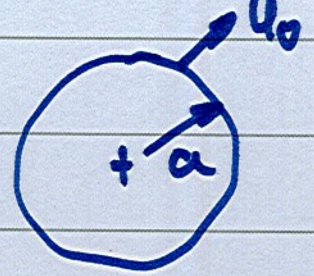
ME 513 Session 33 11/6/19

<u>Sources</u>

* point sources



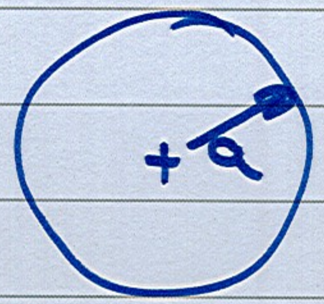
$ka \ll 1$

$$\hat{p} = j\rho_0 c \frac{kQ}{4\pi r} e^{-jkr} \checkmark$$

Q = volume velocity

point monopole

$$\lim_{ka \to 0} I_r = \frac{\rho c}{2} \frac{k^2 Q^2}{(4\pi r)^2}$$



$$Q = 4\pi a^2 U_0$$

$$I_r \propto a^4$$

rapid increase in intensity as source size increases

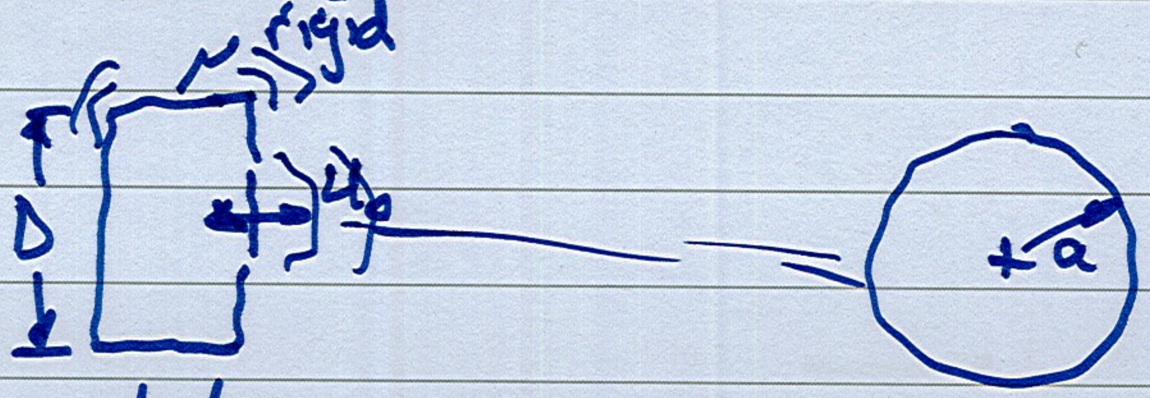
- small sources very poor radiators

- Use a point monopole to represent any compact source that periodically changes volume and displaces Q volume/s

- for a compact source ($D \ll \lambda$) the sound radiation does not depend on the spatial distribution of the source velocity

$D \ll \lambda$ ✓

$$Q = \int_S u_n \, dS \quad [m^3/s]$$



sealed enclosure

circular piston of radius a
- diaphragm

$$Q = \pi a^2 U_0$$

so long as $D \ll \lambda$

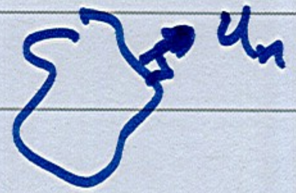
replace the l/s by a point monopole.

$$\tilde{p}(r) = j\rho_0 c \frac{kQ}{4\pi r} e^{-jkr}$$ free field

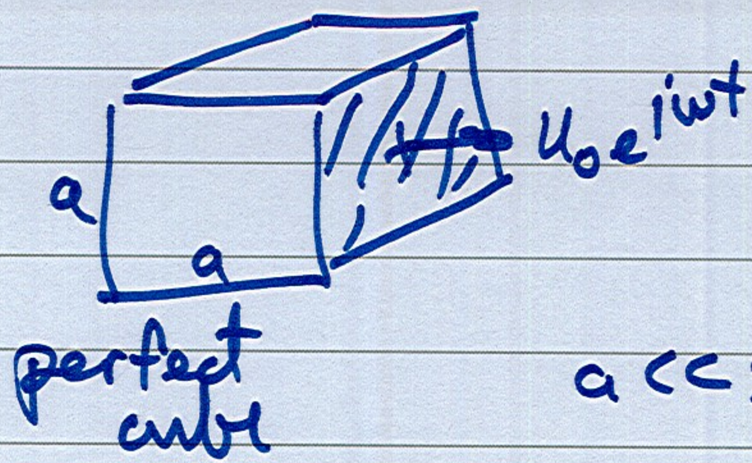
$$W = \rho_0 c \frac{k^2 Q^2}{8\pi}$$

5.3.2 Simple Volume Source

Any source that displaces Q m^3/s and is small compared to a wavelength radiates like a point monopole



