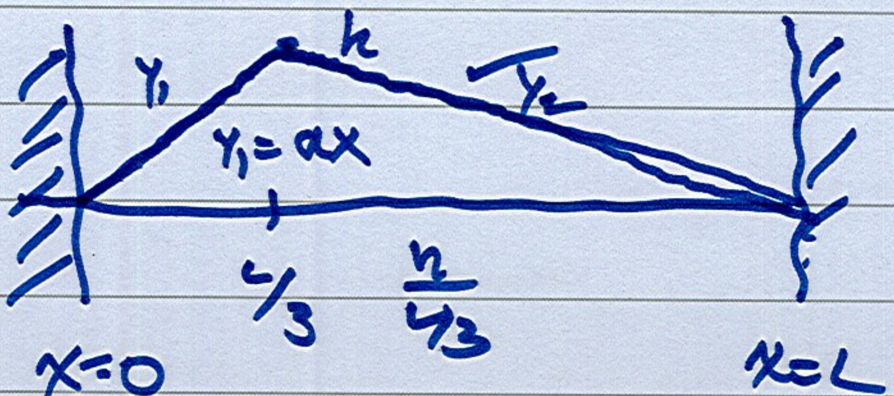
ME 513 Session 16 9/25/19

$\underline{\omega_n t} = \frac{n\pi c}{L} t$

2.10.1



$y_1 = \frac{3h}{L} x$

$t=0$

$t = L/c$

initial velocity = 0

$$y(x,t) = \sum_{n=1}^{\infty} (A_n \cos\omega_n t + \cancel{B_n \sin\omega_n t}) \sin k_n x$$

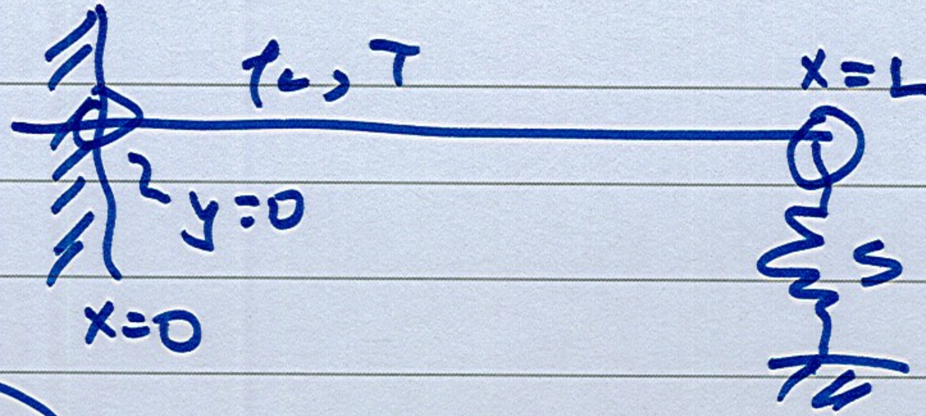
0

$$A_n = \frac{2}{L} \int^{L} y(x,0) \sin k_n x \, dx$$

$$k_n = \frac{n\pi}{L} = \frac{\omega_n}{c}$$

$$= \frac{2}{L} \left\{ \int_0^{L/3} y_1(x,0) \sin k_n x\, dx + \int_{L/3}^{L} y_2(x,0) \sin k_n x\, dx \right]$$

