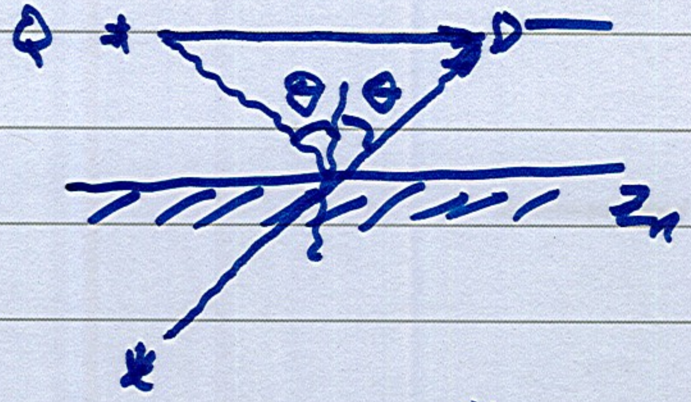
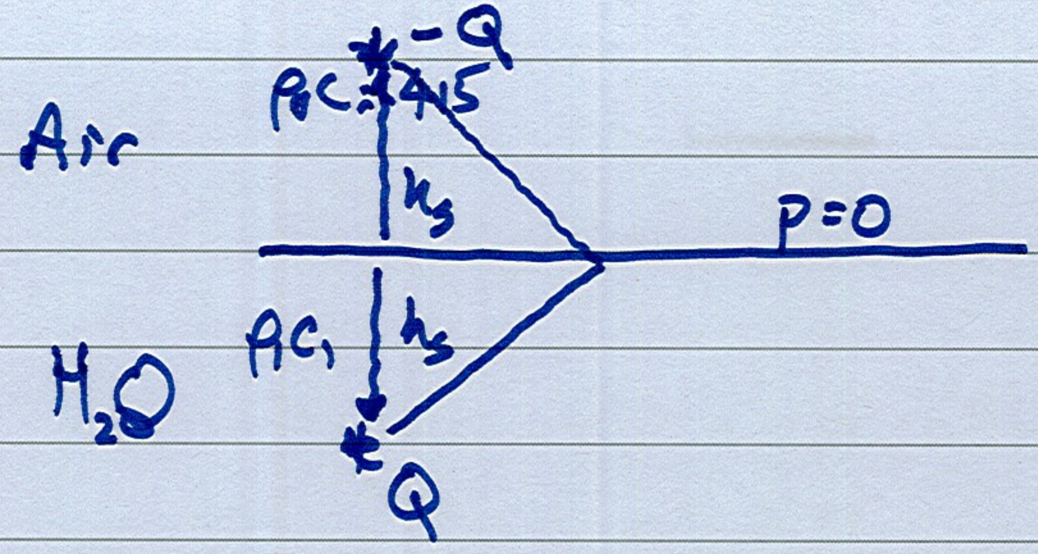
ME 513 Session 36 11/13/14



$$\hat{p}(r) \simeq A\left\{\frac{e^{-jkr_1}}{r_1} + R(\theta)\frac{e^{-jkr_2}}{r_2}\right\}$$

