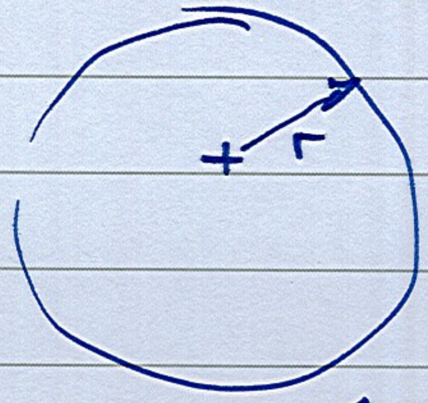
ME 513 Session 20 10/4/19






$$\tilde{p}_+ \propto \frac{1}{r}$$

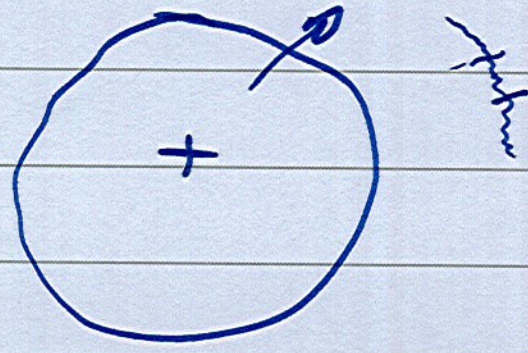
$$\tilde{u}_{r_+} = \frac{1}{\rho c}\left(1 + \underbrace{\frac{1}{jkr}}_{\text{nearfield}}\right)\tilde{p}_+$$

Cylindrical Wave

$kr \gg 1$ $\tilde{p}_+ \propto \frac{1}{r^{1/2}}$

Specific Acoustic Impedance

$$\tilde{z} = \frac{\text{pressure}}{\text{particle velocity}}$$



farfield

$kr \gg 1$

$$\frac{\tilde{p}_+}{\tilde{u}_+} = \rho_0 c$$

nearfield $\tilde{z} \simeq j\omega\rho_0 r$ mass-like

General spherical case

$$\tilde{p}(r) = \frac{A}{r} e^{-jkr} + \frac{B}{r} e^{+jkr}$$

$$\tilde{u}_r(r) = -\frac{1}{j\omega\rho_0}\frac{\partial \tilde{p}}{\partial r}$$

$\rightarrow$ 4 terms

$$\tilde{z}(r) = \frac{\tilde{p}(r)}{\tilde{u}_r(r)} \rightarrow \text{complicated}$$

Notes:

(i) specific impedance usually expressed as

$$\hat{z} = r + jx$$

$r$: specific acoustic resistance

$x$: specific acoustic reactance

(ii) $\rho_0 c$ characteristic impedance

$\neq$ s.a. impedance

(iii) outward going spherical + cylindrical waves

$kr \gg 1 \qquad \hat{z}_+ \to \rho_0 c$

## 3.5 Acoustic Intensity

In Mechanics

Force x Distance = Work (Joules)

Force x Velocity = Power (Watts)

In Acoustics

$$\underbrace{\text{pressure} \times \text{velocity}}_{} = \frac{\text{Power}}{\text{unit area}} \quad [\text{Watts}/m^2]$$

Force/area — acoustic intensity

Instantaneous Intensity

$$= p(t)\bar{u}(t) = \bar{I}_t(t)$$

velocity is a vector

$\rightarrow$ Intensity is vector

Time-Averaged Acoustic Intensity

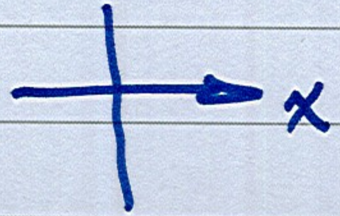
- time-averaged rate of energy flow Through a unit area

For a periodic signal

$$\bar{I} = \frac{1}{T}\int_0^T \underbrace{p(t)}_{\text{real}}\,\underbrace{\bar{u}(t)}_{\text{real}}\,dt = \text{real quantity}$$

T = period

+ve going plane wave




$$\frac{p_+}{u_+} = \rho_0 c \qquad u_+ = \frac{p_+}{\rho_0 c}$$

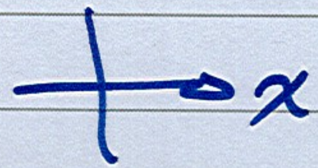
$$I_x = \frac{1}{T}\int_0^T \frac{p_+^2}{\rho_0 c}\, df$$

$$= \frac{1}{\rho_0 c}\underbrace{\left[\frac{1}{T}\int_0^T p_+^2\, dt\right]}_{\text{mean square pressure}}$$

$$= \frac{p_{rms}^2}{\rho_0 c}$$

Consider The harmonic case

$$\tilde{p}_+ = A e^{j(\omega t - kx)}$$




$$\tilde{u}_+ = \frac{A}{\rho_0 c} e^{j(\omega t - kx)}$$

$$\tilde{p}_+ = (A_r + jA_i)(\cos\omega t + j\sin\omega t)$$

$$\tilde{u}_+ = \frac{(A_r + jA_i)}{\rho_0 c}(\cos\omega t + j\sin\omega t)$$

$$\mathrm{Re}\{\tilde{p}_+\} = A_r\cos\omega t - A_i\sin\omega t$$

$$\mathrm{Re}\{\tilde{u}_+\} = \frac{A_r\cos\omega t - A_i\sin\omega t}{\rho_0 c}$$

$$I = \frac{1}{T}\int_0^T \mathrm{Re}\{\tilde{p}_+\}\,\mathrm{Re}\{\tilde{u}_+\}\,dt$$

$$\vdots$$

$$= \frac{1}{2\rho_0 c}\underbrace{\left[A_r^2 + A_i^2\right]}_{|A|^2 = |\tilde{p}_+|^2}$$