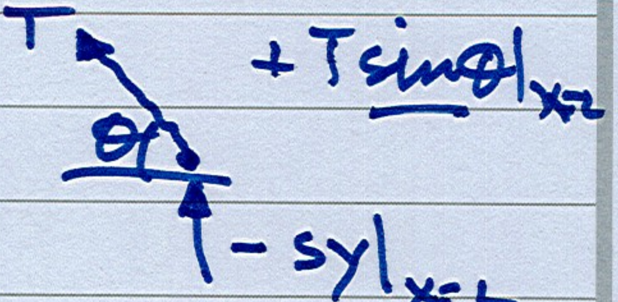
2.11.1



$$\boxed{T = sL}$$

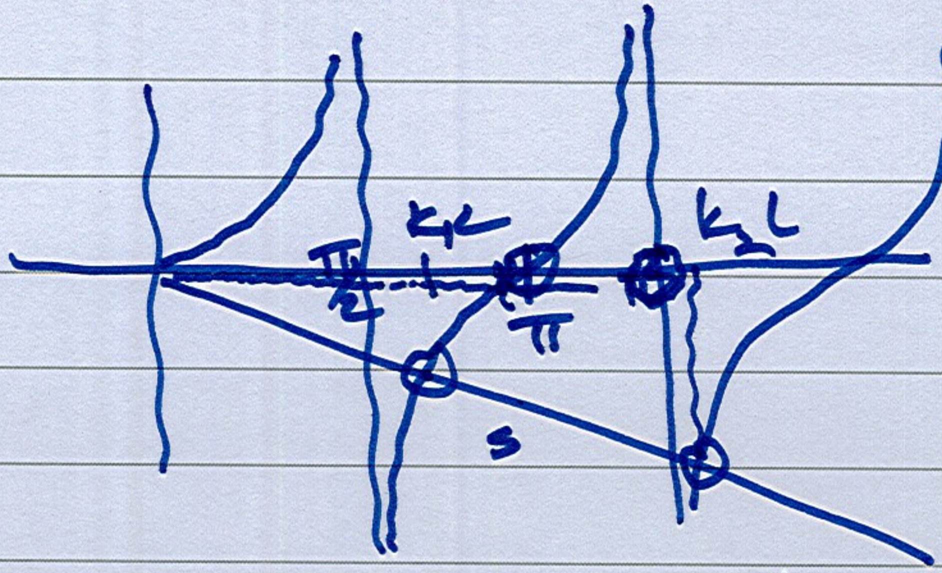
$$y = \left(Ae^{-jkx} + Be^{+jk}\right)e^{i\omega t}$$

$$\sim -2jA\sin kx$$

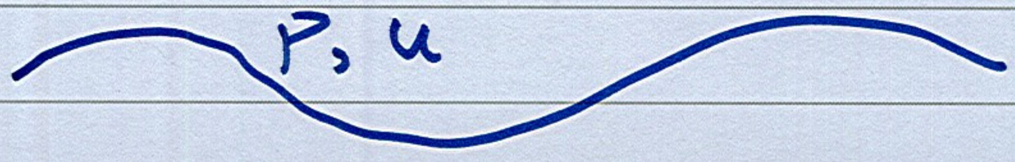


$$\Sigma f_y = 0$$

$$\sin\theta \simeq -\frac{dy}{dx} \qquad \sim(kL) = \tan kL$$



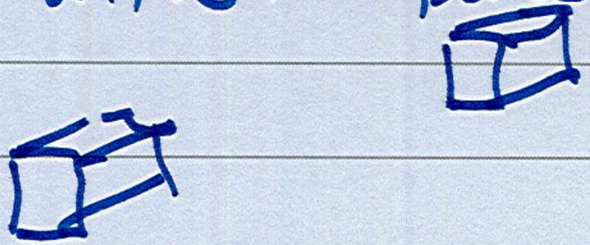
4 Same as 3 $T \neq sL$



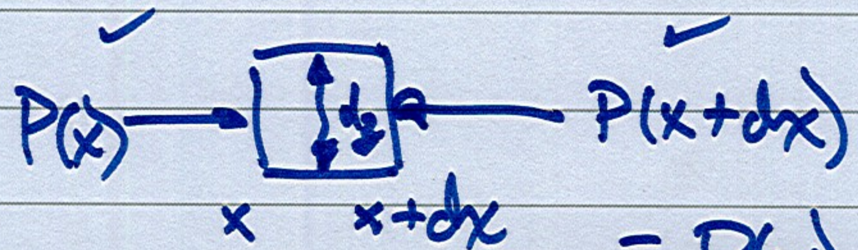
$$\frac{1}{B}\frac{dp}{dt} + \nabla\cdot\bar{u} = 0$$

3.2.2 Pressure - Velocity (II)

apply $f = ma$
to a fixed mass moving
with the fluid



1-D



$$= P(x) + \left.\frac{dP}{dx}\right|_{x} dx + \cancel{\ldots}^{\,0}$$

net force in the x-direction

$$df_x = P(x)\,dy\,dz - P(x+dx)\,dy\,dz$$

$$df_x = -\left.\frac{dP}{dx}\right|_x \underbrace{dx\,dy\,dz}_{\text{volume of cube}}$$

