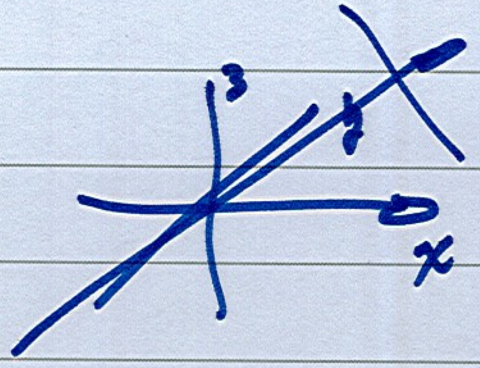
ME 513 Session 19 10/2/19

One-D

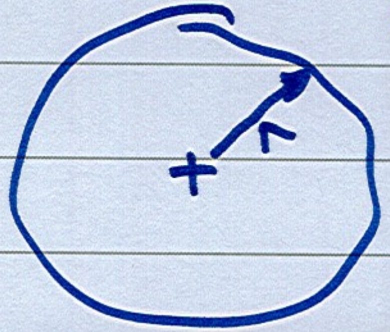


$$\tilde{p}(x,y,z,t) = Ae^{j(\omega t - k_x x - k_y y - k_z z)}$$

$$k^2 = k_x^2 + k_y^2 + k_z^2$$

$$k_z = \sqrt{k^2 - \underbrace{k_x^2 - k_y^2}}$$

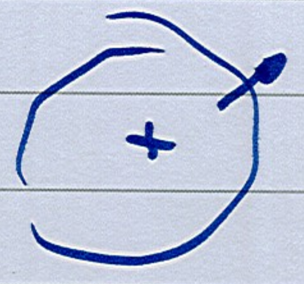
evanescent
wave



spherical symmetry
θ, φ

$$\nabla^2 \quad \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}$$

$$\underbrace{\frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r}}_{\frac{1}{r}\frac{\partial^2 (rp)}{\partial r^2}} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$



$$\frac{\partial^2 (rp)}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2 (rp)}{\partial t^2} = 0$$

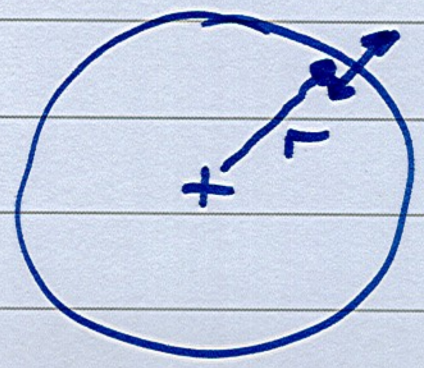
harmonic soln

$$\frac{\partial^2 (r\tilde{p})}{\partial r^2} + k^2 (r\tilde{p}) = 0$$

$$r\tilde{p} = Ae^{-jkr} + Be^{+jkr}$$

$$\tilde{p} = \underbrace{\frac{Ae^{-jkr}}{r}}_{\text{outward}} + \underbrace{\frac{B}{r}e^{+jkr}}_{\text{inward}}$$

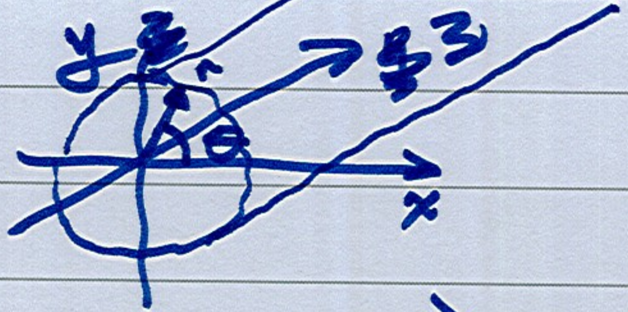
particle velocity



$$\tilde{u}_r = -\frac{1}{j\omega\rho}\frac{d\tilde{p}}{dr}$$

$$\underbrace{\tilde{u}_{r_+}}_{\text{expanding wave}} = \frac{1}{\rho c}\left(1 + \underbrace{\frac{1}{jkr}}_{\text{nearfield}}\right)\tilde{p}_+$$

