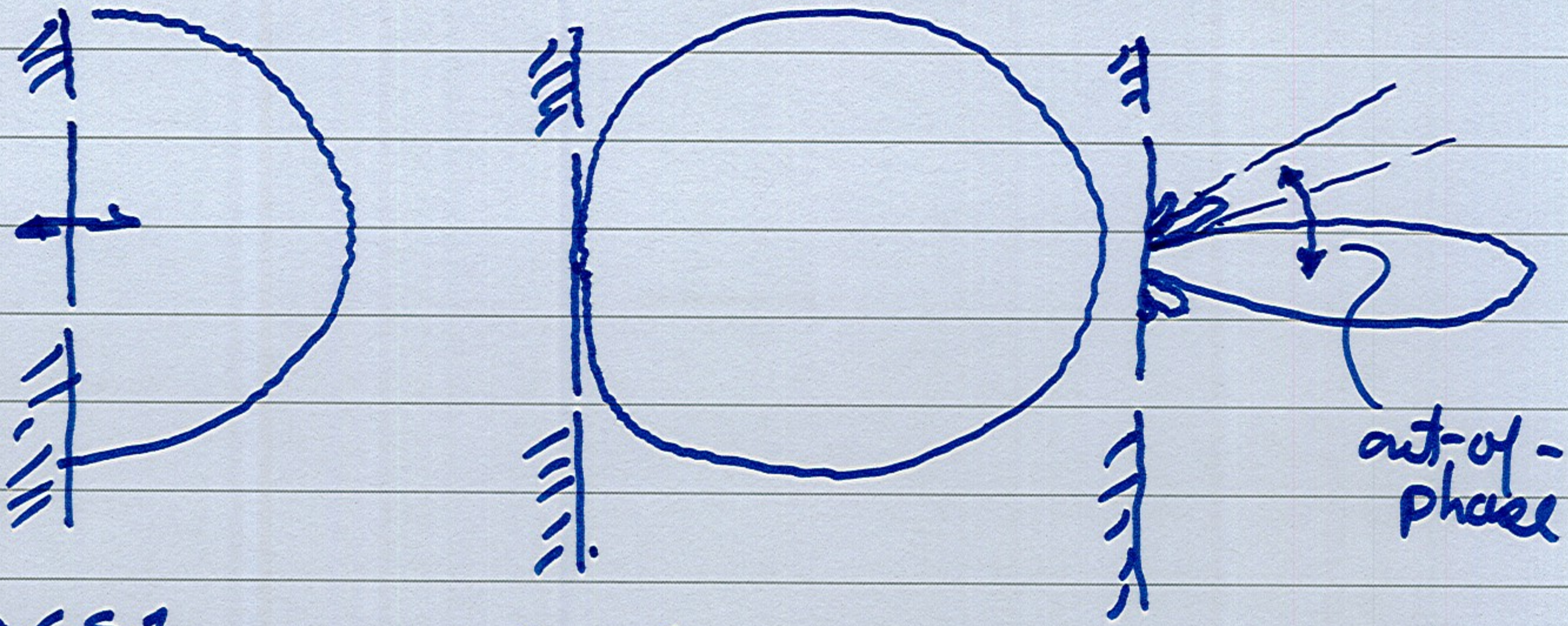
we have the possibility of sidelobes and nulls



main lobe becomes narrower as the frequency increases

$ka \sin\theta_1 = 4$] solve for θ_1

$ka \gg 1$ highly directional radiation



$ka << 1$ | $ka = 4$ | $ka >> 1$

omnidirectional radiation
- monopole-like

First null happens at θ_1

$ka \sin\theta_1 = j_{11}$
solve for θ

Angular values for nulls
- zeros of the Bessel function
see Appendix A5