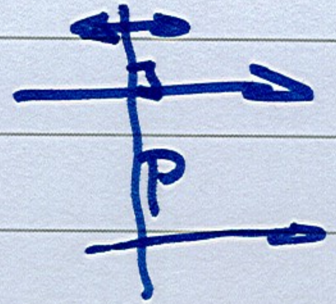
One-Dimensional Solutions

- plane wave



- harmonic

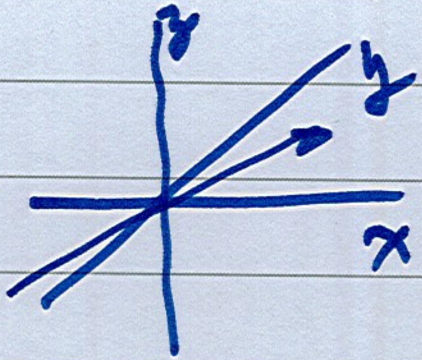
$$\nabla^2 \tilde{p} + k^2 \tilde{p} = 0$$

$$\tilde{p}(x) = \underline{A e^{-jkx}} + \underline{B e^{+jkx}}$$

$$\tilde{u}_x = -\frac{1}{j\omega\rho_0}\frac{d\tilde{p}}{dx}$$

$$\frac{\tilde{p}}{\tilde{u}_x} = \rho_0 c \qquad \tilde{u}_x = \frac{A}{\rho_0 c} e^{-jkx} - \frac{B}{\rho_0 c} e^{+jkx}$$

3.3.1.2 Arbitrary Direction of Propagation



$$\nabla^2 p + k^2 p = 0 \qquad k = \frac{\omega}{c}$$

$$\tilde{p}(x,y,z,t) = A e^{j(\omega t - k_x x - k_y y - k_z z)}$$

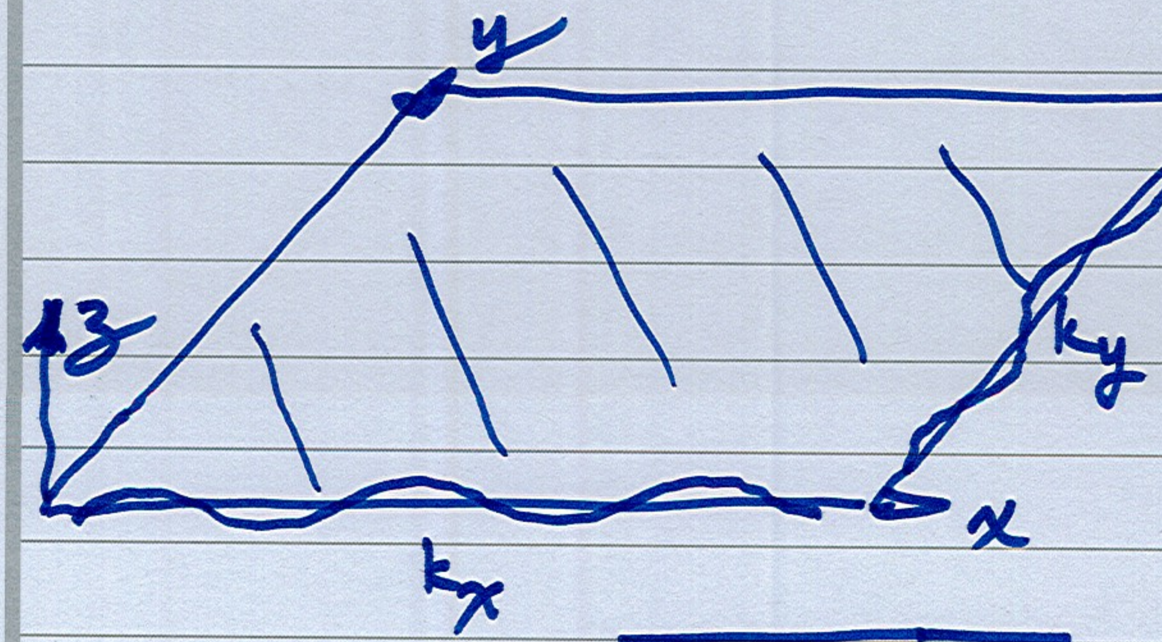
component wave numbers

$\left.\begin{matrix} k_x \\ k_y \\ k_z \end{matrix}\right]$ rates of change of phase w/ position in the coord directions

