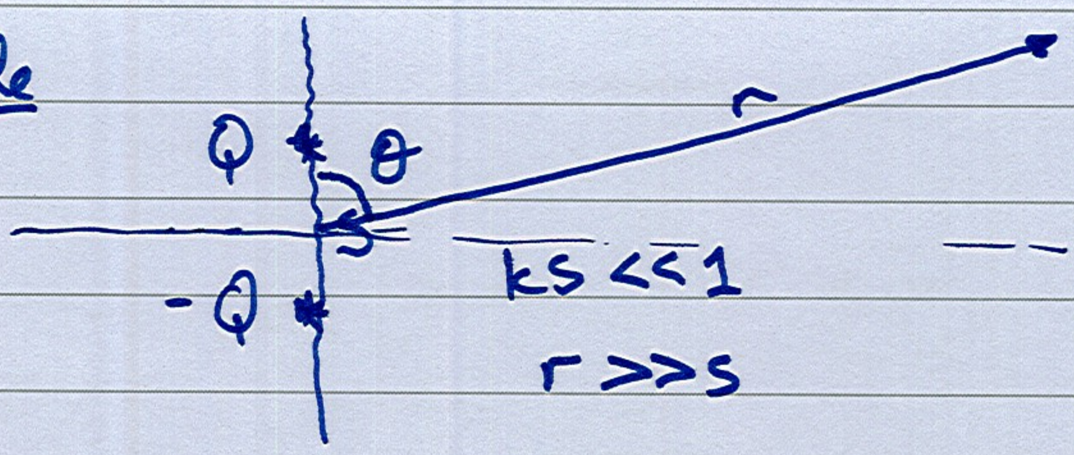
MR 513 Session 37 11/15/19

1

(L) 5cm Additional problem 1.

Dipole



$ks \ll 1$

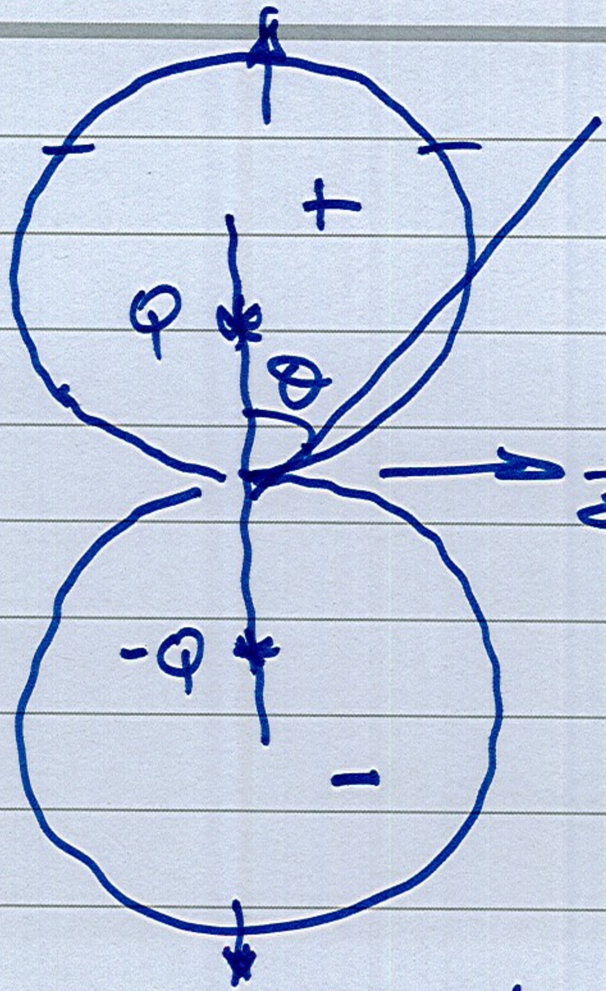
$r \gg s$

$$\tilde{p}(r) = -\rho_0 c k^2 (Qs) \frac{e^{-jkr}}{4\pi r} \cos\theta$$

same radial dependence as monopole

$$\frac{|\tilde{p}(r)|_{dipole}}{|\tilde{p}(r)|_{monopole}} \ll 1$$

Converting monopole → dipole



zero radiation

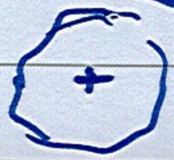
max radiation

pressure is a max
when $\theta = 0$ or π

pressure is a minimum
when $\theta = \pi/2$

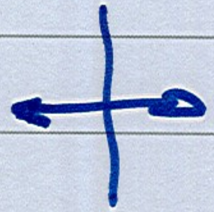
$\hat{p}(\theta=0) = -\hat{p}(\theta=\pi)$

