ME 513 Session 23 10/14/19

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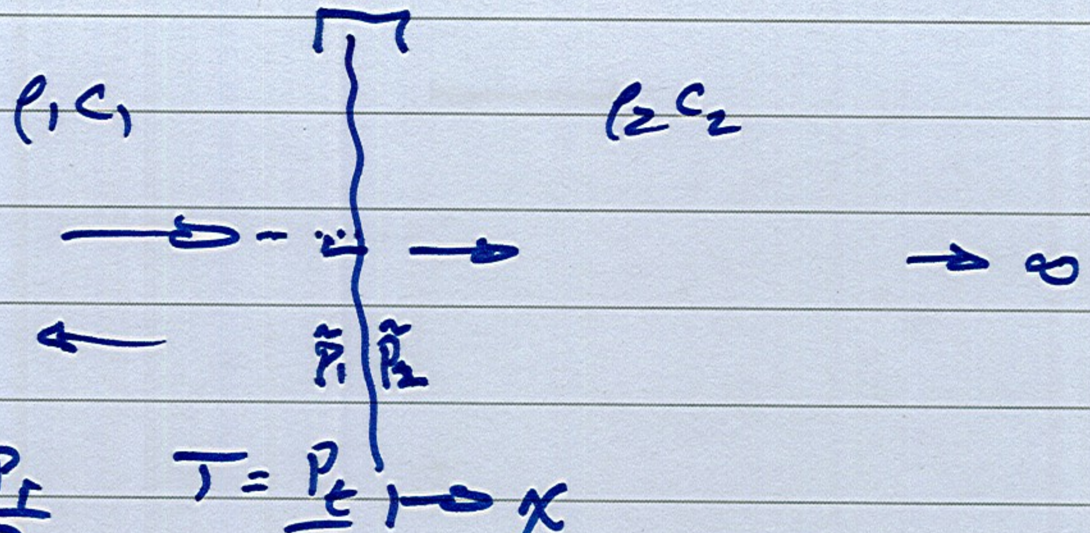
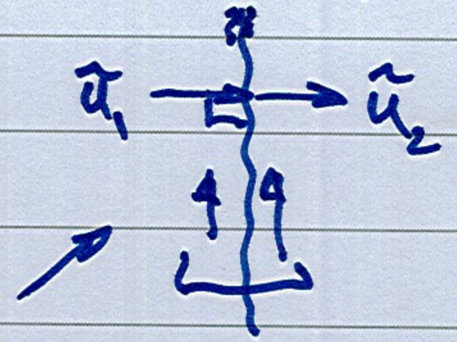
4.0 Fundamentals of Reflection and Transmission



$R = \frac{P_r}{P_i}$ $T = \frac{P_t}{P_i}$

$\tilde{p}_1(0) = \tilde{p}_2(0)$

(ii) Velocity Continuity B.C.



$\tilde{u}_{1_n}(0) = \tilde{u}_{2_n}(0)$ Required for the two fluids to remain in ~~cont~~ contact

slip is allowed in the tangential direct - since fluids are assumed to be inviscid

harmonic case

$$j\omega\tilde{\xi}_{1_n}(0) = j\omega\tilde{\xi}_{2_n}(0)$$

displacements are continuous

Note:

1. $$\frac{\tilde{p}_1(0) = \tilde{p}_2(0)}{\tilde{u}_{1n}(0) = \tilde{u}_{2n}(0)}$$

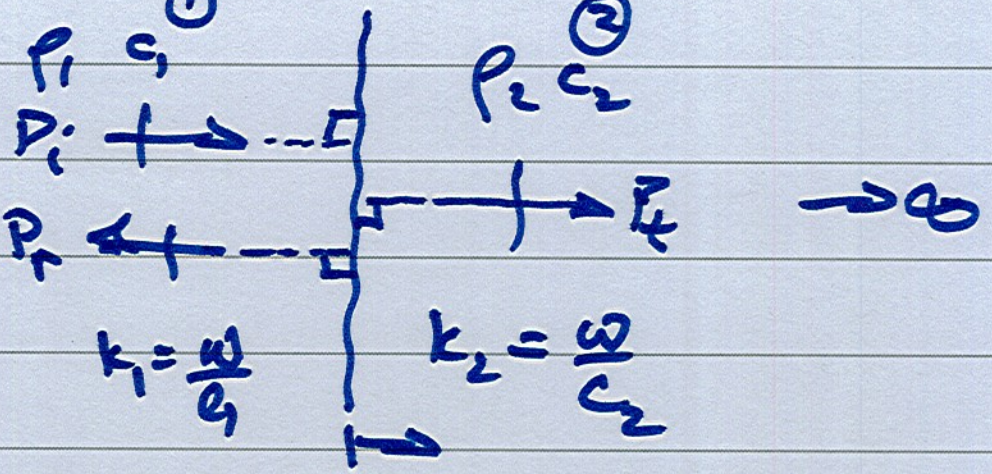
normal specific acoustic impedance is continuous at the interface

$$\left.\frac{\tilde{p}_1}{\tilde{u}_{1n}}\right|_{x=0} = \left.\frac{\tilde{p}_2}{\tilde{u}_{2n}}\right|_{x=0} = \tilde{z}_1\Big|_{x=0} = \tilde{z}_2\Big|_{x=0}$$