$$\theta < 75^\circ$$

5.3.4.2 Pressure Release Surface

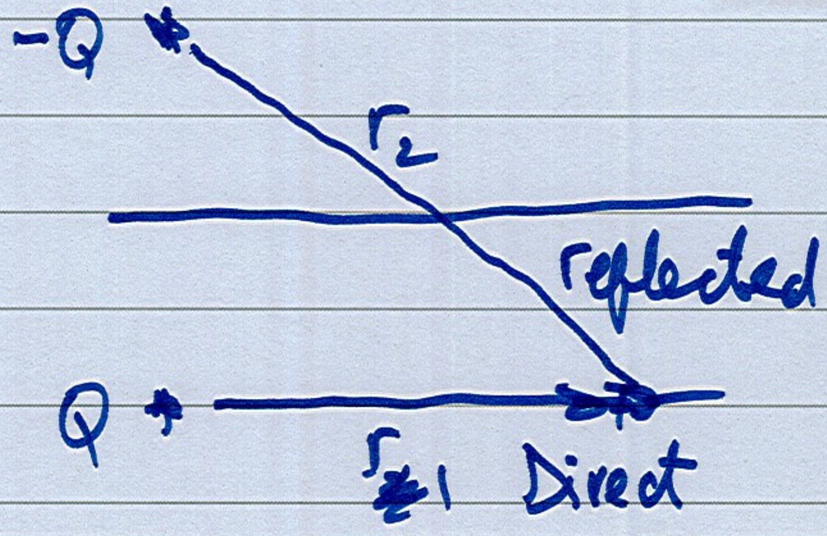


$\rho_1 c_1 : 1.5 \times 10^6$

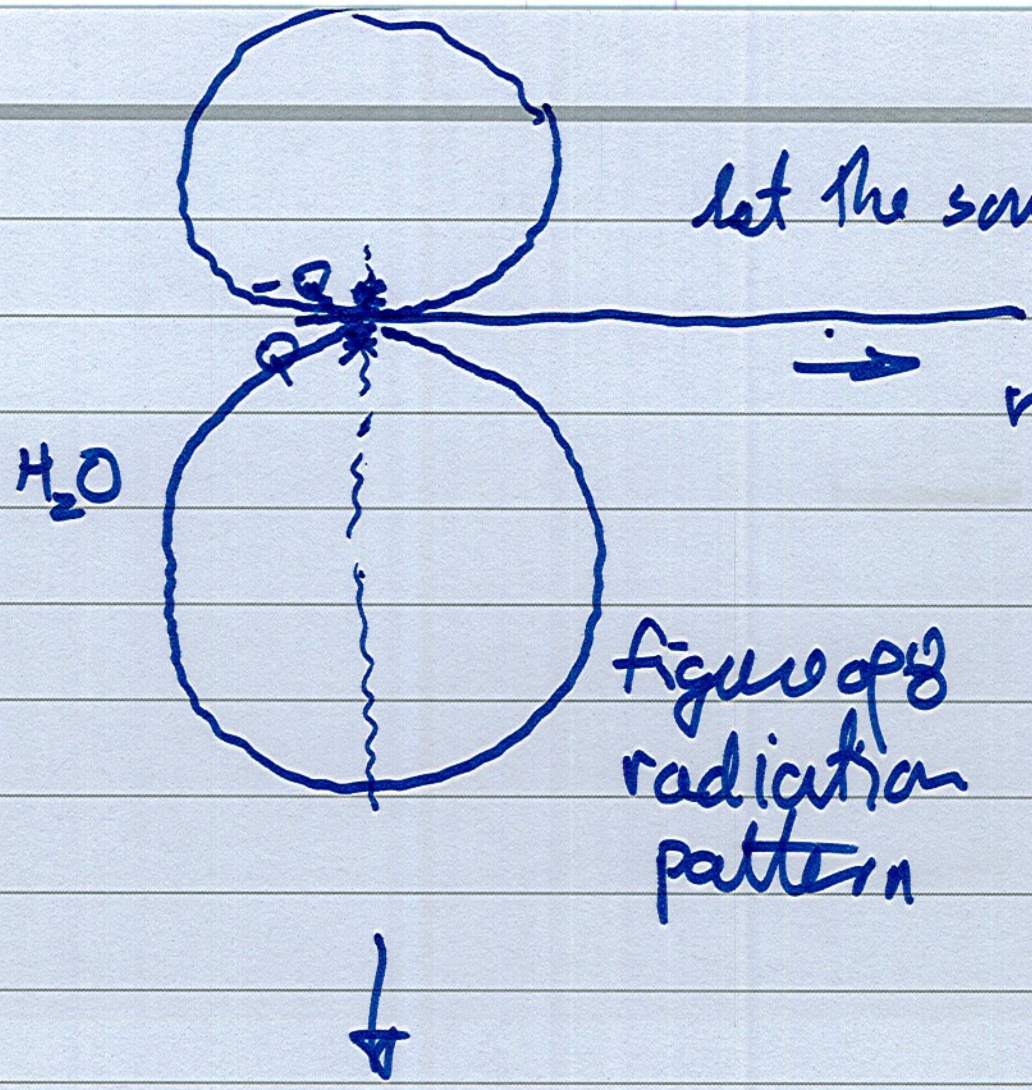
$\rho_1 c_1 \gg \rho_0 c_0$

negative image required to satisfy $p=0$ B.C.