sound ~~radiation~~ radiation
is axisymmetric



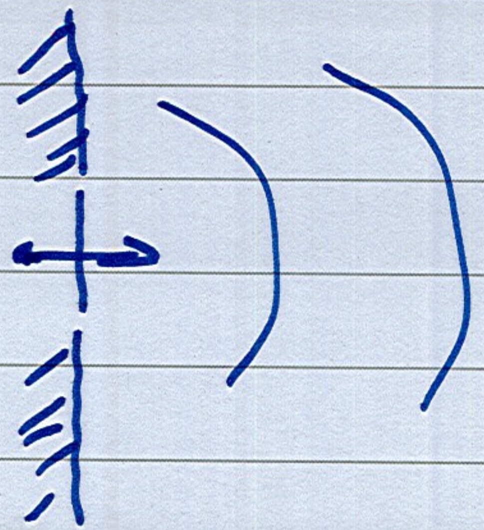
Dipole - no net volume change

unbaffled l/s



Axial Fan

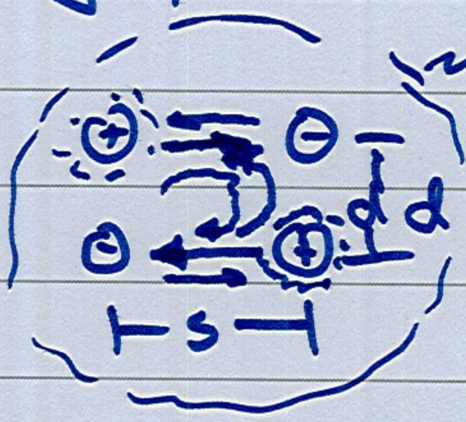
increase the radiation efficiency



monopole radiation

5.3.6 Quadrupoles, etc.

- Array of two dipoles



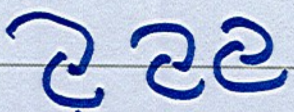
~ no net force application

~~far~~ Compact

$ks \ll 1$

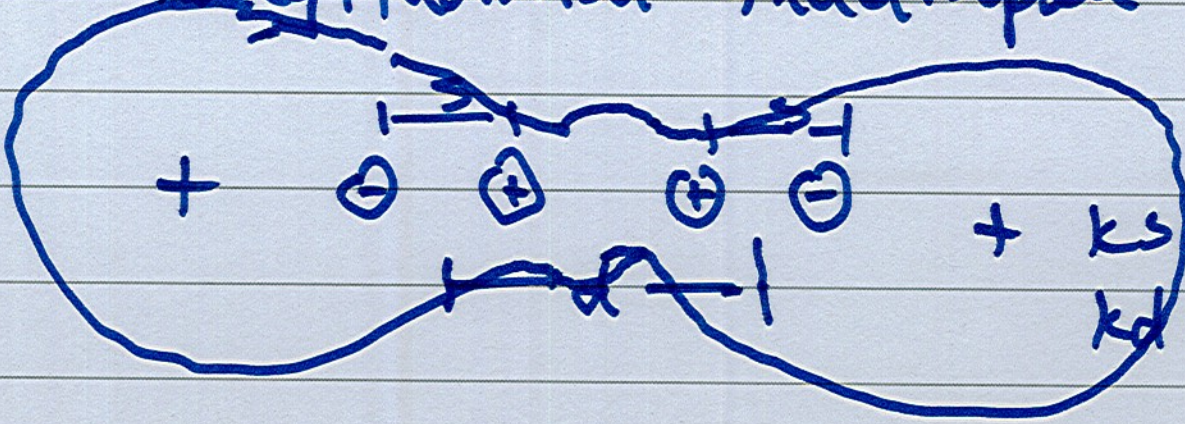
$kd \ll 1$

lateral quadrupoles are used to represent compact sources that apply an oscillatory moment to the fluid

M.J. Lighthill 

use quadrupoles to represent sound radiation from homogeneous turbulence.

