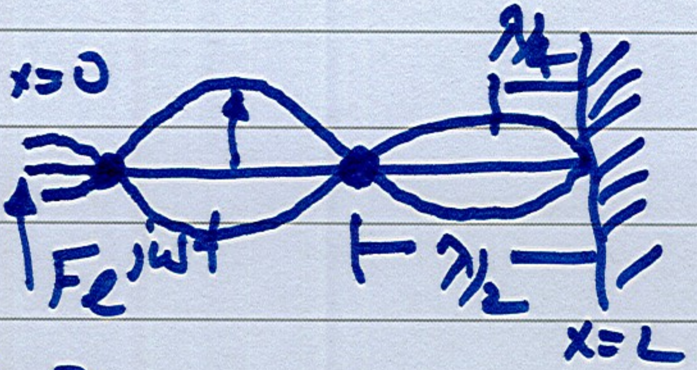
2.9.8C



Eqn 2.9.7

$$y(x,t) = \frac{F}{kT}\,\frac{\sin k(L-x)}{\cos kL}\,e^{j\omega t} \qquad k = \frac{\omega}{c}$$

"envelope" $t=0$ & $t=\frac{T}{2}$

1 -1

(a)

$-j\rho c \cot kL$

(b) plot $|y|$ at $x=0$ vs freq k

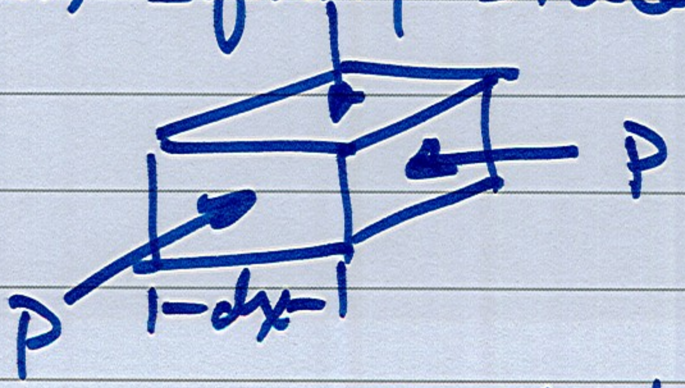
plot max disp. amp

plot the input mechanical impedance

3.2 Derivation of The Wave Eqn

3.21 Pressure - Velocity (I)

(1) Eqn of state



Ambient Pressure

$P_0 \simeq 1 \times 10^5$ Pa

Ambient Air density

$\rho_0 \simeq 1.2$ kg/m^3

for an ideal gas

- pressure is a function of density (and density is a function of particle velocity)

pressure change from $P_0 \rightarrow P$

as density $\rho_0 \rightarrow \rho$

Taylor's series

$$P = P_0 + \left(\frac{dP}{d\rho}\right)_{\rho_0}(\rho - \rho_0) + \cancel{\frac{1}{2}\left(\frac{d^2P}{d\rho^2}\right)_{\rho_0}}(\rho-\rho_0)^2 + \cdots$$

linear acoustics

- 2nd order terms ignored
- small fluctuations compared to ambient

Bulk modulus

$$(P - P_0) = \rho_0 \left(\frac{dP}{d\rho}\right)_{\rho_0} \left(\frac{\rho - \rho_0}{\rho_0}\right)$$

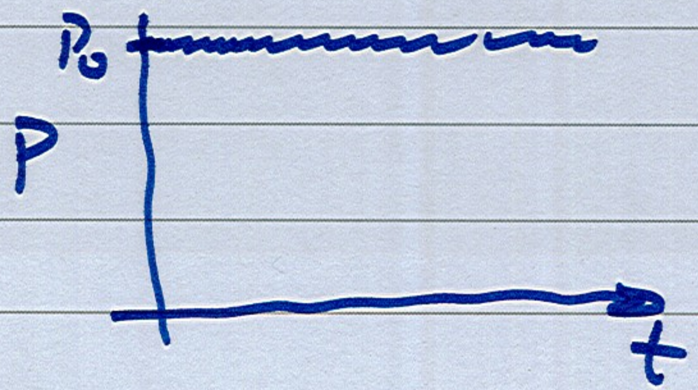
- condensation
- non-dimensional density change

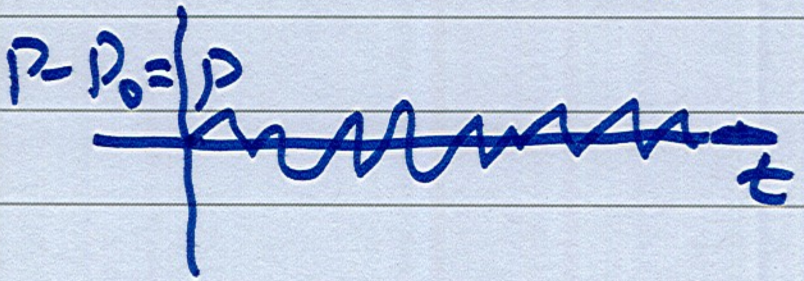
sound pressure

$P - P_0 = p$

$\rho > \rho_0$ $\quad s > 0$ compressed

$\rho < \rho_0$ $\quad s < 0$ expanded





Bulk modulus

$$p = \rho_0 \left(\frac{dP}{d\rho}\right)_{\rho_0} s \qquad \beta = \text{adiabatic}$$

$$\boxed{p = \beta s} \qquad (1)$$

stress = modulus × strain

what is β for adiabatic compression

Two states $\quad P, P_0 \quad \rho, \rho_0$

From thermo

$$\left(\frac{P}{P_0}\right) = \left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