$$\tilde{p}(r) = A\left\{\frac{e^{-jk_1 r_1}}{r_1} - \frac{e^{-jk_1 r_2}}{r_2}\right\}$$



$$k_1 = \frac{\omega}{c_1}$$



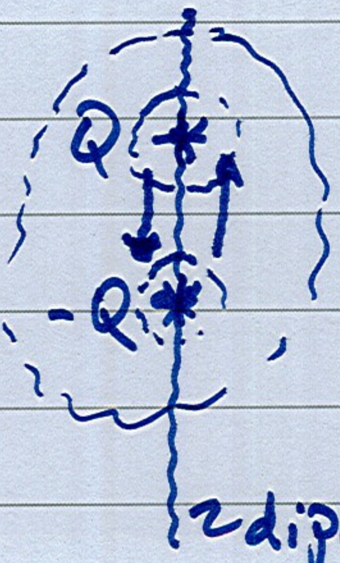
let the source approach
the surface
minimum radiation
due to nearly
perfect cancellation

figure of 8
radiation
pattern

maximum radiation
$\perp$ to the surface

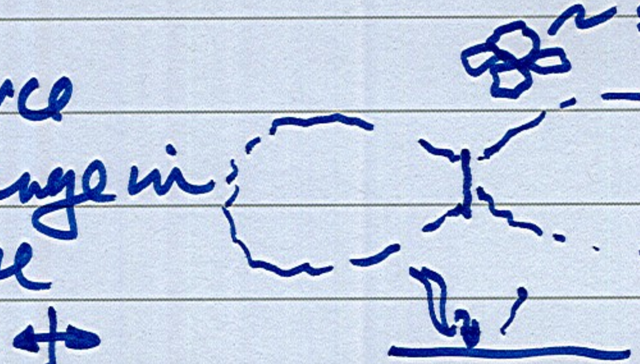
Dipole - two closely-spaced monopoles of equal strength running out of phase

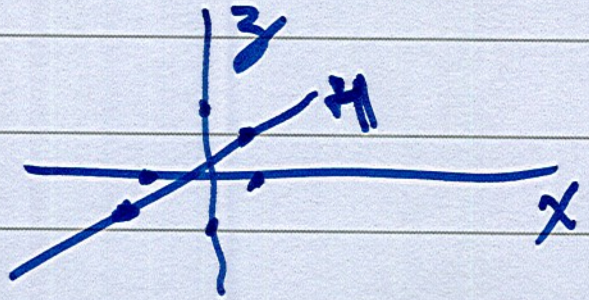
monopole
- mass or volume source



- two monopoles creat~~ing~~e an oscillatory flow
- pt force acting on the fluid to accelerate it back & forth