for a propagating function to be sound - must be true

$$k^2 = k_x^2 + k_y^2 + k_z^2 \;\Big]\ \text{sound}$$

only 2 of the k_x, k_y & k_z can be chosen independently if the solution is "sound"

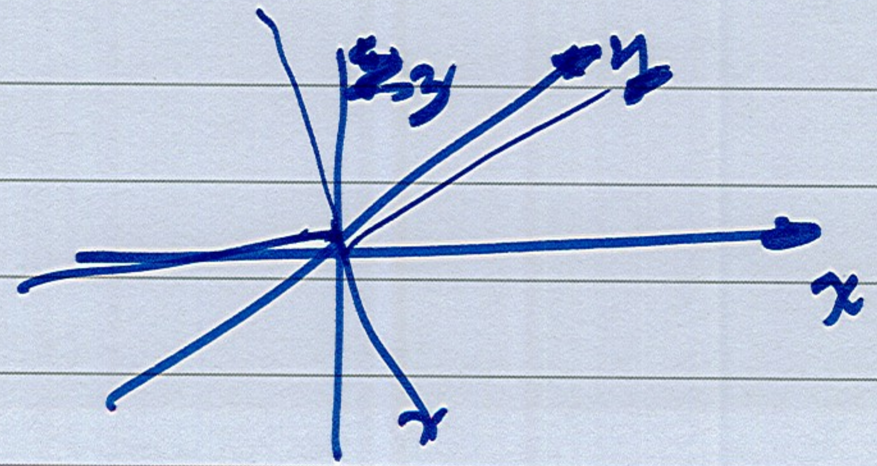


$$k_z = \sqrt{k^2 - k_x^2 - k_y^2}$$

$$e^{-ik_z z}$$

$$\tilde{p}(x,y,z,t) = A e^{i(\omega t \pm k_x x \pm k_y y \pm k_z z)}$$

8 possible combinations



always true that

$$k_x^2 + k_y^2 + k_z^2 = k^2$$

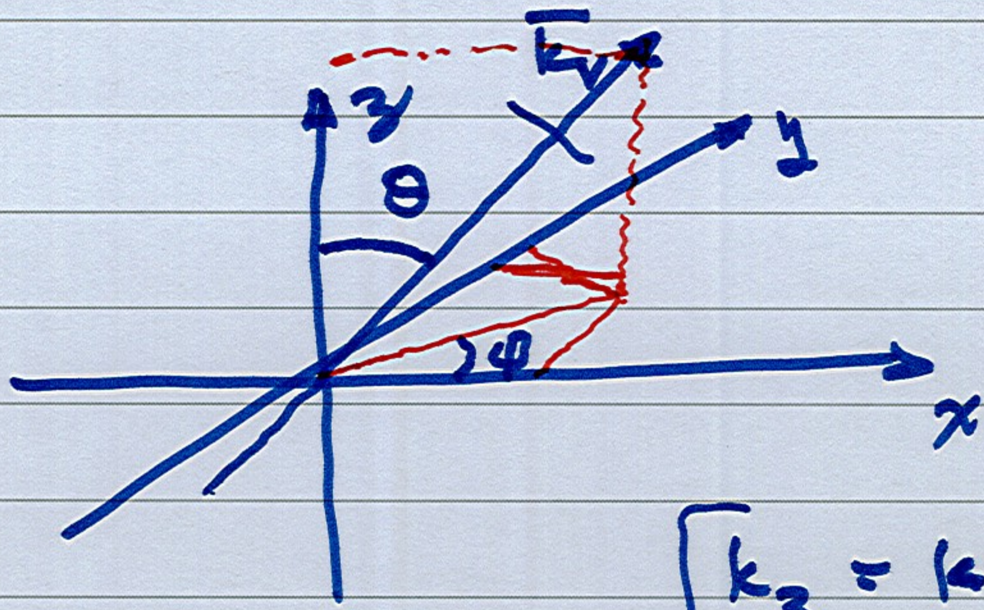
<u>Wave vector</u>

- vector direction of wave prop

$$\bar{k}_v = k_x \bar{i} + k_y \bar{j} + k_z \bar{k}$$

$$|\bar{k}_v| = k = \frac{\omega}{c}$$

$$p(x,y,z,t) = A e^{j(\omega t - k_v \cdot \bar{x})}$$



θ = polar angle
ϕ = azimuth

$$\begin{cases} k_z = k\cos\theta \\ k_x = k\sin\theta\cos\phi \\ k_y = k\sin\theta\sin\phi \end{cases}$$

$$k_z^2 = k^2 - \underbrace{k_x^2 - k_y^2}_{\text{fixed by b.c.'s (vibrating plate)}}$$

fixed by frequency

$$k_z = \sqrt{k^2 - k_x^2 - k_y^2}$$

(i) $k_x^2 + k_y^2 > k^2$ $\sqrt{-ve}$

$$k_z = \pm j \underbrace{\sqrt{k_x^2 + k_y^2 - k^2}}_{\alpha \text{ real number}}$$

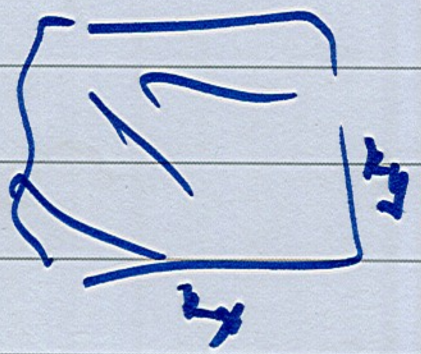
$$k_z = \pm j\alpha$$

$k_z = \pm j\alpha \qquad e^{\mp jk_z z} \rightarrow \underbrace{e^{\pm\alpha z}}_{\text{pure exponential}}$

$e^{+\alpha z}$ exponential growth ×

$e^{-\alpha z}$ exponential decay]