$$\text{force/unit volume} = dV$$

In 3-D $\quad \overline{df} = -\nabla P \, dV$

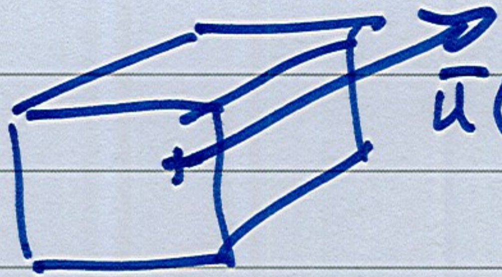
∇P = gradient of the pressure

$$= \frac{\partial P}{\partial x}\bar{i} + \frac{\partial P}{\partial y}\bar{j} + \frac{\partial P}{\partial z}\bar{k}$$

steepest or fastest
change in pressure
with position

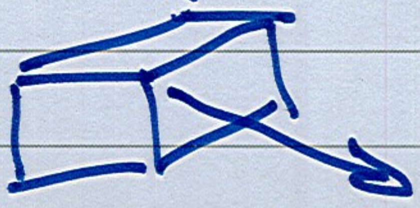
$\check{f} = m\check{a}$

(iii) Acceleration of a moving fluid element



$\bar{u}(x, y, z, t)$

u_i



$\bar{u}(x+dx, y+dy, z+dz, t+dt)$

u_f

$$\frac{u_f - u_i}{dt}$$

$$\bar{u}_i$$

$$\bar{u}_f = \bar{u}(x,y,z,t) + \left.\frac{\partial \bar{u}}{\partial x}\right|_{x,y,z,t} dx + \cdots\cdots + \left.\frac{\partial \bar{u}}{\partial t}\right|_{x,y,z,t} dt$$

$$dx = u_x dt$$

$$dy = u_y dt$$

$$dz = u_z dt$$

$$\bar{a} = \frac{\bar{u}_f - \bar{u}_i}{dt} = \left[\underbrace{u_x \frac{\partial \bar{u}_x}{\partial x} + u_y \frac{\partial \bar{u}}{\partial y} + u_z \frac{\partial \bar{u}}{\partial z}}_{\text{convective acceleration}} + \frac{\partial \bar{u}}{\partial t} \right]$$

- products of small quantities are negligible

$$\bar{a} = \cancel{(\bar{u} \cdot \nabla)\bar{u}} + \frac{\partial \bar{u}}{\partial t}$$

neglect non-linear terms

$$d\bar{f} = -\nabla P dV$$

mass of the element

$$\bar{a} = \frac{\partial \bar{u}}{\partial t}$$

$$m = \rho dV$$

$$d\bar{f} = m\bar{a}$$

$$-\nabla P\, dV = \rho\, dV \frac{d\bar{u}}{dt}$$

(iv) Pressure - Velocity Relation

$$P = P_0 + p \quad \Big] \qquad \nabla P = \nabla p$$

$$\rho = \rho_0 (s + 1) \qquad s = \frac{\rho - \rho_0}{\rho_0} \ll 1$$

$$\boxed{-\nabla p = \rho_0 \frac{d\bar{u}}{dt}}$$

(4) Euler Eqn
linearized
momentum
eqn

2nd Eqn relating p + u

3.2.3 Linear Wave Eqn

(3) $\frac{1}{\beta}\frac{\partial p}{\partial t} + \nabla \cdot \bar{u} = 0$ $\qquad \frac{\partial}{\partial t}$

(4) $\nabla p + \rho_0 \frac{\partial \bar{u}}{\partial t} = 0$ $\qquad \frac{1}{\rho_0}\nabla \cdot$

subtract (3) from (4)

$$\nabla^2 p - \left(\frac{\rho_0}{\beta}\right)\frac{\partial^2 p}{\partial t^2} = 0$$

$c = \sqrt{\frac{\beta}{\rho_0}}$ sound speed

$$\boxed{\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0}$$

linear wave eqn

$$c = \sqrt{\frac{\beta}{\rho_0}} = \sqrt{\frac{\gamma P}{\rho_0}}$$

stationary media
+ ideal fluids

3.2.4 speed of sound

c = speed of wave propagation

$$= \sqrt{\frac{\gamma P_0}{\rho_0}} = 331.6 \text{ m/s at } 0^\circ C$$

c_0

$$\approx 340 \text{ m/s at } 20^\circ C$$

c is directly to absolute temperature to the ½ power

$$c = c_0 \sqrt{\frac{T_K}{273}} \qquad T_K \, [^\circ K]$$

$$= c_0 \sqrt{1 + T_C/273} \qquad T_C \, [^\circ C]$$

is c dependent on height
in the atmosphere?