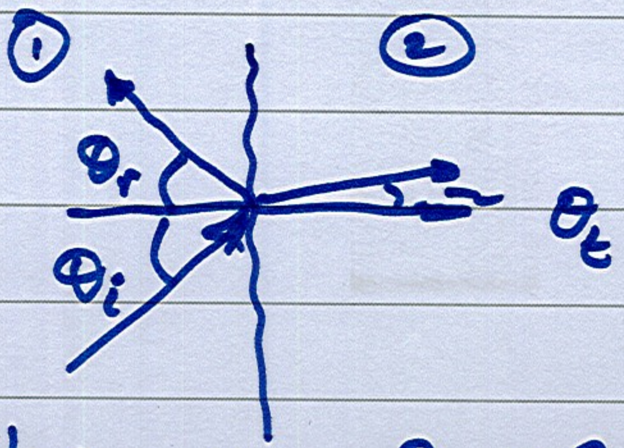
ME 513 Session 27 10/23/19



Snell's Law $\theta_r = \theta_i$

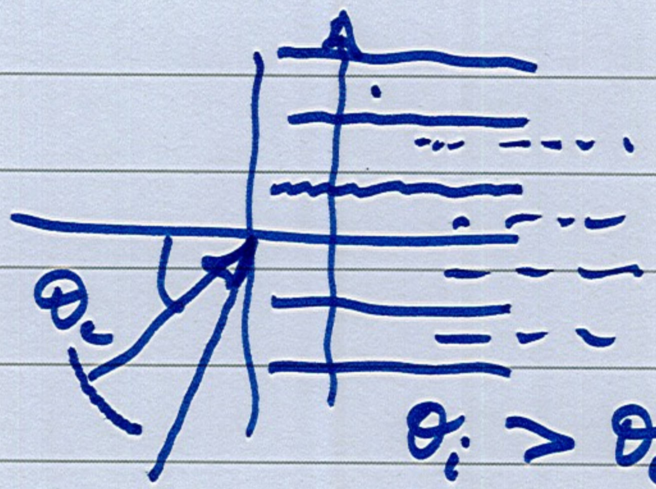
$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{c_2}{c_1}$$

$$R = \frac{\zeta_{21} - \frac{\cos\theta_t}{\cos\theta_i}}{\zeta_{21} + \frac{\cos\theta_t}{\cos\theta_i}} \qquad \sqrt{1-\left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i}$$

$$= \frac{\zeta_{21}\cos\theta_i - \cos\theta_t}{\zeta_{21}\cos\theta_i + \cos\theta_t}$$

$$R = \frac{\zeta_{21}\cos\theta_i - \sqrt{1-\left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i}}{\zeta_{21}\cos\theta_i + \sqrt{1-\left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i}}$$

$$\zeta_{21} = \frac{\rho_2 c_2}{\rho_1 c_1}$$



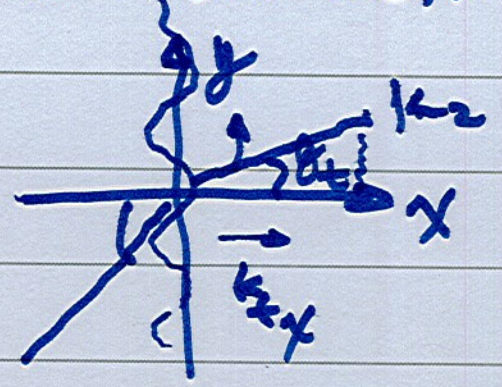
$= 0$

$R \to 1$ Total Reflection

$\theta_i > \theta_c$ what happens

what is the propagation direction

k_{tx} ? k_{ty} ?



$k_{ty} = k_2 \sin\theta_2 = \underset{k_{iy}}{k_1 \sin\theta_i}$ because of pressure continuity

↑ forcing spatial pattern

real number

$$k_{tx} = k_2 \cos\theta_t \qquad \cos^2\theta_t + \sin^2\theta_t = 1$$

even if θ_t is complex

$$k_{tx} = \pm k_2 \sqrt{1 - \sin^2\theta_t}$$

$$= \pm k_2 \sqrt{1 - \underbrace{\underbrace{\left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i}_{>1}}_{<0}}$$

$c_2 > c_1$

$$= \pm \sqrt{-1}\, k_2 \sqrt{\underbrace{\left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i - 1}_{\text{+ve number when } \theta_i > \theta_c}}$$

$$k_{tx} = \pm j\gamma \qquad \gamma = k_2 \sqrt{\left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i - 1}$$

