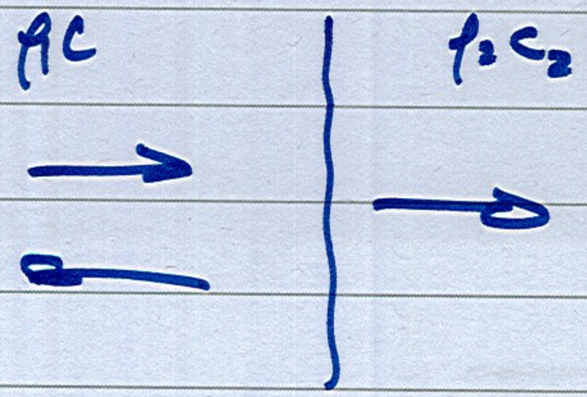
Reflection and Transmission ✓

Midterm Nov 1 Friday

The Atlantic



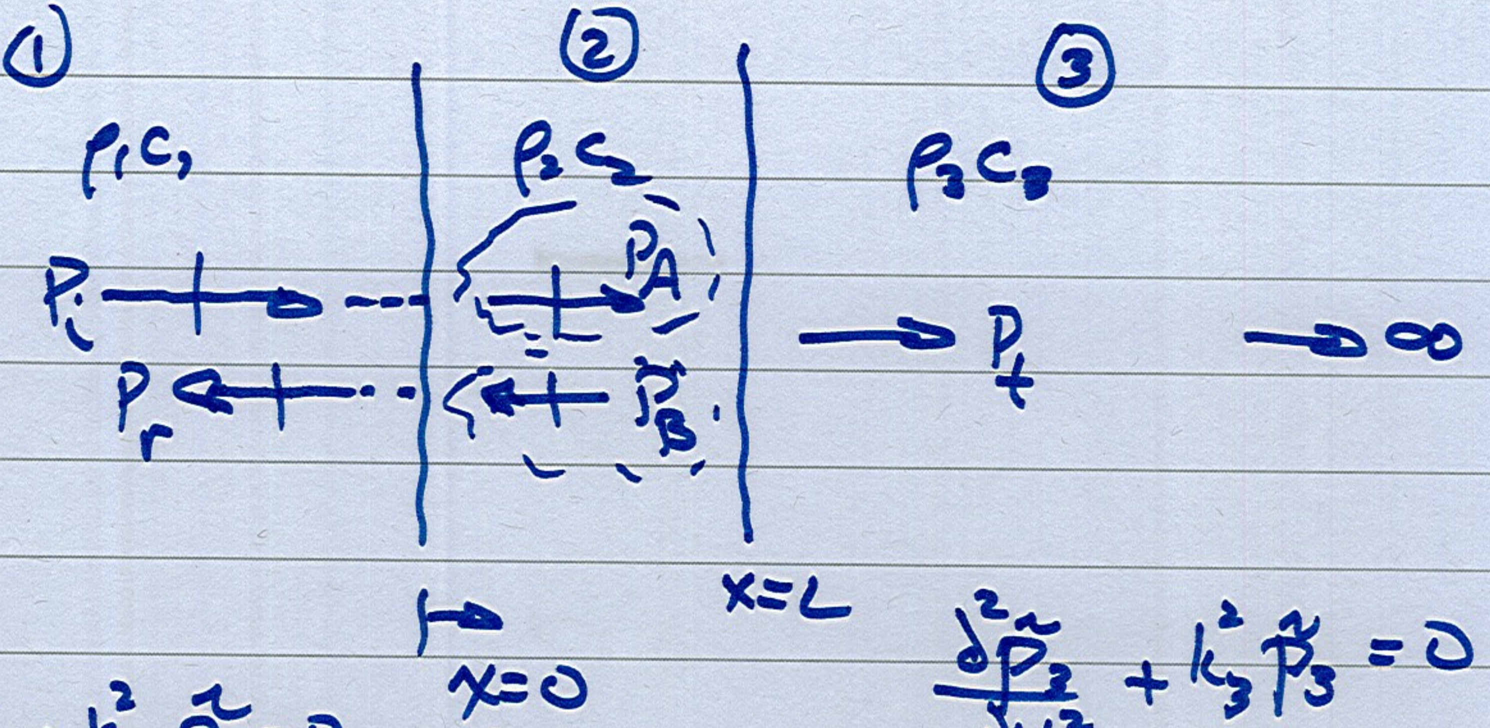
$\rho_2 c_2 > \rho_1 c_1$ R +ve

$\rho_2 c_2 < \rho_1 c_1$ R -ve

$\rho_2 c_2 \gg \rho_1 c_1$ $R \to 1$ $T \to 2$

$\rho_2 c_2 \ll \rho_1 c_1$ $R \to -1$ $T \to 0$

4.2.2



$$\frac{d^2\hat{p}_1}{dx^2} + k_1^2\,\hat{p}_1 = 0 \qquad \frac{d^2\hat{p}_2}{dx^2} + k_2^2\,\hat{p}_2 = 0 \qquad \frac{d^2\hat{p}_3}{dx^2} + k_3^2\,\hat{p}_3 = 0$$