3.3.2 Cylindrical Wave



$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

(r, θ, z)

cylindrical symmetry

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

$$\nabla^2 p = \frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}p$$

harmonic case $\quad p(r,t) = \tilde{p}(r)e^{j\omega t}$

$e^{+jkx} = \cos kx + j \sin kx$

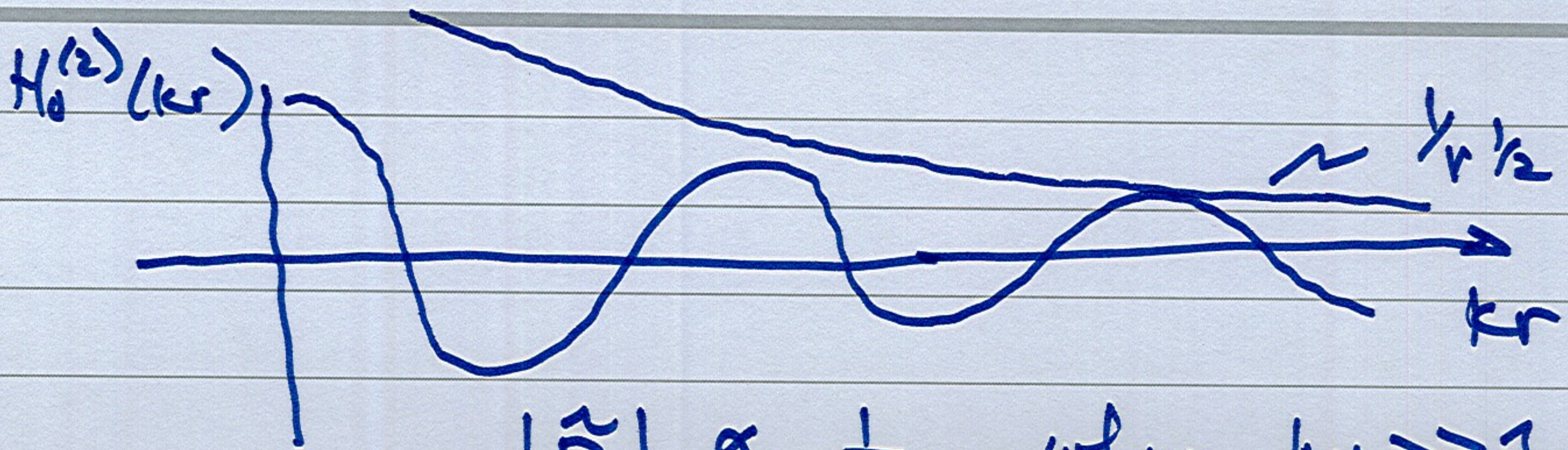
$$\frac{d^2 \tilde{p}}{dr^2} + \frac{1}{r}\frac{\partial}{\partial r}\tilde{p} + k^2 \tilde{p} = 0 \quad \checkmark$$

Bessel Eq of zeroth order

solution in terms of $J_0(kr)$ $Y_0(kr)$

$$\tilde{p} = \underbrace{A H_0^{(2)}(kr)}_{\text{outward going}} + \underbrace{B H_0^{(1)}(kr)}_{\text{inward going}}$$

$$\left.\begin{aligned} H_0^{(1)} &= J_0(kr) + j Y_0(kr) \\ H_0^{(2)} &= J_0(kr) - j Y_0(kr) \end{aligned}\right] \text{Hankel Functions}$$



$|\tilde{p}| \propto \frac{1}{r^{1/2}}$ when $kr \gg 1$

plane wave $|\hat{p}| \propto |e^{-ikx}| \rightarrow$ constant for all x

spherical wave $|\tilde{p}| \propto \left|\frac{1}{r} e^{-ikr}\right| \rightarrow \frac{1}{r}$

cylindrical wave $|\tilde{p}| \propto |H_0^{(2)}(kr)| \rightarrow \frac{1}{r^{1/2}}$

far field when $kr \gg 1$

3.4 Specific Acoustic Impedance

$$\tilde{z} = \frac{\text{Acoustic pressure}}{\text{particle velocity}} \qquad \frac{\tilde{p}}{\tilde{u}}$$

harmic case

Mechanical Impedance

$$= \frac{\text{Force}}{\text{velocity}}$$

plane wave in the +ve

$$\tilde{p}_+(x) = A e^{-ikx}$$

$$\tilde{u}_+(x) = -\frac{1}{i\omega\rho_0}\frac{\partial \tilde{p}_+}{\partial x} = \frac{1}{\rho_0 c}\tilde{p}_+$$