if the z-region is semi-infinite



$\tilde{p} = A e^{-jk_x x} e^{-jk_y y} e^{-\alpha z}$

~~evan~~ **evanescent**

or non-propagating wave

(ii) $k_x^2 + k_y^2 < k^2$ $\qquad k_z \rightarrow$ real

$$\tilde{p} = A e^{j(\omega t - k_x x - k_y y - k_z z)}$$

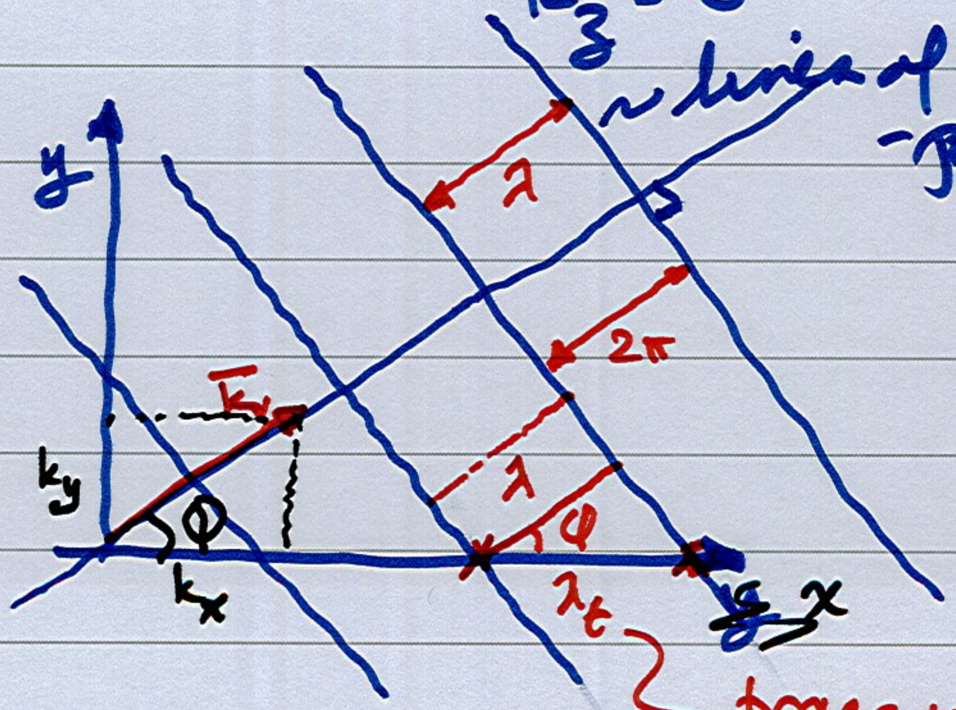
$e^{\pm j k_z z}$ oscillations in the z-direction - no attenuation or decay