Longitudinal Quadrupole



$ks \ll 1$

$kd \ll 1$

Compact Sources – small compared to a wavelength

Simple Compact Sources

– monopole – volume

radiation efficiency – dipole – force

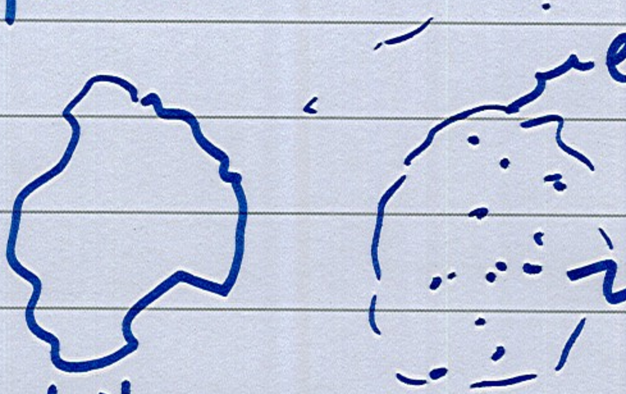
drops – ~~lateral~~ quadrupole – moment

per unit source strength

$|P_{monopole}| \gg |P_{dipole}| \gg |P_{quadrupole}|$

Multipole Decomposition

Equivalent Source Methods



equivalent source array

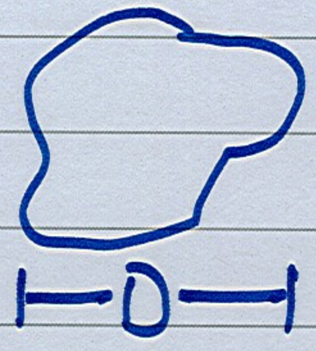
Arbitrary

adjust the strengths of the simple sources to achieve same boundary motion as the real source

$$\tilde{p}(r) = \sum \text{monopoles} + \sum \text{dipoles} + \sum \text{quadrupoles} + \sum \text{octapoles} + \ldots$$

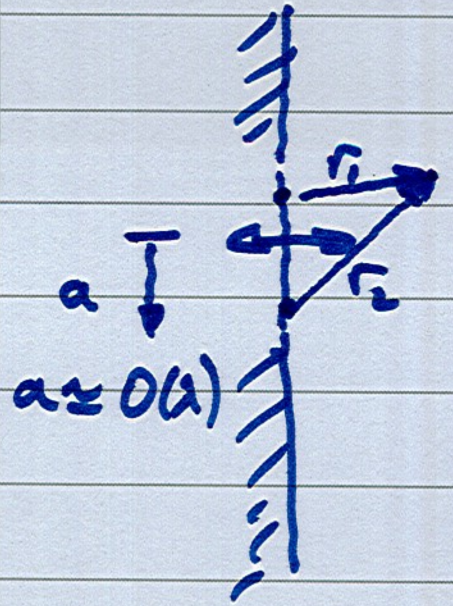
5.4 Sound radiation from extended sources