415 Rayls

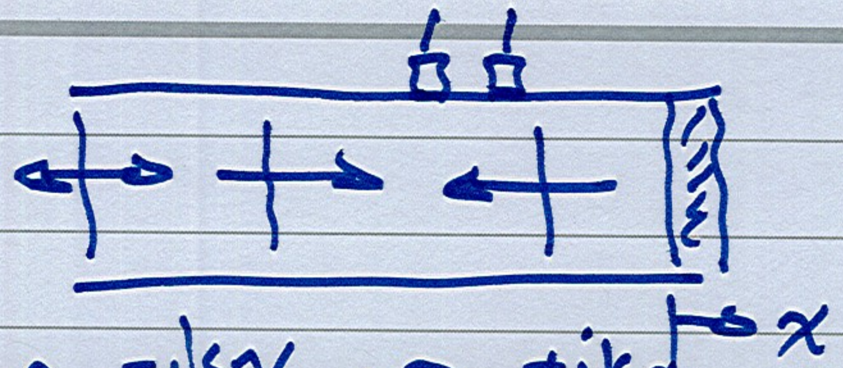
$$\tilde{z} = \frac{\tilde{p}_+}{\tilde{u}_+} = \rho_0 c$$ characteristic impedance

plane wave in -ve x-direction

$$\tilde{z} = \frac{\hat{p}_-}{\hat{u}_{x-}} = -\rho_0 c$$

$\tilde{z}$ is a function of direction

In general $\tilde{z}$ is a function of position



$$\hat{p}(x) = Ae^{-ikx} + Be^{+ikx}$$

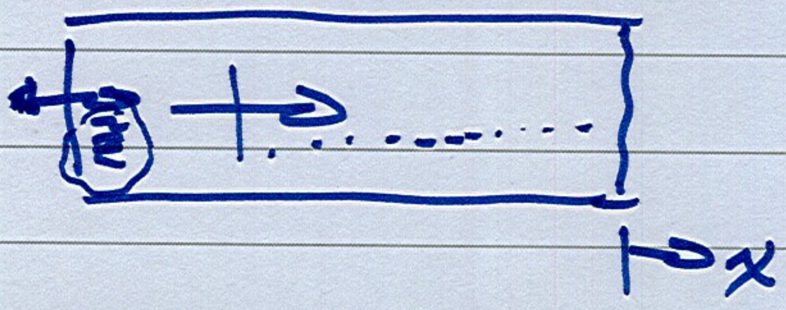
$$\hat{u}(x) = \frac{A}{\rho_0 c}e^{-ikx} - \frac{B}{\rho_0 c}e^{+ikx}$$

$$\hat{z} = \frac{\hat{p}(x)}{\hat{u}(x)} = \rho_0 c \frac{Ae^{-ikx} + Be^{+ikx}}{Ae^{-ikx} - Be^{+ikx}}$$

