3.2.4 Speed of Sound

$$c = \sqrt{\frac{\gamma P_0}{\rho_0}}$$

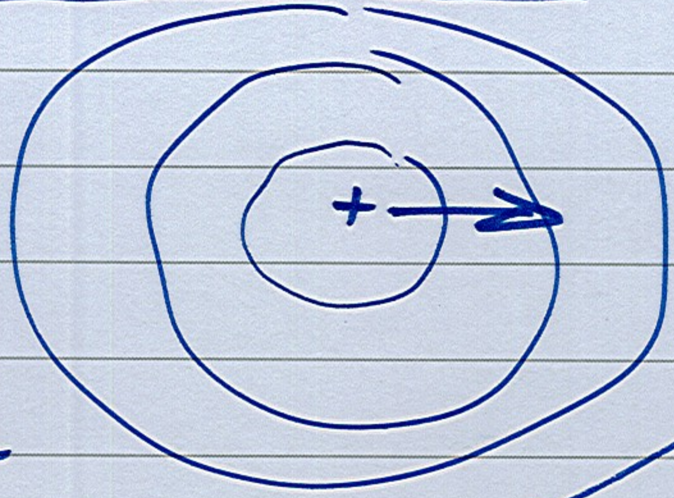
Isothermal Atmosphere

$$\left(\frac{P_0}{\rho_0}\right)$$

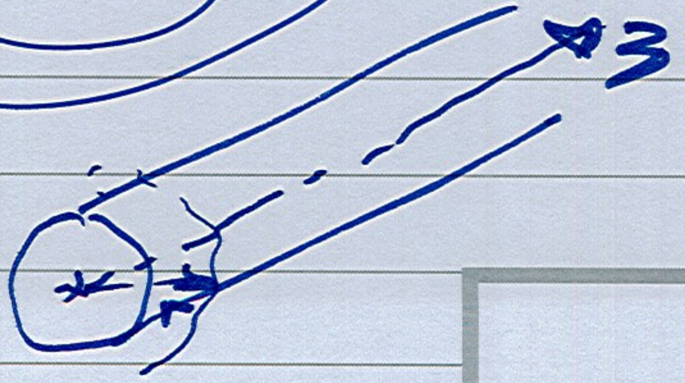
$c \neq$ function of height

3.3 One-Dimensional Solutions

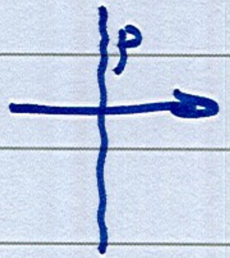
- spherical
    - spherically symmetric



- cylindrical waves

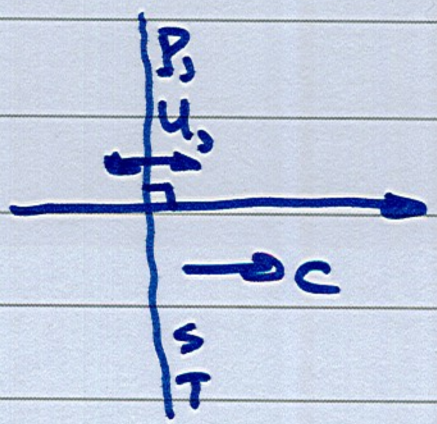


- plane waves



3.3.1 Plane Waves

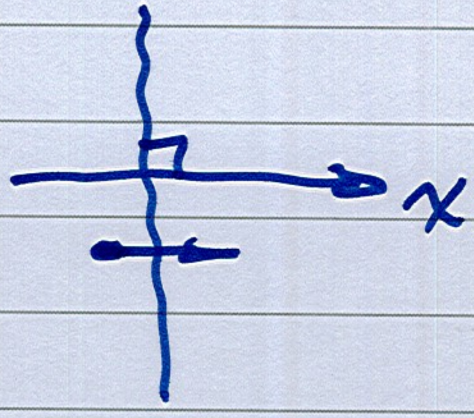
- properties are instantaneously uniform over an infinite plane ⊥ to the direction of wave propagation



longitudinal wave

- particle motion is back and forth in the direction of wave propagation

3.3.1.1 Propagation in the x-direction



no variation of pressure in the y or z directions

$$\frac{\partial}{\partial y} \quad \frac{\partial}{\partial z} = 0$$

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

$$p(x,t) = \underset{+ve}{p_1(ct - x)} + \underset{-ve}{p_2(ct + x)}$$

complex harmonic cases

$$p(x,t) = \underline{\tilde{p}(x)}\,\underline{e^{j\omega t}}$$ separable form

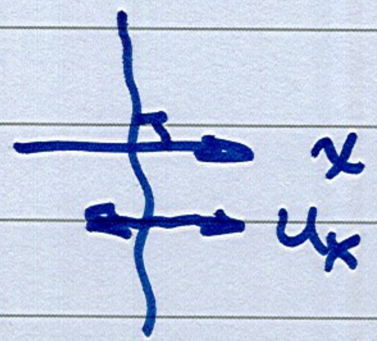
- sub into the wave eqn

$$\left.\frac{d^2\tilde{p}}{dx^2} + k^2\tilde{p} = 0\right]$$ scalar Helmholtz Eqn

$$k = \frac{\omega}{c}$$

$$\tilde{p}(x) = A_1 e^{-jkx} + A_2 e^{+jkx}$$

$$p(x,t) = \tilde{p}(x)\,e^{j\omega t}$$



linearized momentum eqn

$$-\nabla p = \rho_0 \frac{\partial \bar{u}}{\partial t} \qquad \bar{u} = u_x \bar{i} + u_y \bar{j} + u_z \bar{k}$$