- finite-sized sources



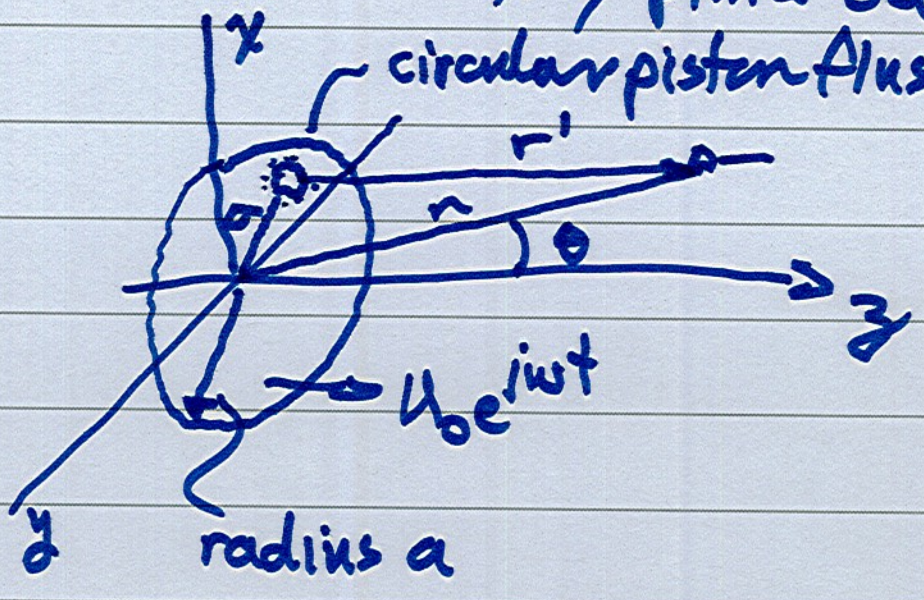
$D \simeq O(\lambda)$

5.4.1 Piston in a baffle



$(r_2 - r_1) \simeq O(\lambda)$

reinforcement & cancellation are possible because of relatively large pld's x-y plane rigid baffle



circular rigid piston

axisymmetric around the z-axis

Incremental Source strength

$dQ = U_0 dS$ ~ incremental source region

$$d\hat{p} = j \rho_0 c k (dQ) \frac{e^{-jkr'}}{2\pi r'}$$

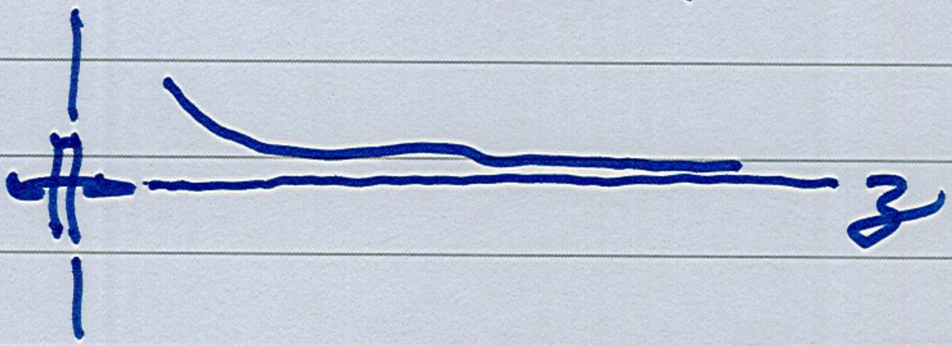
2 monopole on a hard surface

Integrate over the surface of the piston

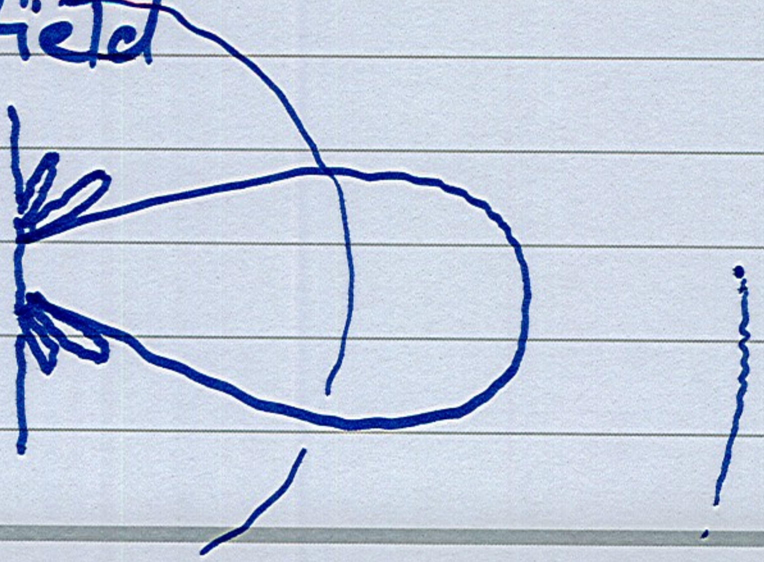
$$\tilde{p}(r,\theta) = \frac{i\rho_0 c U_0 k}{2\pi} \int_S \frac{e^{-ikr'}}{r'} \, dS$$

