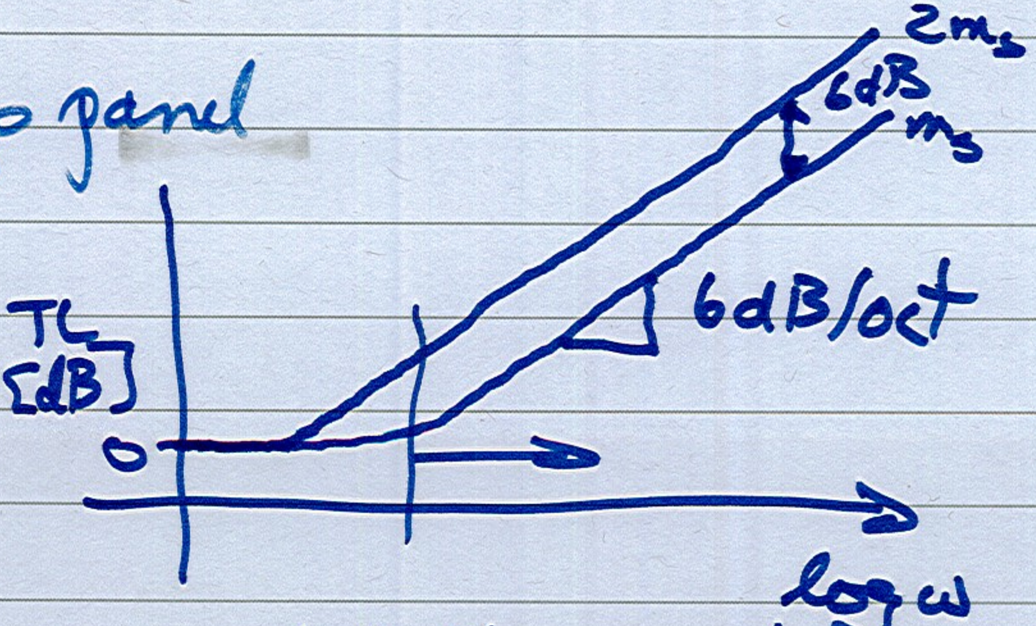
Thin heavy limp panel

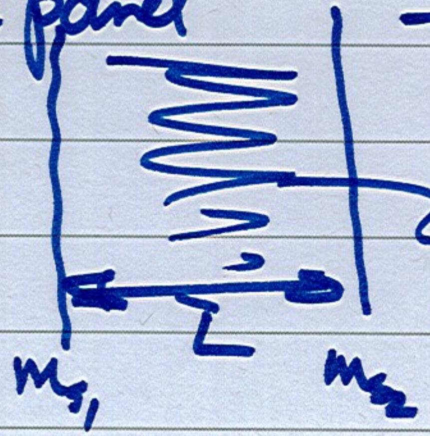
$$T = \frac{2\rho_0 c\, e^{+jkz}}{2\rho_0 c + j\omega m_s}$$

m_s



Mass law

Double panel



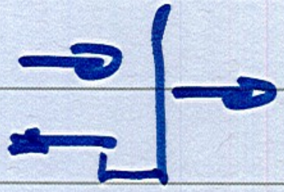
- much higher performance at a given mass/unit area