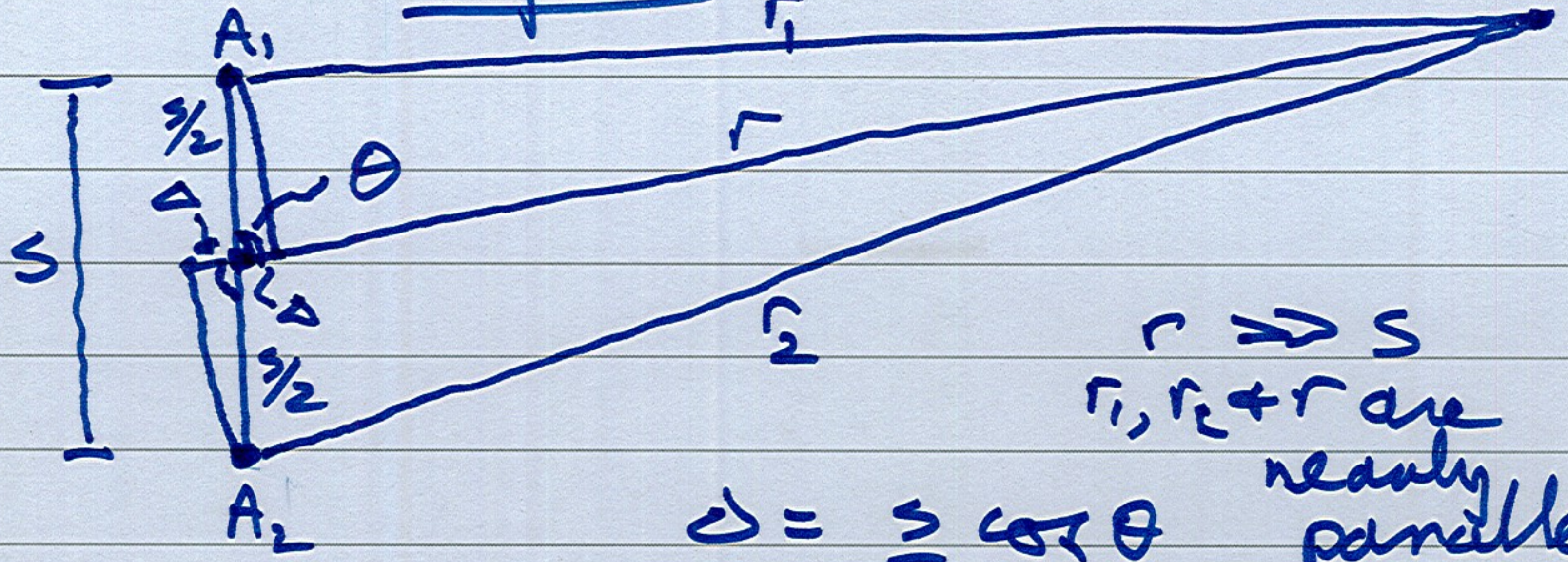
- dipole
  - pt force
  - no change in volume





3 orthogonal dipoles can be used to represent a point force in an arbitrary direction

5.3.5 **Dipole**



$r \gg s$

$r_1, r_2 + r$ are nearly parallel

$$\Delta = \frac{s}{2}\cos\theta$$

θ is the angle from the dipole axis directed towards the positive monopole

$$r_1 \simeq r - \Delta = r - \frac{s}{2}\cos\theta$$

$$r_2 \simeq r + \Delta = r + \frac{s}{2}\cos\theta$$

$$\tilde{p}(r) = A_1 \frac{e^{-jkr_1}}{r_1} + \frac{A_2}{r_2} e^{-jkr_2}$$

asumed $r >> s$

Nxt $A_2 = -A_1$

$A_1 = A$

$$\tilde{p}(r) = A\left\{ \frac{e^{-jk(r-\Delta)}}{r-\Delta} - \frac{e^{-jk(r+\Delta)}}{r+\Delta} \right\}$$

$$= Ae^{-jkr}\left\{ \frac{r\left(e^{+jk\Delta} - e^{-jk\Delta}\right) + \xrightarrow{\ \to 0\ }\cancel{\Delta\left(e^{+jk\Delta} + e^{-jk\Delta}\right)}}{r^2 - \cancel{\Delta^2} \to 0} \right\}$$

receiver is in the far field

$r >> s$

$\Delta << r$

$$\tilde{p}(r) = \frac{A}{r} e^{-jkr}\left(e^{+jk\Delta} - e^{-jk\Delta}\right)$$

$$= 2jA\frac{e^{-jkr}}{r} \sin k\Delta$$

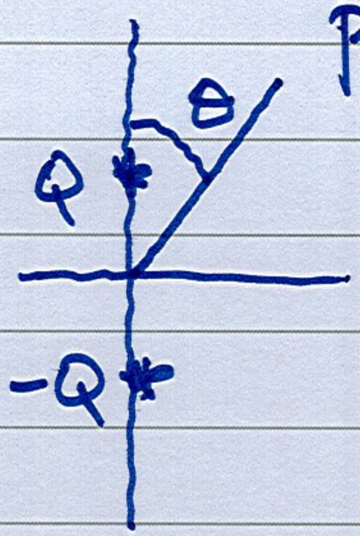
$k\Delta = \frac{ks}{2}\cos\theta$

compact source $ks << 1$

$\frac{2\pi}{\lambda} s \lessapprox 1$

$$\sin k\delta = \sin\Big(\underbrace{\frac{ks}{2}\cos\theta}_{\text{very small}}\Big) \approx \frac{ks}{2}\cos\theta$$