S ~ total piston area

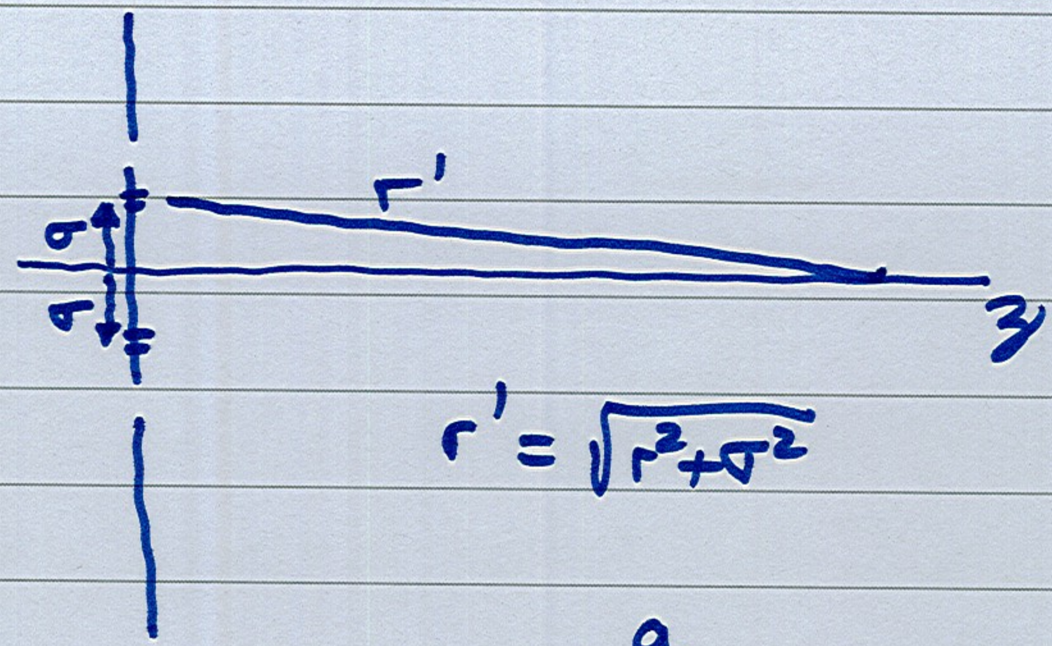
(i) On-axis

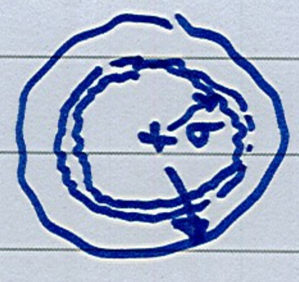


(ii) Farfield



(i) On-axis pressure





$$r' = \sqrt{r^2 + \sigma^2}$$

$$\hat{p}(r,0) = \frac{j\rho_0 c k U_0}{2\pi} \int_0^a \frac{e^{-jk(r^2+\sigma^2)^{1/2}}}{(r^2+\sigma^2)^{1/2}} \, \cancel{2\pi r\,dr} \; 2\pi\sigma\, d\sigma$$

$$\tilde{p}(r,0) = 2j\rho_0 c U_0 e^{-jkr/2} \underbrace{\left(e^{-j\frac{kr}{2}\left[\sqrt{1+\frac{a^2}{r^2}}-1\right]}\right)}_{\substack{\text{magnitude} = 1 \\ \text{phase}}} \underbrace{\sin\left[\frac{kr}{2}\left[\sqrt{1+\frac{a^2}{r^2}}-1\right]\right]}_{\substack{\text{pressure} \\ \text{mag as} \\ r \text{ increases}}}$$