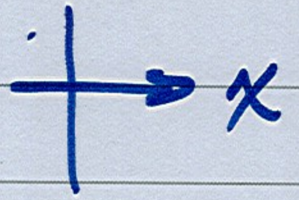
$$I = \frac{|\hat{p}_+|^2}{2\rho_0 c} \text{ - msp}$$

$$= \frac{p_{rms}^2}{\rho_0 c}$$ for a freely propagating plane wave

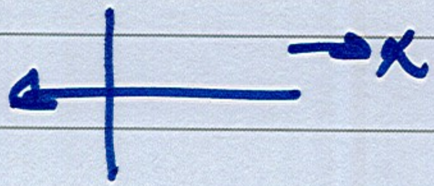
given $\tilde{p}$ what is $p^2_{rms}$?

$$p^2_{rms} = \frac{\tilde{p}\,\tilde{p}^*}{2} \leftarrow \text{conjugate}$$

for the freely prop plane wave



$$I = \frac{p^2_{rms}}{\rho_0 c}$$



$$I = -\frac{p^2_{rms}}{\rho_0 c}$$

$$\frac{\tilde{p}_-}{\tilde{u}_-} = -\rho_0 c$$

~~For propagating case~~

Complex harmonic signals $e^{j\omega t}$

$$\bar{I} = \frac{1}{2} \mathrm{Re}\{\hat{p}_+ \hat{u}_+^*\}$$

Freely propagating case

$$\bar{I} = \frac{1}{2} \mathrm{Re}\left\{ \frac{A e^{-jkx} e^{j\omega t} A^* e^{+jkx} e^{-j\omega t}}{\rho_0 c} \right\}$$

$$= \frac{1}{2} \mathrm{Re}\left\{ \frac{A A^*}{\rho_0 c} \right\}$$

$$= \frac{p_{rms}^2}{2\rho_0 c}$$

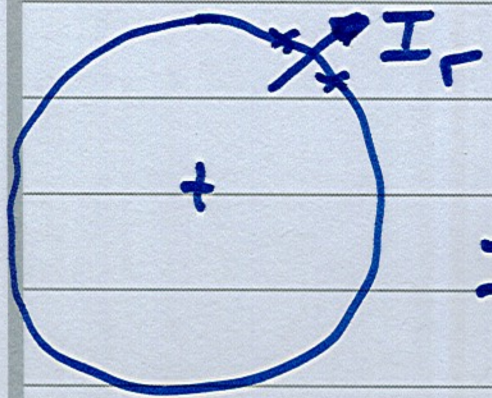
General expression for the time averaged acoustic intensity

$$\overline{I} = \frac{1}{2} \mathrm{Re}\{\hat{p}\,\overline{\tilde{u}}^{*}\}$$

when using complex harmonic representations

## spherical Case

- spherical symmetry
- free space




$$I_r = \frac{1}{2}\mathrm{Re}\{\tilde{p}_+ \tilde{u}_{r_+}^*\} \qquad \tilde{u}_{r_+} = \frac{1}{\rho_0 c}\left(1 + \frac{1}{jkr}\right)\tilde{p}_+$$

$$\tilde{u}_{r_+}^* = \frac{1}{\rho_0 c}\left(1 - \frac{1}{jkr}\right)\tilde{p}_+^*$$

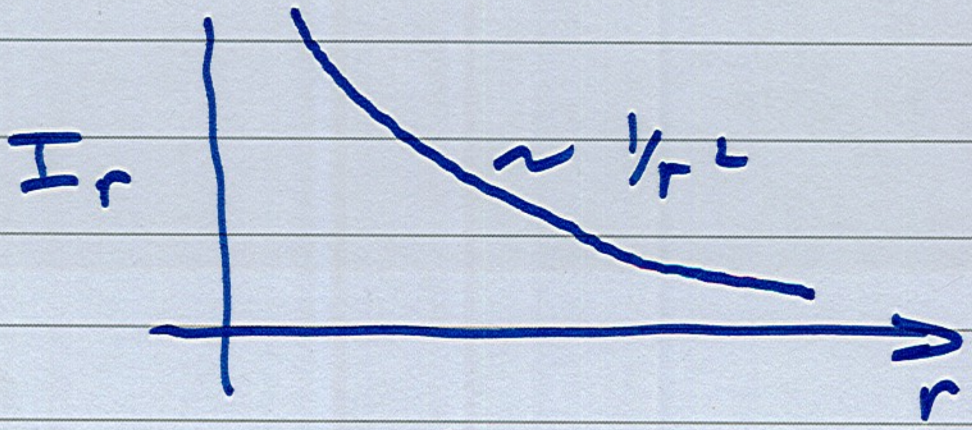
$$\tilde{p}_+ \tilde{u}_{r_+}^* = \frac{\tilde{p}_+ \tilde{p}_+^*}{\rho_0 c}\left(1 \underbrace{- \frac{1}{jkr}}_{\text{nearfield}}\right)$$

$$I_r = \frac{1}{2}\mathrm{Re}\{\tilde{p}_+ \tilde{u}_{r_+}^*\} = \frac{|\tilde{p}_+|^2}{2\rho_0 c} \quad p_{rms}^2$$

nearfield term does not contribute to the acoustic intensity

$$\tilde{p}_t = \frac{A}{r} e^{jkr} \qquad |\tilde{p}_t| \propto \frac{1}{r}$$

$$I_r = \frac{|\tilde{p}_t|^2}{2\rho_0 c} \qquad I_r \propto \frac{1}{r^2}$$




In contrast with particle velocity

$\boxed{\text{no intensity near field}}$

$$\text{Intensity} = \frac{\text{Sound Power}}{\text{Unit area}}$$