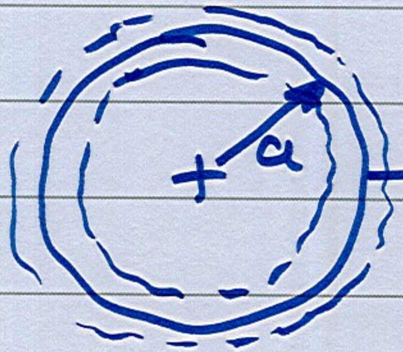
ME 513 Session 32 11/4/19

Sources - monopoles, dipole & quadrapoles

- compact - $D \ll \lambda$



$\rightarrow u_0 e^{j\omega t}$

spherically symmetric

$$\tilde{p}(r) = \frac{\acute{A}}{r} e^{-jkr} + \cancel{\frac{B}{r} e^{+jkr}}$$

outward inward

free space

Apply BC at $r = a$ $\quad \hat{u}_r(a) = U_0$

$$\tilde{u}_r(r) = -\frac{1}{j\omega\rho_0}\frac{d\tilde{p}}{dr} = \frac{A}{\rho_0 c}\frac{e^{-jkr}}{r}\left(1 - \frac{j}{kr}\right)$$

nearfield

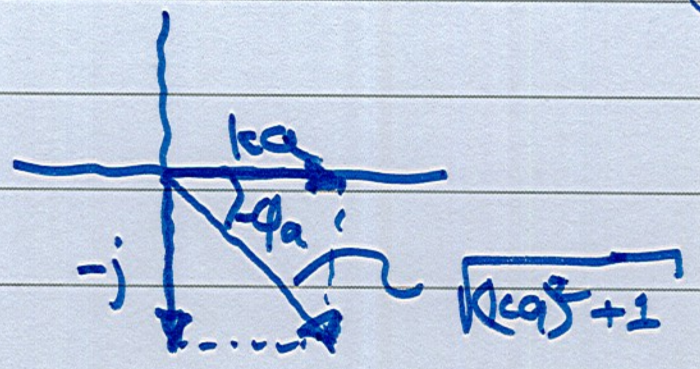
u_0 at $r = a$

$\tilde{u}_r(a) = u_0 \longrightarrow A$

$$A = \rho_0 c a u_0 e^{jka}\left(\frac{1}{1 - \frac{j}{ka}}\right)$$

$$A = \rho_0 c a U_0 e^{jka} \frac{ka}{(ka - j)}$$

$$(ka - j) = \sqrt{(ka)^2 + 1}\, e^{-j\phi_a}$$



non-dimensional radius of the source

$$ka = \frac{2\pi a}{\lambda} = 2\pi\left(\frac{a}{\lambda}\right)$$

$$A = \rho_0 c a U_0 \frac{ka}{\sqrt{(ka)^2+1}} e^{-j\phi_a}$$

$$= \rho_0 c a U_0 \cos\phi_a e^{+j\phi_a}$$

$$\tilde{p}(r) = \rho_0 c a U_0 e^{jka} e^{j\phi_a} \cos\phi_a \frac{e^{-jkr}}{r}$$

pulsating sphere of radius a

5.2.3 Impedance, Intensity & Sound Power

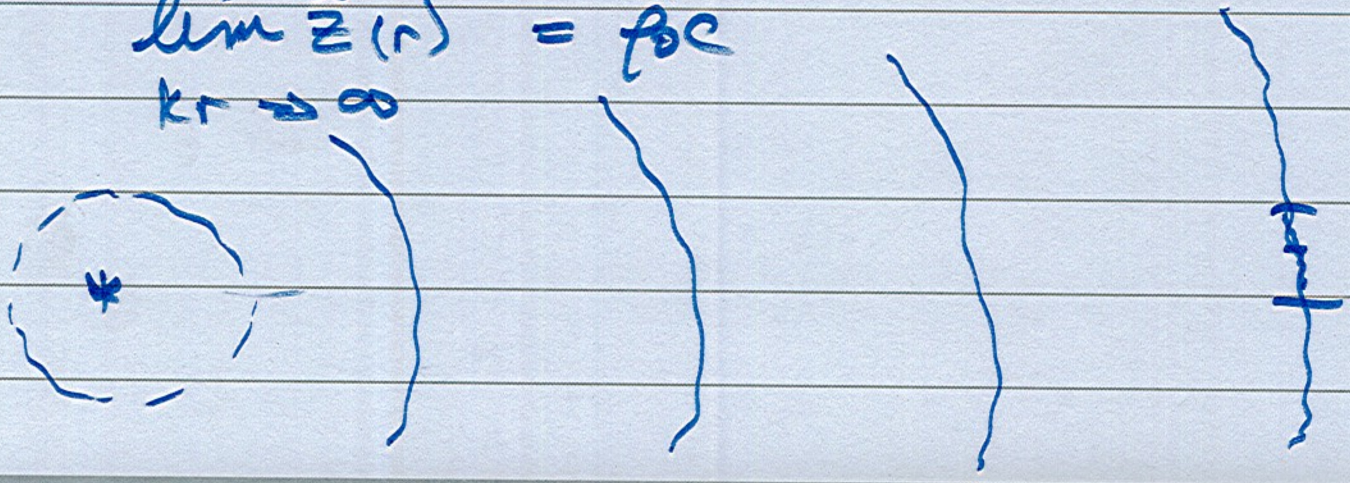
specific acoustic Impedance: $\dfrac{\tilde{p}(r)}{\tilde{u}_r(r)} = \tilde{z}(r) = \rho_0 c \dfrac{kr}{kr - j}$