Application of B.C.'s

$$\tilde{p}_1 = P_i e^{-jk_1 x} + P_r e^{+jk_1 x}$$

$$\tilde{p}_2 = P_t e^{-jk_2 x}$$



Pressure B.C. at $x = 0$

$$\tilde{p}_1(0) = \tilde{p}_2(0)$$

$$\boxed{P_i + P_r = P_t} \quad (1)$$

$$\tilde{u}_1 = \frac{P_i}{\rho_1 c_1} e^{-jk_1 x} - \frac{P_r}{\rho_1 c_1} e^{+jk_1 x}$$

$$\tilde{u}_2 = \frac{P_t}{\rho_2 c_2} e^{-jk_2 x}$$

$$\tilde{u}_3(0) = \hat{u}_3(0)$$

$$\boxed{\frac{P_i}{\rho_1 c_1} - \frac{P_r}{\rho_1 c_1} = \frac{P_t}{\rho_2 c_2}} \quad (2)$$

$\div P_i \to R, T$

$$\zeta_{21} = \frac{\rho_2 c_2}{\rho_1 c_1}$$

(1) $\to$ $1 + R = T$

(2) $\to$ $1 - R = \frac{\rho_1 c_1}{\rho_2 c_2} T = \frac{1}{\zeta_{21}} T$

$$T = \frac{2\zeta_{21}}{\zeta_{21} + 1}$$

$$R = \frac{\zeta_{21} - 1}{\zeta_{21} + 1}$$

Notes:

(i) If there is no dissipation in regions 1 & 2

$\rightarrow \quad \dfrac{\rho_2 c_2}{\rho_1 c_1} = \xi_{21} = \text{real} \qquad T = \dfrac{2\xi_{21}}{\xi_{21}+1}$

R & T are real $\qquad \xi_{21} = \dfrac{\rho_2 c_2}{\rho_1 c_1} \qquad R = \dfrac{\xi_{21}-1}{\xi_{21}+1}$

R can be +ve or -ve

$\xi_{21} > 1$ R is +ve $\qquad \xi_{21} < 1$ R is -ve

$\xi_{21} > 1$ in-phase reflection $\quad \xi$ hard surface

$\xi_{21} < 1$ out-of-phase reflection - soft surface

(ii) $\zeta_{21} = \frac{\rho_2 c_2}{\rho_1 c_1}$

Air ① | Water ②

$(\rho_1 c_1)_{air} \approx 415$ MKS Rayls

$(\rho_2 c_2)_{water} \approx 1.5 \times 10^6$ MKS Rayls

$\zeta_{21} >> 1$

$R = \frac{\zeta_{21} - 1}{\zeta_{21} + 1} \rightarrow 1$ when $\zeta_{21} \rightarrow \infty$

$\tilde{P}_1(0) = \tilde{P}_2(0) = P_i + P_r = P_i(1 + R)$

$= 2P_i = P_t$

when $\zeta_{21} >> 1$ pressure is doubled at the surface

Air / Water Case

$R = 1 \qquad T = 2 \qquad T = \dfrac{2\zeta_{21}}{\zeta_{21} + 1}$

$\underline{R + T \neq 1}$

$I_2 = \dfrac{(\hat{p}^2_{rms})_2}{\rho_2 c_2} \simeq 0$ when $\zeta_{21} \to \infty$