Sound field in region ②

$$\hat{p}_t = P_t\, \underbrace{e^{-jk_{t_x}x}}\, \underbrace{e^{-jk_{t_y}y}}$$

k_{t_y} = real number

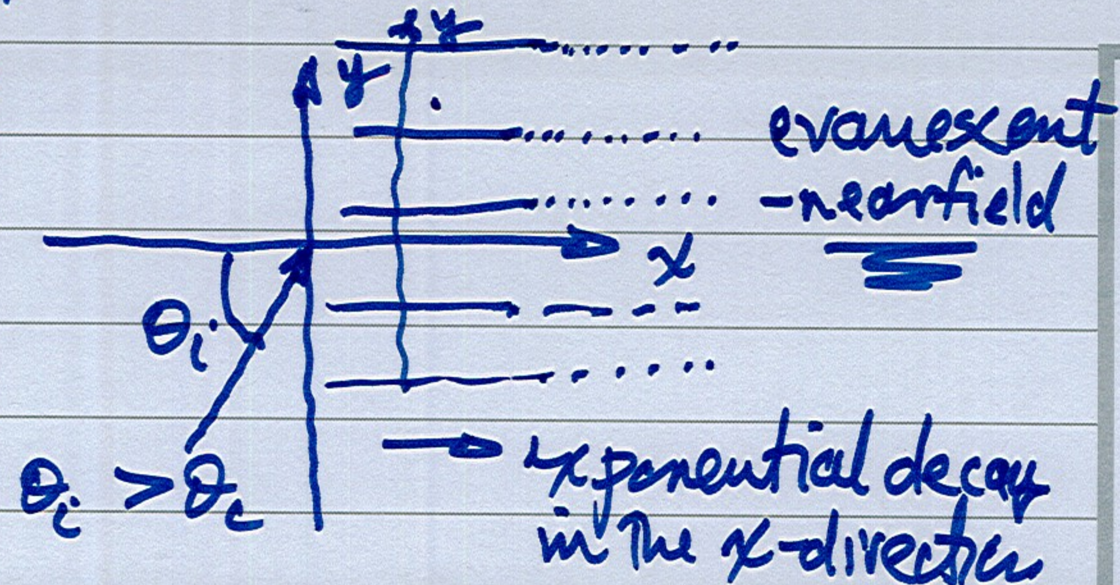
$e^{-jk_{t_y}y}$ → propagation

$e^{-jk_{t_x}x}$ → $e^{-j(\pm j\gamma)x}$

$e^{+\gamma x}$ ← discard in case of infinite region ②

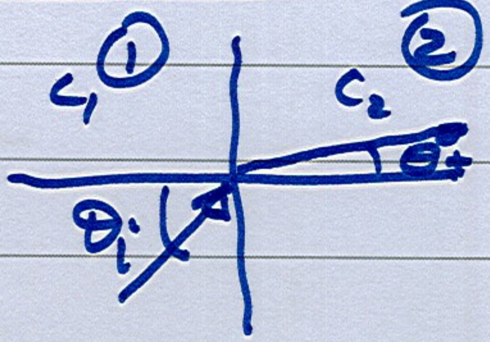
$e^{-\gamma x}$

$e^{+\gamma x}$, $e^{-\gamma x}$ → exponential growth or decay



Consequences of Snell's Law

1. From Snell's Law



when $\frac{c_2}{c_1} < 1$

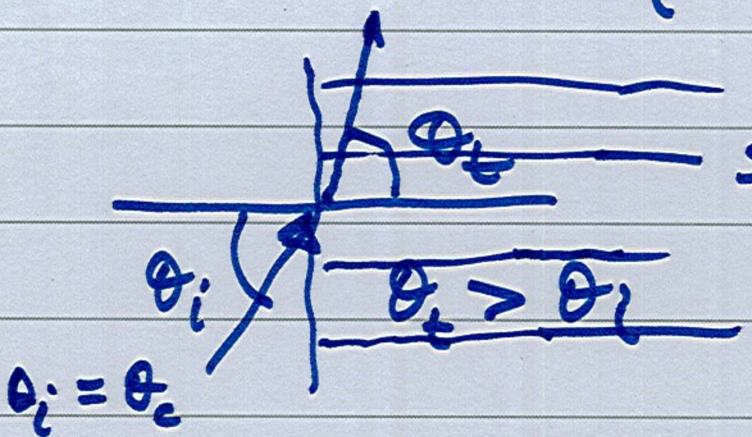
$$\sin\theta_t < \sin\theta_i$$

$$\theta_t < \theta_i$$

"sound refracts towards the normal"