$$Q = \int_S u_n \, dS$$

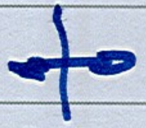


$$Q = u_0 a^2$$

$a << \lambda$

- some sources do not exhibit a volume change

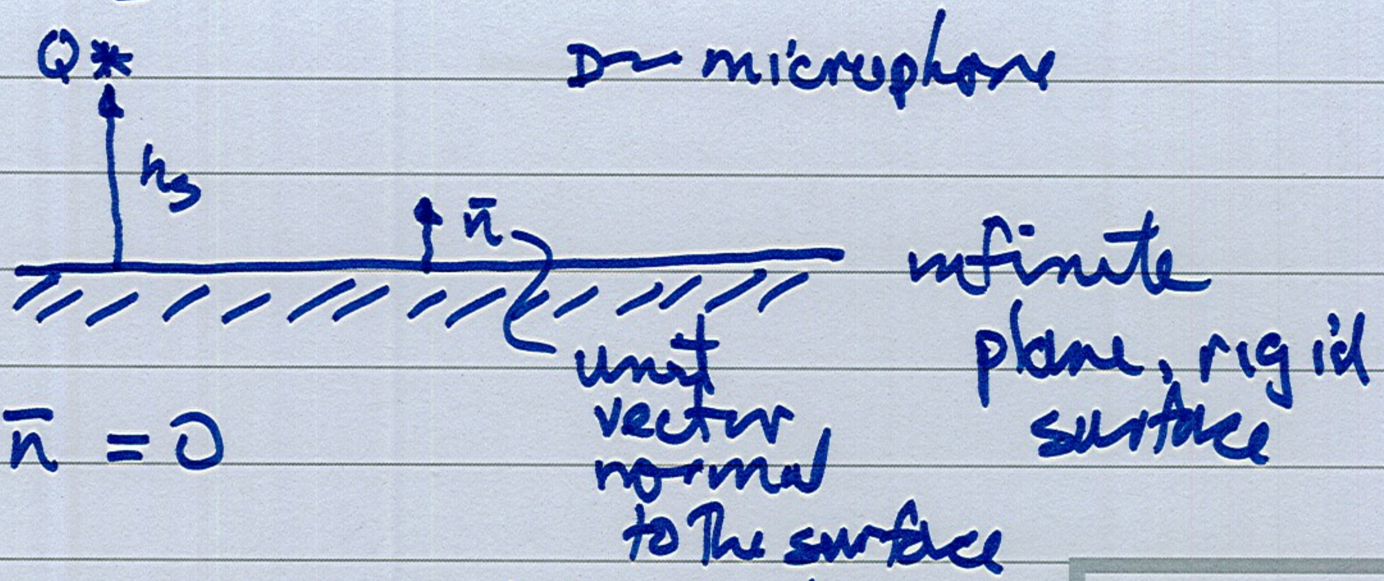
 fan

 unbaffled l/s

not volume sources

5.3.3 Reflection at a hard surface

5.3.3.1 Single Reflection

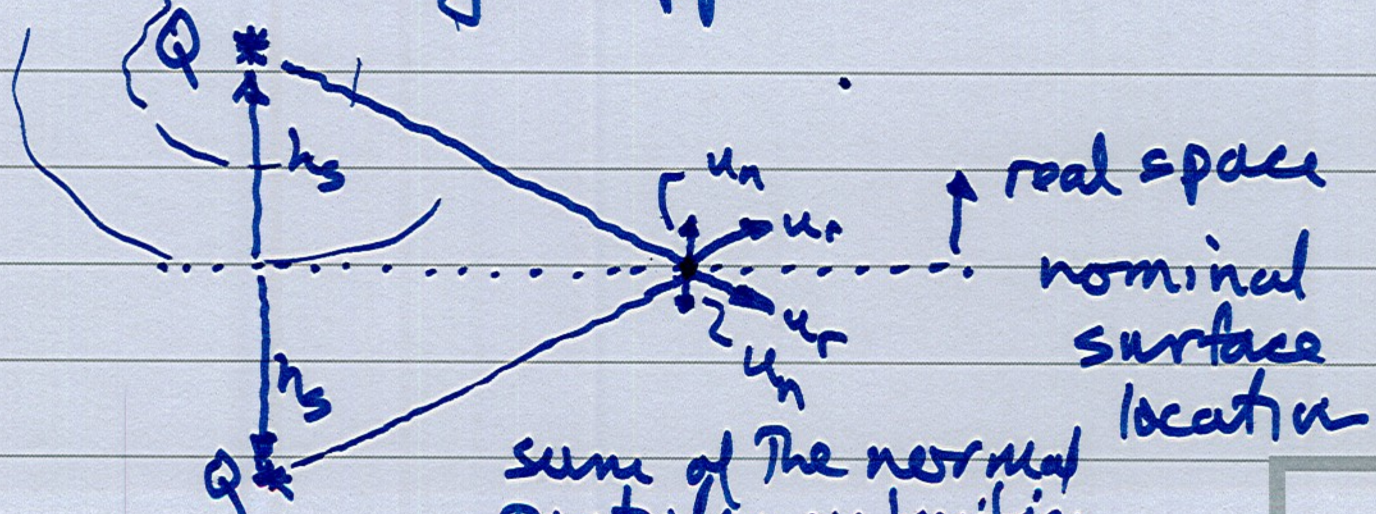


$\bar{u} \cdot \bar{n} = 0$

zero normal particle velocity

Equivalent system in free space
which satisfies the b.c.

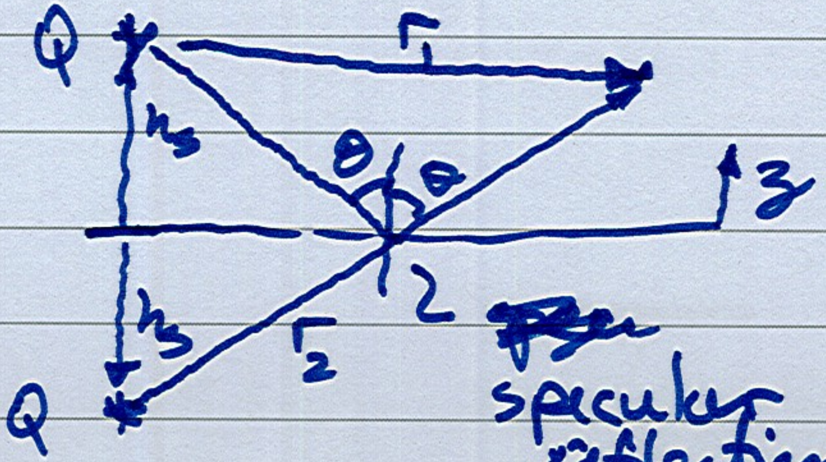
– image approach



sum of the normal
particle velocities
cancel at the
surface location

$\bar{u} \cdot \bar{n} = 0$
is satisfied

This image source arrangement
is mathematically equivalent to the
real case in region above the surface.



~~spec~~ specular reflection

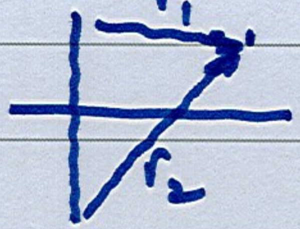
Solution applies in the region $z \geq 0$

$$\tilde{p}_t = \tilde{p}_1 + \tilde{p}_2$$

$$\tilde{p}_t = j\rho_0 c \frac{kQ}{4\pi r_1} e^{-jkr_1} + j\rho_0 c \frac{kQ}{4\pi r_2} e^{-jkr_2}$$

direct $\uparrow$ (first term) — reflected $\uparrow$ (second term)

$$\tilde{p}_t = i\rho_0 c \frac{kQ}{4\pi r_1} e^{-ikr_1}\left[1 + \underbrace{\left(\frac{r_1}{r_2}\right)e^{-ik(r_2-r_1)}}_{\text{reflected component}}\right]$$