not transmitted

Essentially, no energy transmission at a hard surface

$I_{1_i} = \dfrac{(\hat{p}^2_{rms})_{1_i}}{\rho_1 c_1}$

- almost all is reflected

(iv) $R = \dfrac{\xi_{21} - 1}{\xi_{21} + 1}$

$\underline{\xi_{21} = 1}$ $\quad R \to 0 \quad\quad T \to 1$

ξ

$\rho_1 = \rho_2$

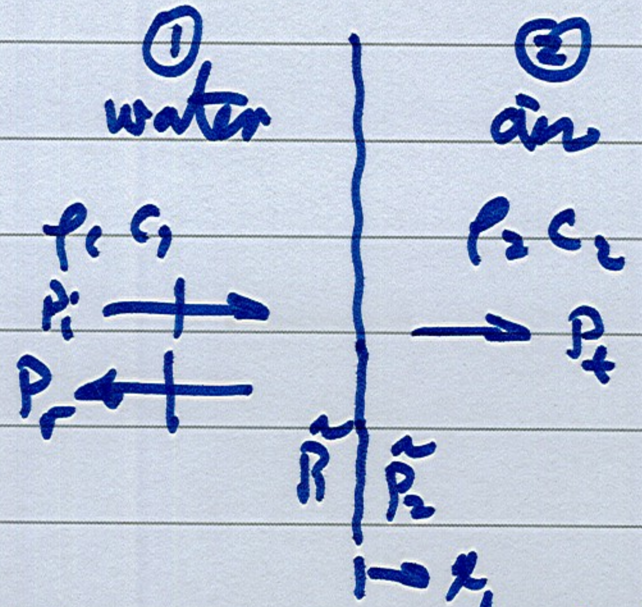
$c_1 = c_2$

$\rho_1 c_1 = \rho_2 c_2$

$\left(\dfrac{\rho_1}{\rho_2}\right) = \left(\dfrac{c_2}{c_1}\right)$ if this condition is satisfied

zero reflection + perfect transmission

(v) $\zeta_{21} \ll 1$ $\qquad \zeta_{21} = \dfrac{\rho_2 c_2}{\rho_1 c_1} \ll 1$



$$R = \frac{\zeta_{21} - 1}{\zeta_{21} + 1} \simeq -1$$ out-of-phase reflection

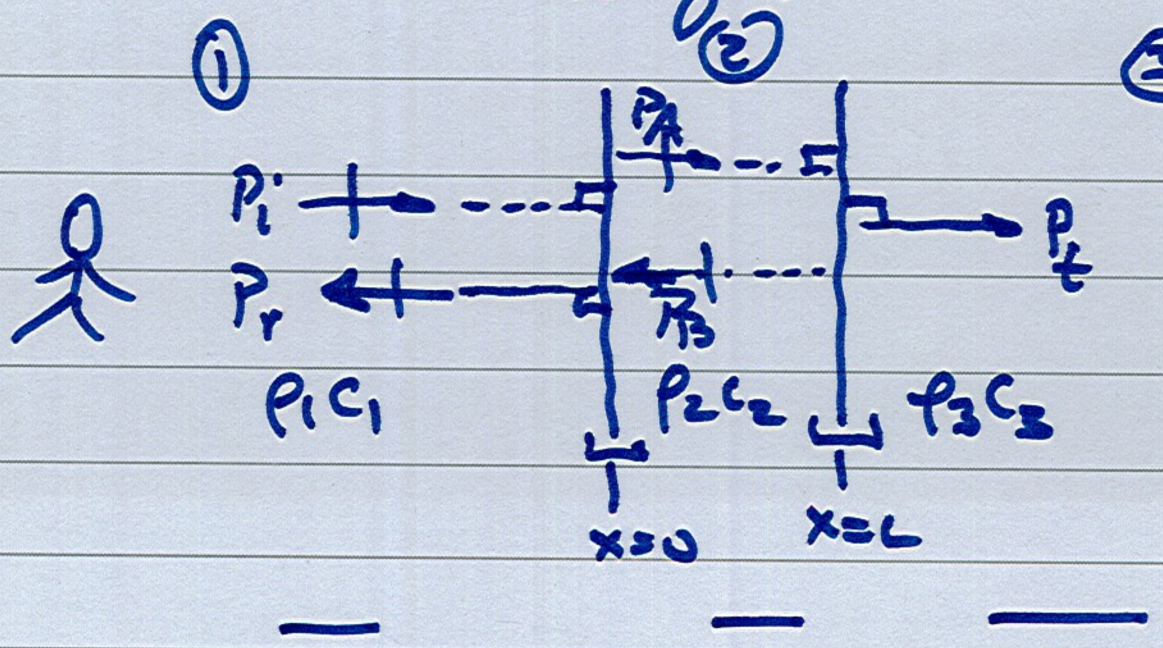
$$\tilde{p}_1(0) = p_i + p_r = p_i(1 + R) \simeq 0 = \tilde{p}_2(0)$$

no transmission $\quad T \to 0$

pressure release surface

$\tilde{p} = 0$ b.c. $\to$ pressure release b.c.

4.2.2 Normal Incidence Sound Transmission Through a fluid layer



Frequency Domain

$$\int_{-\infty}^{\infty} f(t)\, e^{-j2\pi f t}\, dt$$