Function of x

R = plane wave reflection coefficient

$$\left(\frac{B}{A}\right)_{x=0} = R$$



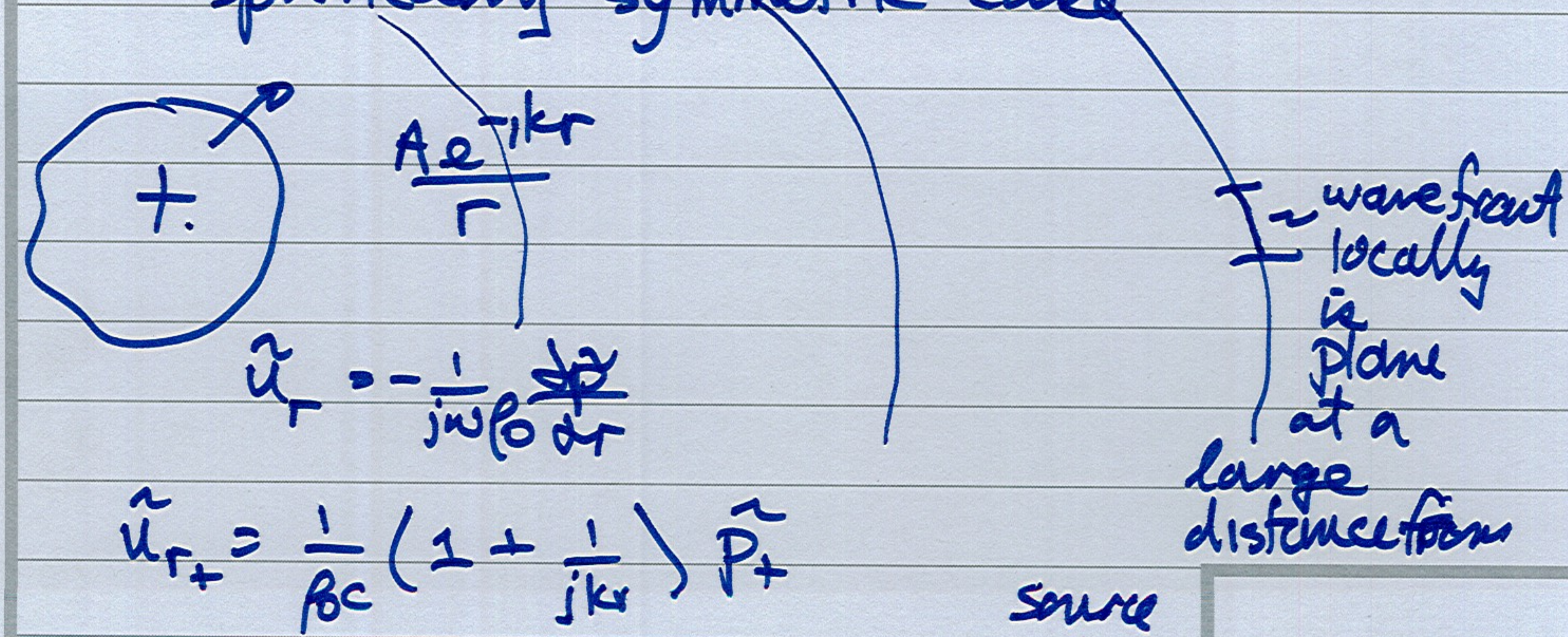
$$\tilde{z} = \frac{\tilde{p}(x)}{\tilde{u}(x)} = \rho_0 c \, \frac{e^{-ikx} + R e^{+ikx}}{e^{-ikx} - R e^{+ikx}}$$

$$0 \leqslant |R| \leqslant 1$$

$R \to 0 \qquad \tilde{z} \to \rho_0 c$

$R \to 1 \qquad \tilde{z} \to -i\rho_0 c \, \underline{\cot kx}$

spherically symmetric case



$$\frac{Ae^{-jkr}}{r}$$

$$\tilde{u}_r = -\frac{1}{j\omega\rho_0}\frac{\partial\tilde{p}}{\partial r}$$

$$\tilde{u}_{r+} = \frac{1}{\rho_0 c}\left(1 + \frac{1}{jkr}\right)\tilde{p}_+$$

$$\tilde{z}_+ = \frac{\tilde{p}_+}{\tilde{u}_{r+}} = \frac{\rho_0 c}{1 + \frac{1}{jkr}}$$

$kr \rightarrow \infty \quad \tilde{z}_+ \rightarrow \rho_0 c \quad$ farfield nearfield $kr << 1$

$\tilde{z}_t \Rightarrow j(kr)\rho_0 c$ $\qquad k = \frac{\omega}{c}$

$j\left(\frac{\omega}{\cancel{c}} r\right)\rho_0 \cancel{c}$

$\underline{j\omega\rho_0 r}$

+ve
imaginary
linearly prop to freq

<u>masslike</u>