$\left(\frac{r_1}{r_2}\right)$ = relative spherical spreading attenuation

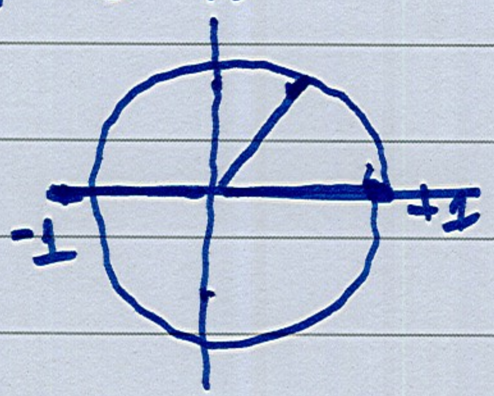
$(r_2 - r_1)$ path length difference

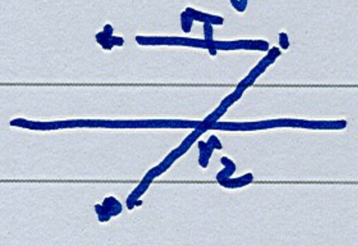
$k(r_2-r_1)$ = phase difference between direct + reflected signals

$$0 \leq \left(\frac{r_1}{r_2}\right) \leq 1$$

reflected sound is attenuated wrt the direct sound due to spherical spreading

$e^{-jk(r_2 - r_1)}$





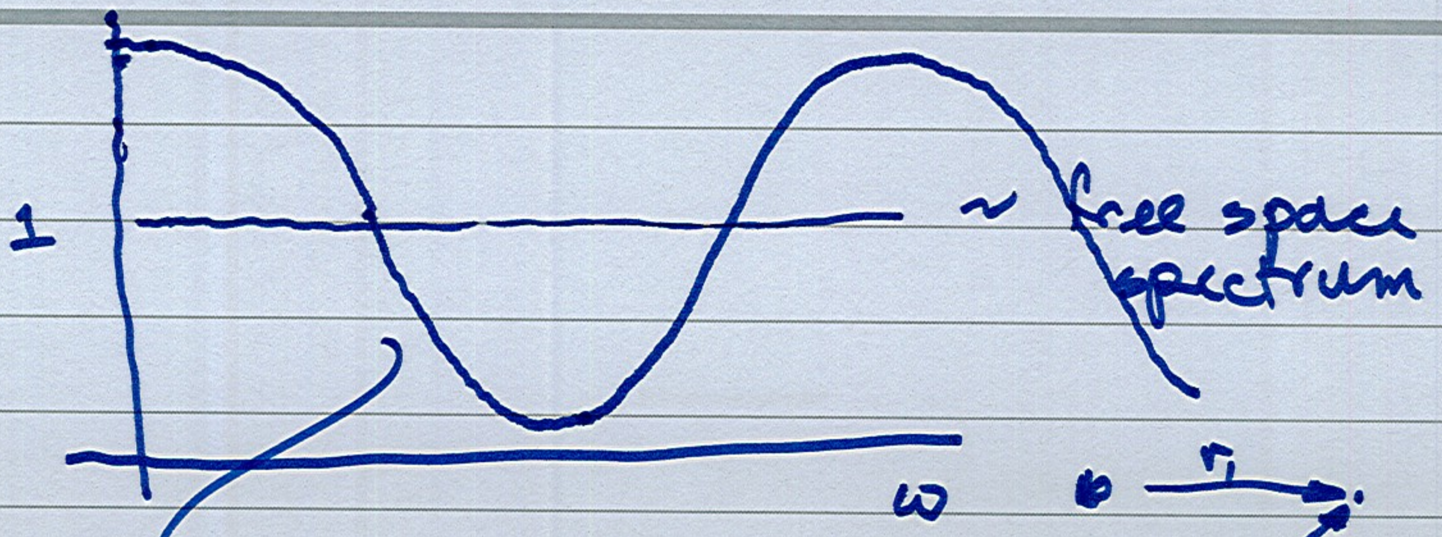
$$k(r_2 - r_1) = 0,\ 2\pi,\ 4\pi \qquad e^{-jk(r_2 - r_1)} = 1$$

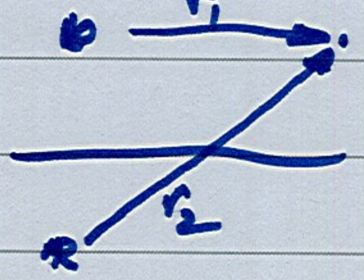
$$\frac{\cancel{2\pi}}{\lambda}(r_2 - r_1) = \cancel{2\pi}$$

$$(r_2 - r_1) = \lambda$$ approximate maximum

$$k(r_2 - r_1) = \pi,\ 3\pi,\ 5\pi \ldots$$

$$e^{-jk(r_2 - r_1)} = -1 \qquad (r_2 - r_1) = \lambda/2,\ 3\lambda/2,$$ minimum





magnitude of the ripple depends on $\left(\frac{r_1}{r_2}\right)$

Effect of a single reflector is ~~to~~ to add a ripple to the received spectrum.