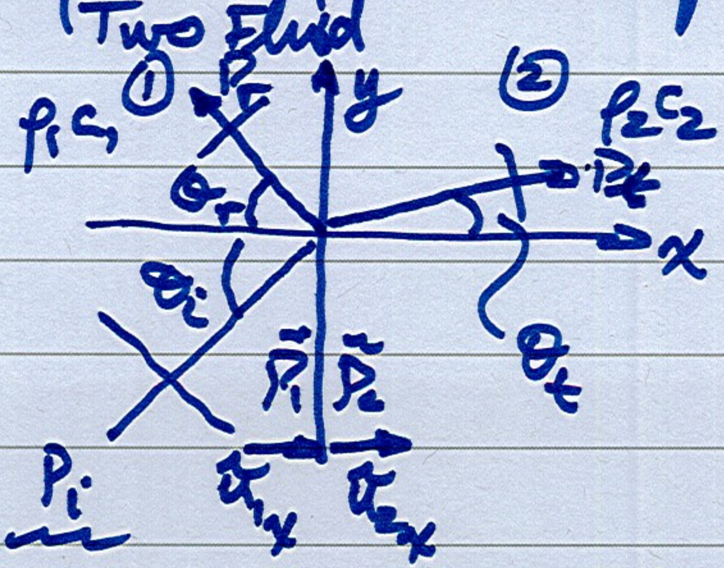
decoupling

T_I R_I $\underline{T_I + R_I = 1}$

4.3 Oblique Incidence Reflection + Transmission

Two Fluid





$\to \infty$

$\tilde{u}_{1y} \neq \tilde{u}_{2y}$ slip is allowed because no viscosity

2-D problem – no prop in 3-direction

$\nabla^2 \hat{p}_1 + k_1^2 \hat{p}_1 = 0 \qquad k_1 = \frac{\omega}{c_1}$

$\nabla^2 \tilde{p}_2 + k_2^2 \hat{p}_2 = 0 \qquad k_2 = \frac{\omega}{c}$

$\tilde{p}_1 \big|_{x=0} = P_i + P_r \qquad \tilde{p}_2 \big|_{x=0} = P_t$

① $$\tilde{p}_1 = P_i e^{-i(\overset{\sim k_{ix}}{k_{ix}} x + k_{iy} y)} + P_r e^{+i(k_{rx} x - k_{ry} y)}$$

$$k_{ix}^2 + k_{iy}^2 = k_1^2 \qquad k_{ix} = k_1 \cos\theta_i \quad k_{iy} = k_1 \sin\theta_1$$

$$k_{rx} = k_1 \cos\theta_r \quad k_{ry} = k_1 \sin\theta_r$$

$$\tilde{u}_{1x} = -\frac{1}{j\omega\rho_1}\frac{\partial \tilde{p}_1}{\partial x}$$

$$= \frac{P_i}{\rho c_1}\cos\theta_i e^{-i(k_{ix}x + k_{iy}y)} - \frac{P_r}{\rho_1 c_1}\cos\theta_r e^{+i(\underbrace{k_{rx}x - k_{ry}y}_{k_r})}$$

4

(2)

$$\tilde{p}_2 = P_t e^{-j(k_{t_x} x + k_{t_y} y)}$$

$$k_{t_x} = k_2 \cos\theta_t \qquad k_{t_y} = k_2 \sin\theta_t$$

$$\tilde{u}_{2_x} = -\frac{1}{j\omega\rho_2} \frac{\partial \tilde{p}_2}{\partial x}$$

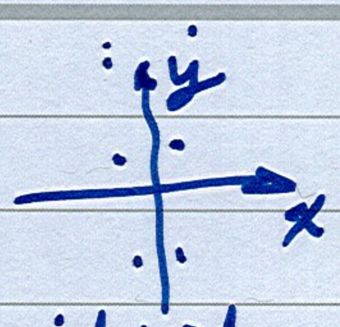
$$= \frac{P_t}{\rho_2 c_2} \cos\theta_t \, e^{-j(k_{t_x} x + k_{t_y} y)}$$

Two B.C.'s

$$\tilde{p}_1(0) = \tilde{p}_2(0)$$

$$\tilde{u}_{1_x}(0) = \tilde{u}_{2_x}(0)$$

B.C. ① Pressure $\tilde{P}_1(0,y) = \tilde{P}_2(0,y)$



infinite in y-direction

$$P_i e^{-jk_{iy}y} + \underline{P_r e^{-jk_{ry}y}} = P_t e^{-jk_{ty}y}$$

This b.c. <u>must</u> be independent of position in the y-direction

$\therefore$ $$\underbrace{k_{iy} = k_{ry} = k_{ty}}$$

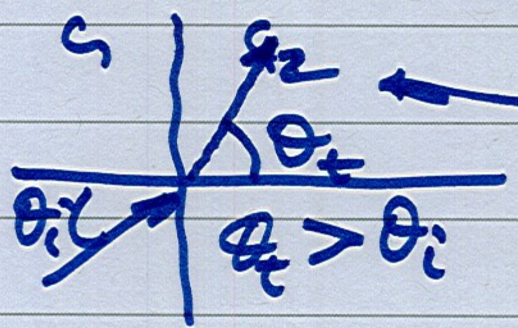
$$k_1 \sin\theta_i = k_1 \sin\theta_r$$

$$\therefore \theta_r = \theta_i$$

$$k_1 \sin\theta_i = k_2 \sin\theta_t$$

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{k_2}{k_1} = \frac{c_1}{c_2} \qquad \underline{\text{Snell's Law}}$$