$\sin\theta \downarrow \theta$ when θ is small



$$\tilde{p}(r) = 2jA\frac{e^{-jkr}}{r}\frac{ks}{2}\cos\theta$$

$$A = j\rho_0 c\frac{kQ}{4\pi}$$ appropriate for the monopole

Q is the volume source strength
m^3/s

$$\boxed{\tilde{p}(r) = -\rho_0 c k^2 (Qs)\frac{e^{-jkr}}{4\pi r}\cos\theta}$$

sound field radiated by a point dipole

Qs = dipole source strength
= D dipole moment

Assumptions

- compact source $\quad ks \ll 1$
- receiver is in the farfield

$$s \ll r \qquad \lambda \ll r$$

For a monopole

$$\tilde{p}(r) = i\rho_0 c \frac{kQ}{4\pi r} e^{-ikr}$$

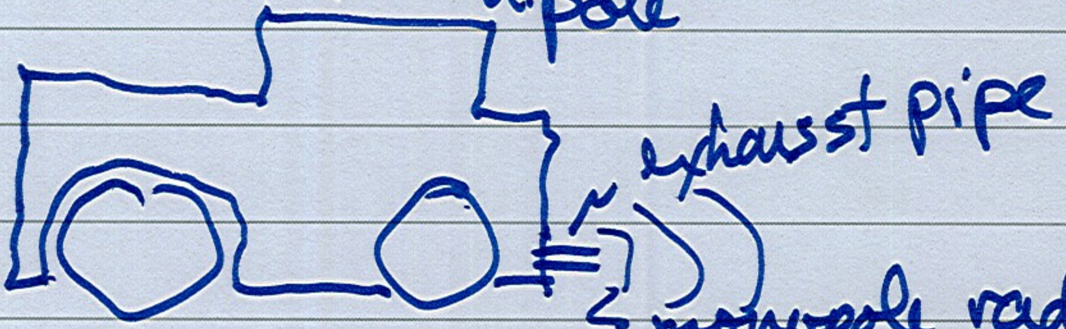
For a dipole

$$\tilde{p}(r) = -\rho_0 c k^2 (Qs) \frac{e^{-ikr}}{4\pi r} \cos\theta$$

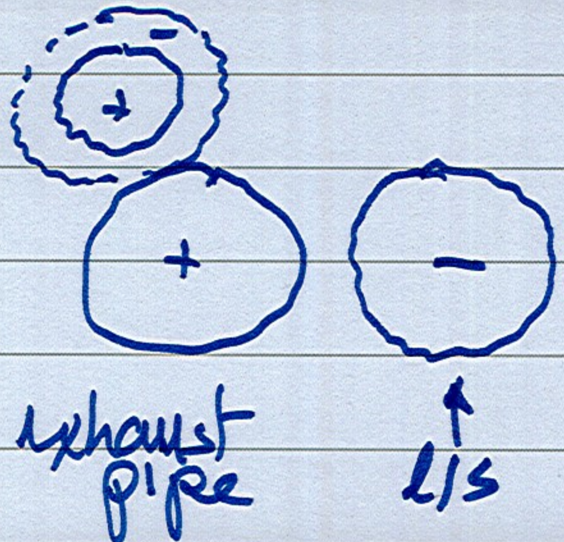
$$\frac{|\tilde{p}(r)|_{dipole}}{|\tilde{p}(r)|_{monopole}} = ks\cos\theta \qquad ks \ll 1$$

for a given monopole source strength Q,
dipole is a much weaker source

$$|\hat{p}(r)|_{dipole} \ll |\hat{p}(r)|_{monopole}.$$



monopole radiation from
The opening of the
pipe



turn monopole
into a dipole

- greatly reduce The sound radiation by reducing radiation efficiency