2. if $c_2 > c_1$

$$\sin\theta_t = \frac{c_2}{c_1}\sin\theta_i$$



sound refracts away from the normal

maximum possible real θ_t is $\pi/2$

defn of θ_c $\sin\theta_c = \dfrac{c_1}{c_2}$

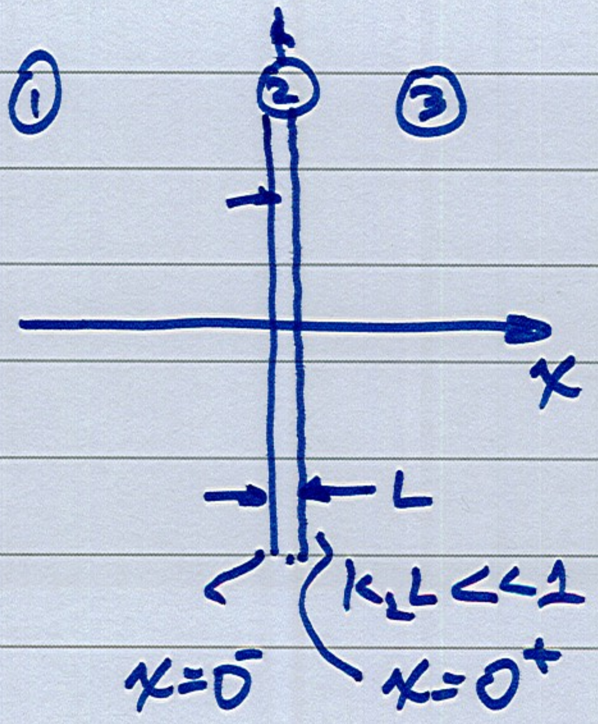
$\sin\theta_t = 1$

3. If $\theta_i > \theta_c$

exponential decay into the second medium

θ_t is then complex

4.3.2 Reflection and Transmission at a Thin, limp panel

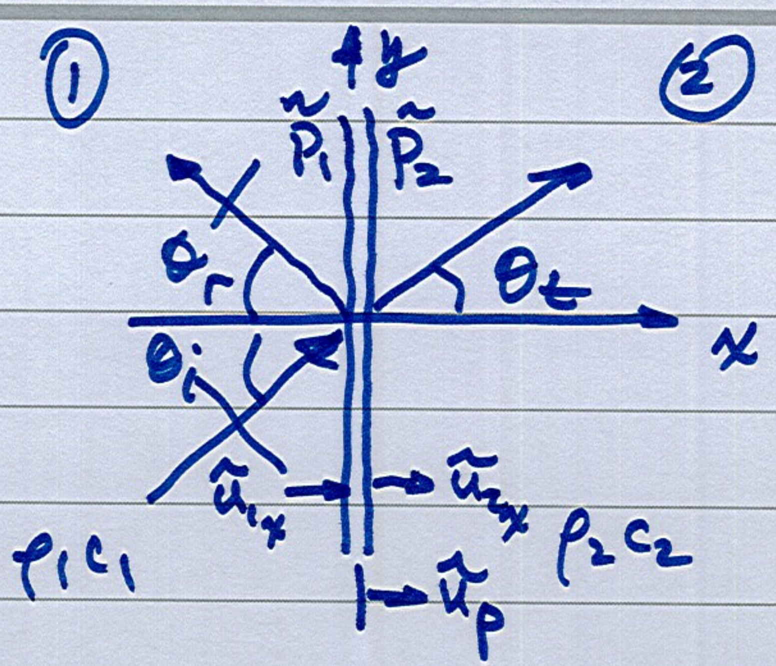


limp — no flexural stiffness

- no wave propagation within the panel in the x-direction – panel does not compress in the x-direction
- no free wave prop possible in region ② in the y-direction

↑ local reaction

- only consider solid body transverse motion



same medium on both sides

$\rho_1 = \rho_2 \qquad c_1 = c_2$

m_s – mass/unit area

$\tilde{u}_p$ – transverse velocity of the panel

– panel is limp – no free wave prop in the y-direction

① region

$$\tilde{P}_1 = P_i e^{-i(k_x x + k_y y)} + P_r e^{+i(k_x x - k_y y)}$$

$$\theta_i = \theta_r = \theta_t \qquad k_x = k\cos\theta_i \qquad k = \frac{\omega}{c}$$

$$k_y = k\sin\theta_i$$

$$\tilde{u}_{1x} = \frac{P_i}{\rho_0 c}\cos\theta_i e^{-i(k_x x + k_y y)} - \frac{P_r}{\rho c}\cos\theta_i e^{+i(k_x x - k_y y)}$$

$$\rho_1 c_1 = \rho_2 c_2 = \rho_0 c$$

in region ②

$$\tilde{P}_2 = P_t e^{-i(k_x x + k_y y)}$$

$$\tilde{u}_{2x} = \frac{P_t}{\rho_0 c}\cos\theta e^{-i(k_x x + k_y y)}$$