$$k_1 = \frac{\omega}{c_1} \qquad k_2 = \frac{\omega}{c_2} \qquad k_3 = \frac{\omega}{c_3}$$

$$\hat{p}_1(x) = P_i e^{-jk_1x} + P_r e^{+jk_1x}$$

$$\hat{p}_2(x) = P_A e^{-jk_2x} + P_B e^{+jk_2x}$$

$$\hat{p}_3(x) = P_t e^{-jk_3x}$$

B.C.'s

at $x=0$ $\qquad \tilde{p}_1 = \tilde{p}_2 \qquad (1)$

$\tilde{u}_{1x} = \tilde{u}_{2x} \qquad (2)$

at $x=L$ $\qquad \tilde{p}_2(L) = \tilde{p}_3(L) \qquad (3)$

$\tilde{u}_{2x}(L) = \tilde{u}_{3x}(L)$

(1) $P_i + P_r = P_A + P_B \quad \div P_i$

$$\boxed{1 + R = A + B} \qquad R = \frac{P_r}{P_i} \quad A = \frac{P_A}{P_i} \quad B = \frac{P_B}{P_i}$$

(2) $1 - R = \frac{1}{\zeta_{21}}(A - B) \qquad \zeta_{21} = \frac{\rho_3 c_2}{\rho_1 c_1}$

$$T = \frac{P_t}{P_i}$$

(3) $Ae^{-ik_2L} + Be^{+ik_2L} = Te^{-ik_3L}$

(4) $Ae^{-ik_2L} - Be^{+ik_2L} = \frac{T}{\zeta_{32}} e^{-ik_3L}$

$$\zeta_{32} = \frac{\rho_3 c_3}{\rho_2 c_2}$$

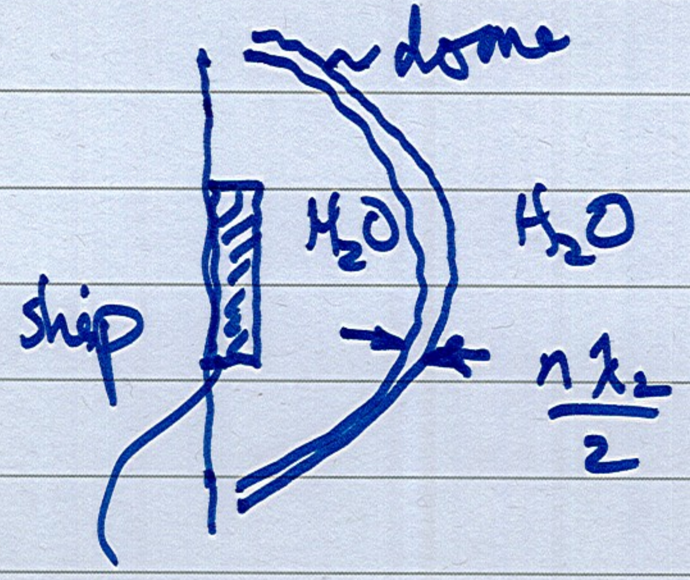
$$R = \frac{\left(1 - \frac{\rho_1 c_1}{\rho_3 c_3}\right)\cos k_2 L + j\left(\frac{\rho_2 c_2}{\rho_3 c_3} - \frac{\rho_1 c_1}{\rho_2 c_2}\right)\sin k_2 L}{\left(1 + \frac{\rho_1 c_1}{\rho_3 c_3}\right)\cos k_2 L + j\left(\frac{\rho_2 c_2}{\rho_3 c_3} + \frac{\rho_1 c_1}{\rho_2 c_2}\right)\sin k_2 L}$$