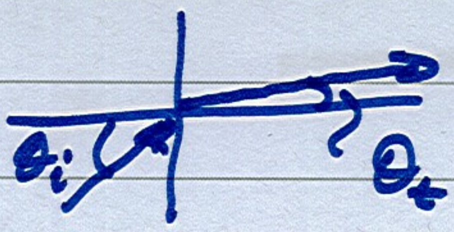
$$\sin\theta_t = \frac{c_2}{c_1}\sin\theta_i$$



$c_2 > c_1$

$\theta_t > \theta_i$

$c_2 < c_1$



$\theta_t < \theta_1$

Given c_1, c_2 & $\theta_i \rightarrow \theta_t$

Pressure B.C.

$$P_i + P_r = P_t \qquad R = \frac{P_r}{P_i}$$

$$\boxed{1 + R = T} \qquad T = \frac{P_t}{P_i}$$

Velocity B.C. $\hat{u}_{1x}(0,y) = \hat{u}_{2x}(0,y)$

$$\frac{P_i}{\rho_1 c_1}\cos\theta_i - \frac{P_r}{\rho_1 c_1}\cos\theta_i = \frac{P_t}{\rho_2 c_2}\cos\theta_t$$

$$\theta_r = \theta_i$$

$$\boxed{1 - R = \frac{\cos\theta_t}{\cos\theta_i}\frac{1}{\zeta_{21}}T}$$

$\zeta_{21} = \dfrac{\rho_2 c_2}{\rho_1 c_1}$ 2 eqn in 2 unknowns

$$R = \frac{\zeta_{21} - \frac{\cos\theta_t}{\cos\theta_i}}{\zeta_{21} + \frac{\cos\theta_t}{\cos\theta_i}} \qquad T = \frac{2\zeta_{21}}{\zeta_{21} + \frac{\cos\theta_t}{\cos\theta_i}}$$

when $\theta_i = 0$ normal - reduce to previous results

when $\theta_i = \frac{\pi}{2}$ $\quad R \to 1$ & $T \to 1$

θ_t from $\sin\theta_t = \frac{c_2}{c_1} \sin\theta_i$

write R & T in terms of θ_i only

Snell's Law $\sin\theta_t = \frac{c_2}{c_1}\sin\theta_i$

$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t} = \sqrt{1 - \left(\frac{c_2}{c_1}\right)^2 \underline{\sin^2\theta_i}}$$

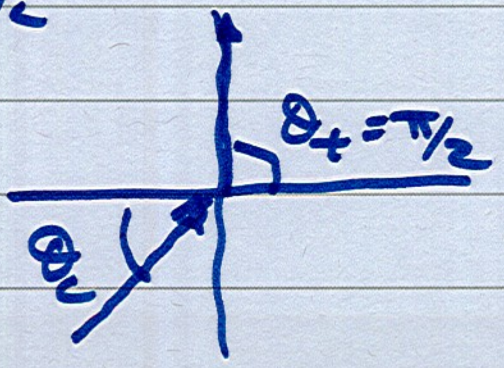
$$\Sigma$$

when $1 - \left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i = 0$

defines θ_c - critical angle

angle of total reflection

Critical Incidence Angle θ_c

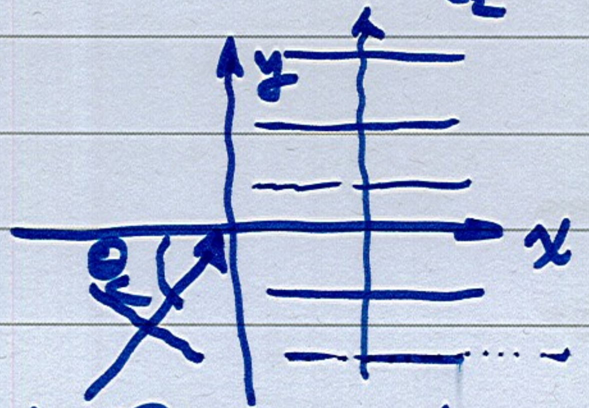


$\theta_i = \theta_c$ when $\theta_t = \frac{\pi}{2}$

$$\sin\theta_c = \frac{c_1}{c_2} \cancelto{1}{\sin\theta_t}$$

$$= \frac{c_1}{c_2}$$

$R \to 1$



in region ② sound prop ∥ to y-axis

what if $\theta_i > \theta_c$

$$\sin\theta_t = \frac{c_2}{c_1}\sin\theta_i > 1$$

if θ_t is complex $\sin\theta_t$ can be > 1

$$\sin\theta_t = \frac{e^{j\theta_t} - e^{-j\theta_t}}{2j}$$

when $\theta_i > \theta_c$

The result is that

k_{t_x} = imaginary number

- exponential decay away from interface