propagating wave

consider 2-D case

– no variation in the z-direction

$k_z = 0$



~ lines of constant phase ~~propat~~ prop at speed c

trace wavelength

$k_x = k\cos\phi$

$k_y = k\sin\phi$

$k_x^2 + k_y^2 + \cancel{k_z^2} = k^2$

$\lambda_t \cos\phi = \lambda$

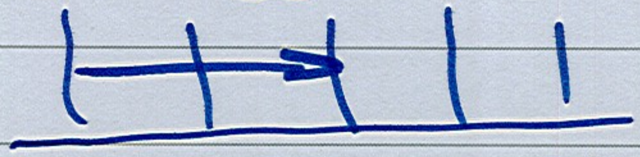
$\lambda_t = \dfrac{\lambda}{\cos\phi}$

$k_y = \sqrt{k^2 - k_x^2}$

$$\lambda_t = \frac{\lambda}{\cos\phi}$$

$\lambda_t \to \lambda \quad \phi \to 0$

grazing incidence case



$\lambda_t \to \infty$

$\phi \to \frac{\pi}{2}$

in that case $\lambda_t \geq 0$

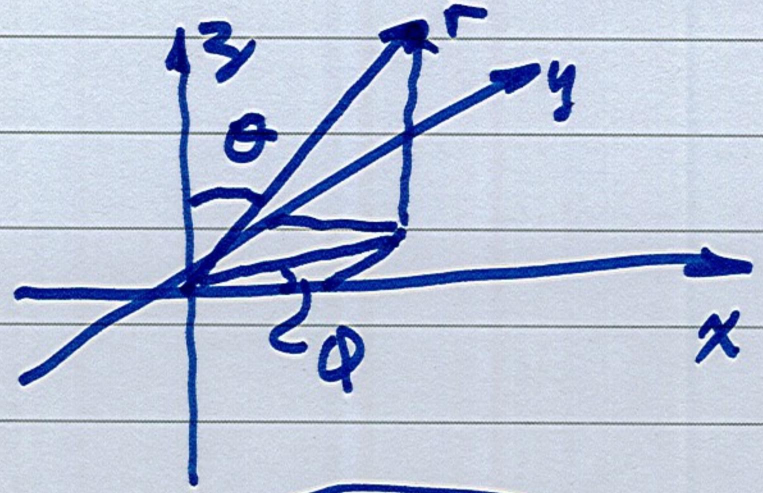
trace wave speed

$$c_t = \frac{\lambda_t}{T} > c$$

$\phi = 0 \qquad c_t = c$