$$k_2 L = \frac{2\pi}{\lambda_2} L = 2\pi\left(\frac{L}{\lambda_2}\right)$$

non-dimensional layer depth

$R \to 0$ when $\rho_1 c_1 = \rho_3 c_3$

when $\sin k_2 L = 0$



Sonar Dome Case

$R \to 0 \quad |T| \to 1$ perfect transmission

$\rho_1 c_1 = \rho_3 c_3$

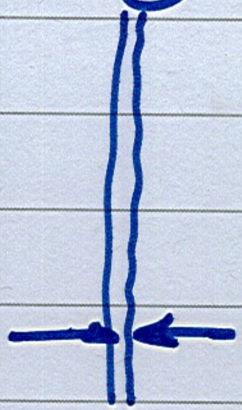
$\sin k_2 L \to 0$

choose

$k_2 L = n\pi$

$L = n\left(\frac{\lambda_2}{2}\right)$

Thin, heavy barrier case



① $\rho_1 c_1$

②

③ $\rho_3 c_3 = \rho_1 c_1$

very thin compared to a wavelength
- limp (no flexural stiffness)
- heavy

$k_2 L << 1$

$\sin k_2 L \approx k_2 L$

$\cos k_2 L \approx 1$

$\rho_2 c_2 >> \rho_1 c_1$ heavy

$$R = \frac{\left(1 - \cancelto{0}{\frac{\rho_1 c_1}{\rho_3 c_3}}\right) 1 + j\left(\frac{\rho_2 c_2}{\rho_3 c_3} - \cancelto{0}{\frac{\rho_1 c_1}{\rho_2 c_2}}\right) k_2 L}{\left(1 + \cancelto{1}{\frac{\rho_1 c_1}{\rho_3 c_3}}\right) 1 + j\left(\frac{\rho_2 c_2}{\rho_3 c_3} + \cancelto{0}{\frac{\rho_1 c_1}{\rho_2 c_2}}\right) k_2 L}$$

$$R = \frac{j \frac{\rho_2 \cancel{c_2}}{\rho_3 c_3} \frac{\omega}{\cancel{c_2}} L}{2 + j \frac{\rho_2 \cancel{c_2}}{\rho_3 c_3} \frac{\omega}{\cancel{c_2}} L} \qquad (\rho_2 L) = m_s$$

mass/unit area

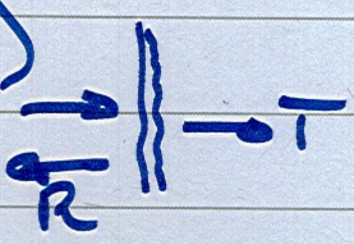
$\rho_1 c_1 = \rho_3 c_3 = \rho_0 c$

physical parameter = mass/unit area

$$\boxed{R = \frac{j \omega m_s}{2 \rho_0 c + j \omega m_s}}$$

$\omega \to \infty \qquad R \to 1 \qquad \omega \to 0 \qquad R \to 0$

$m_s \to \infty \qquad R \to 1 \qquad (T \to 1)$



$$T = \frac{2e^{ik_3L}}{\left(1 + \frac{\rho_1 c_1}{\rho_3 c_3}\right)\cos k_2L + j\left(\frac{\rho_2 c_2}{\rho_3 c_3} + \frac{\rho_1 c_1}{\rho_2 c_2}\right)\sin k_2L}$$

(i) Sonar $\quad k_2L = n\pi$

$\sin k_2L \to 0$

$\cos k_2L \to 1$

$\rho_1 c_1 = \rho_3 c_3$

$$T \to \frac{\cancel{2e}\; 2e^{+ik_3L}}{\pm 2} \qquad |T| \to 1$$

perfect transmission except for phase shift

(ii) Thin ~~too~~ heavy barrier

$\rho_1 c_1 = \rho_3 c_3$ $\qquad \dfrac{\rho_2 c_2}{\rho_1 c_1} \gg 1$ $\qquad k_2 L \ll 1$

$$T \to \frac{2e^{+ik_3L}}{2 + j\left(\frac{\rho_2 c_2}{\rho_3 c_3}\right)k_2 L}$$

$k_1 = k_3 = k$

$\rho_1 c_1 = \rho_3 c_3 = \rho_0 c$

$\rho_2 L = m_s$

$$\boxed{T = \frac{2\rho_0 c\, e^{+ikL}}{2\rho_0 c + j\omega m_s}}$$

Thin, heavy barrier - below the critical frequency

$\omega \to 0 \qquad |T| \to 1$

high freq $\quad \omega m_s \gg 2\rho_0 c$

In The high frequency region

$$\boxed{|T| = \frac{2\rho c}{\omega m_s}}$$

<u>mass law</u>

$$|T| \propto \frac{1}{m_s}$$

$$|T| \propto \frac{1}{f}$$

"limp" → flexural stiffness is negligible

sheet of AL 0.05" limp below 10 kHz

Transmission Loss

$$TL = 10 \log_{10} \frac{1}{|T|^2} \quad dB$$

$|T| \downarrow \qquad TL \uparrow$