1-D)

$$-\frac{\partial p}{\partial x}\cancel{\bar{i}} = \rho_0 \frac{\partial u_x}{\partial t}\cancel{\bar{i}}$$

$$p(x,t) = \tilde{p}(x)\, e^{j\omega t}$$
$$u_x(x,t) = \tilde{u}_x(x)\, \underline{e^{j\omega t}}$$

$$-\frac{\partial \tilde{p}}{\partial x} = j\omega \rho_0 \tilde{u}_x$$

$$\boxed{\tilde{u}_x = -\frac{1}{j\omega\rho_0}\frac{\partial \tilde{p}}{\partial x}}$$

Given a pressure distribution

- can derive the particle velocity

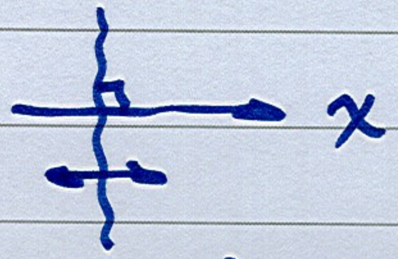
$$\hat{u}_x = -\frac{1}{j\omega\rho_0}\frac{\partial \tilde{p}}{\partial x}$$

$$\hat{u}_y = -\frac{1}{j\omega\rho_0}\frac{\partial \tilde{p}}{\partial y}$$

$$\hat{u}_z = -\frac{1}{j\omega\rho_0}\frac{\partial \tilde{p}}{\partial z}$$

Particle Velocity associated with

$$\tilde{p}_+ = \underline{A_1 e^{-jkx}}$$



$$\tilde{u}_{x_+} = -\frac{1}{j\omega\rho_0}\frac{\partial \tilde{p}_+}{\partial x} = -\frac{1}{j\omega\rho_0}(-jk)\,A_1 e^{-jkx}$$

$$= \frac{k}{\omega\rho_0} A_1 e^{-jkx} \qquad k = \frac{\omega}{c}$$

$$= \frac{A_1 e^{-jkx}}{\rho_0 c}$$

$\rho_0 c$ ~ characteristic impedance

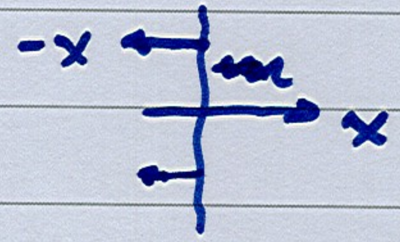
$$\tilde{u}_{x_+} = \frac{\tilde{p}_+}{\rho_0 c}$$

$(\rho_0 c) \simeq 415$ [Rayls]

<u>specific</u> acoustic impedance

$$\frac{\tilde{p}_+}{\tilde{u}_{x_+}} = \rho_0 c \quad \text{characteristic impedance}$$

$$\tilde{p}_- = A_2 e^{+jkx}$$



$$\tilde{u}_{x_-} = -\frac{1}{j\omega\rho_0}\frac{\partial \tilde{p}_-}{\partial x} = -\frac{1}{j\omega\rho_0}(jk) A_2 e^{+jkx}$$

$$= -\frac{A_2 e^{+jkx}}{\rho_0 c} \sim \tilde{p}_-$$

$$\tilde{u}_{x_-} = -\frac{\tilde{p}_-}{\rho_0 c} \qquad \frac{\tilde{p}_-}{\tilde{u}_{x_-}} = -\rho_0 c$$

sign of the impedance depends on direction of prop use these results to calculate the particle velocity for any given sound field

$$\boxed{\bar{u} = -\frac{1}{j\omega\rho_0}\nabla\tilde{p}}$$

harmonic motion

$$\tilde{p}(x) = \underbrace{A_1 e^{-ikx}}_{\tilde{p}_+} + \underbrace{A_2 e^{+ikx}}_{\tilde{p}_-}$$

$$\tilde{u}_x(x) = \frac{\tilde{p}_+}{\rho_0 c} - \frac{\tilde{p}_-}{\rho_0 c}$$

$$= \frac{A_1}{\rho_0 c} e^{-ikx} - \frac{A_2}{\rho_0 c} e^{+ikx}$$

Typical Velocites

- plane wave prop in the +ve x-dir

$$|\tilde{p}_+| = 1\ \text{Pa} \quad \hat{=}\ 94\ \text{dB}$$

$$|\tilde{u}_x| = \left|\frac{\tilde{p}_+}{\rho_0 c}\right| = 2.4\ \text{mm/s}$$

$$c = 340\ \text{m/s}$$

$$|\tilde{u}_{x_+}| \ll c$$

xi

ξ displacement

for harmonic fields

$$j\omega \tilde{\xi}_{x_+} = \tilde{u}_{x_+}$$

$$|\tilde{\xi}_{x}| = \frac{|\tilde{u}_{x_+}|}{\omega}$$

particle displacement

$|\tilde{\xi}_{x_+}| \doteq \frac{|\tilde{a}_{x_+}|}{\omega}$ at $\underline{1\ \mathrm{kHz}}$ $\omega \simeq 6000$

$\underline{94\ \mathrm{dB}}$ $|\tilde{\xi}_{x_+}| = \frac{2.4 \times 10^{-3}}{6 \times 10^{+3}} \simeq 0.25 \times 10^{-6}$

$O(10^{-6})$

$\underline{\text{micron}}$

74 dB $|\tilde{\xi}_{x_+}|$ $O(10^{-7})$

54 dB $|\tilde{\xi}_{x_+}|$ $O(10^{-8})$

34 dB $|\tilde{\xi}_{x_+}|$ $O(10^{-9})$

nanometers