$$= \rho_0 c \cos\phi_r \, e^{j\phi_r}$$

$$\cos\phi_r = \frac{kr}{\sqrt{(kr)^2 + 1}}$$

far field $kr \gg 1$

$$\lim_{kr \to \infty} \tilde{z}(r) = \rho_0 c$$



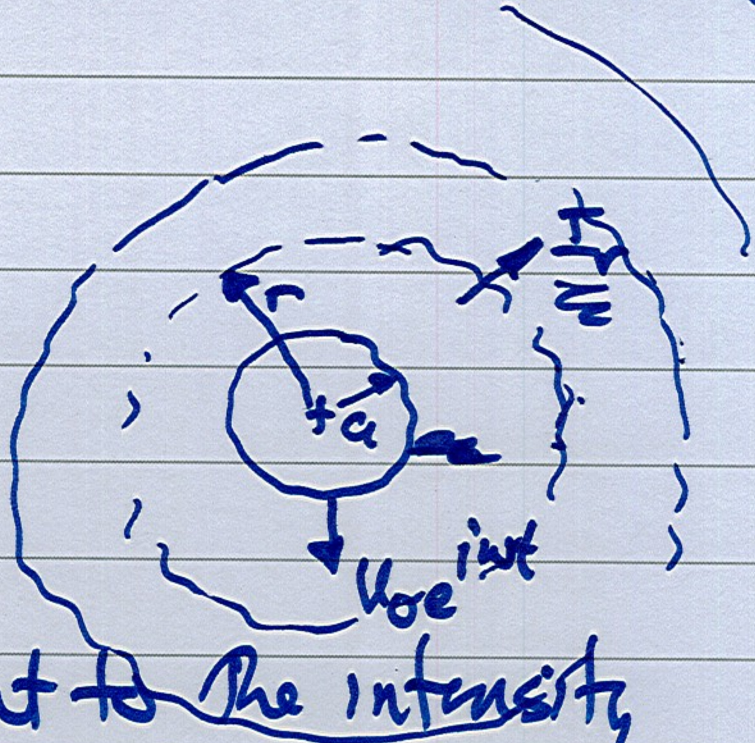
$kr \ll 1$ $\quad \vec{z}(r) \simeq + i \rho_0 c (kr)$

nearfield solution $\quad = j\omega\rho_0 r$ + small real part

mass-like impedance

- solid body motion of fluid

Intensity



$$I_r = \frac{1}{2}\mathrm{Re}\{\tilde{p}(r)\tilde{u}_r^*(r)\}$$

$$= \frac{1}{2}\left(\frac{a}{r}\right)^2 \rho_0 c\, U_0^2 \cos^2\phi_a$$

only a radial component to the intensity
- free space

$I_r \propto \frac{1}{r^2}$ Inverse square Law

Sound Power $W = \int_S I_r \, dS$

$$W = 4\pi r^2 I_r$$

independent of radius

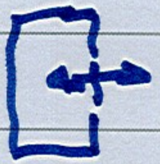
$$W = 2\pi a^2 \rho_0 c U_0^2 \cos^2 \theta_a$$

$$W \propto a^2$$

large sources
radiate more
effectively than
small sources

5.3 Simple Sources

5.3.1 Point Monopole - compact source that changes volume

 volume source

$$\tilde{p} = \frac{Ae^{-jkr}}{r}$$

$$A = j\rho_0 c a U_0 e^{jka} \frac{ka}{ka - j}$$

$\underline{4\pi a^2 U_0}$ = volume displaced by the sphere per unit time

$[m^3/s]$ Volume Velocity

= Q monopole source strength

$$A = \omega \rho_0 \frac{Q}{4\pi ka} e^{jka} \frac{ka}{ka - j}$$ ~ sphere of arbitrary size

$$\lim_{(ka) \to 0} A \rightarrow j\omega \rho_0 \frac{Q}{4\pi}$$

"point source"

$$\tilde{p}(r) = j\omega\rho_0 \frac{Q}{4\pi r} e^{-jkr}$$

$$\boxed{\tilde{p}(r) = j\rho_0 c \frac{kQ}{4\pi r} e^{-jkr}}$$

point monopole.