$\phi = \frac{\pi}{2} \qquad c_t \to \infty$

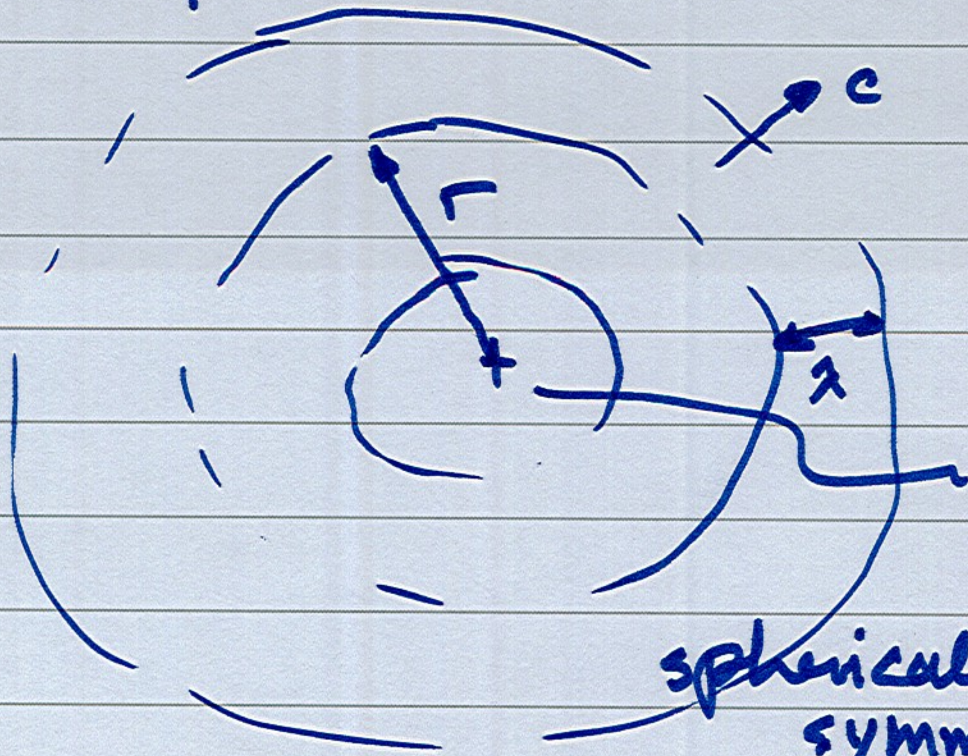
3.3.2 Spherical Waves



polar angle

(r, θ, ϕ) azimuth

spherically symmetric
no variation with
θ or ϕ



omnidirectional source

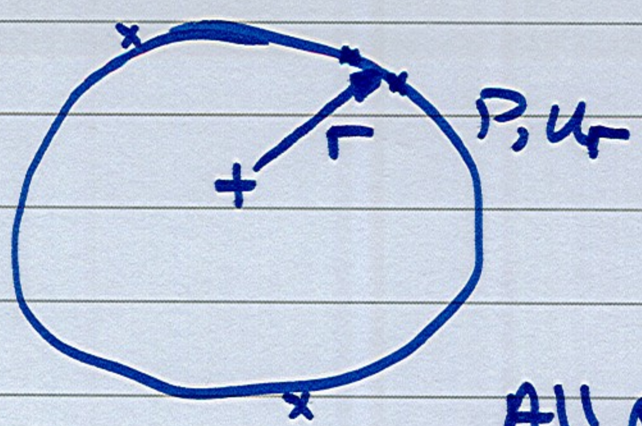
spherically symmetric

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

In spherical coordinates

$$\nabla^2 = \underbrace{\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}} + \cancelto{0}{\frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial}{\partial \Phi}\right)}$$

$\tilde{p}$ is a function of r only



All quantities are instantaneously constant

All particle velocity is in the radial direction

$$p \propto \frac{1}{r}$$