$\gamma =$ ratio of ~~spec~~ specific heats

$= 1.4$

for adiabatic compression

$$\beta = \rho_0 \left(\frac{dP}{d\rho}\right)_{\rho_0} = \gamma P_0$$

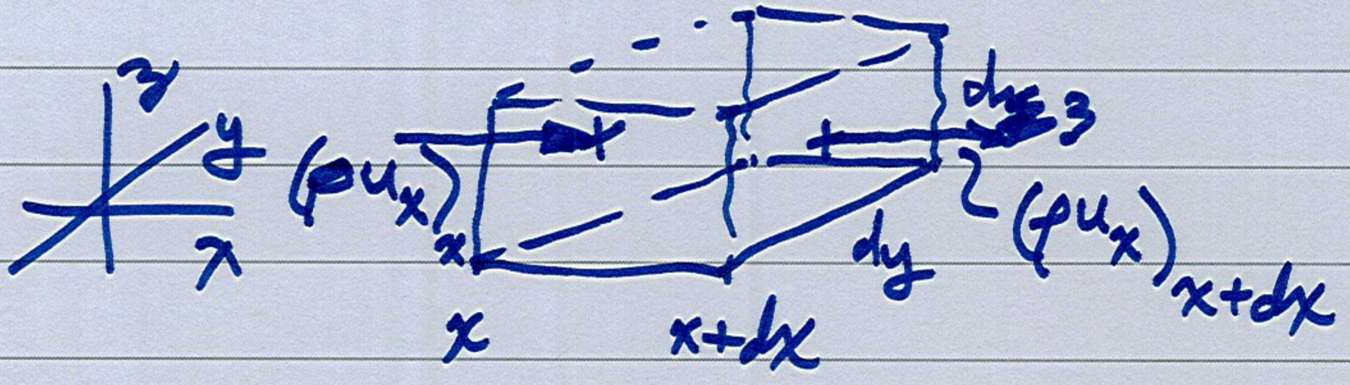
$$= \underline{\underline{1.4 \times 10^5}}$$

$P = \beta s$

2 relate to velocity

(iii) Continuity Eqn (conservation of mass)

control volume that is fixed in space



start with
1-D
- motion in x-direction

Rate at which mass flows in

(a) $(\rho u_x)_x \, dy\,dz$ kg/s

Rate at which mass flows out

(b) $(\rho u_x)_{x+dx} \, dy\,dz$ kg/s

$$= \left[(\rho u_x)_x + \frac{\partial(\rho u_x)}{\partial x}\bigg|_x dx + \cancel{\ldots} \right] dy\,dz$$

net rate of mass inflow

$$(a) - (b) = -\left.\frac{\partial(\rho u_x)}{\partial x}\right|_x dx\,dy\,dz$$

Rate of change of mass in the control volume

$$\frac{\partial \rho}{\partial t}\,dx\,dy\,dz \qquad \text{kg/s}$$

$$\frac{\partial \rho}{\partial t}\,\cancel{dx\,dy\,dz} = -\left.\frac{\partial(\rho u_x)}{\partial x}\right|_x \cancel{dx\,dy\,dz}$$

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_x)}{\partial x} = 0}$$ conservation of mass

both ρ & u_x are functions of x

$$\frac{\partial \rho}{\partial t} + u_x \frac{\partial \rho}{\partial x} + \rho \frac{\partial u_x}{\partial x} = 0$$

$$s = \frac{\rho - \rho_0}{\rho_0} \qquad \rho = s\rho_0 + \rho_0 = \rho_0(s+1)$$

$$\rho_0 \frac{\partial (s+\cancel{1})}{\partial t} + \rho_0 u_x \frac{\partial (s+\cancel{1})}{\partial x} + \rho_0(s+1)\frac{\partial u_x}{\partial x} = 0$$

$$\rho_0 \frac{\partial s}{\partial t} + \cancel{\rho_0 \underset{\varepsilon}{u_x} \frac{\partial s}{\partial x}} + \rho_0(\cancel{s}+1)\underset{\varepsilon}{\frac{\partial u_x}{\partial x}} = 0$$

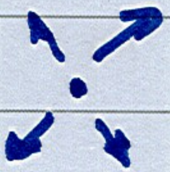
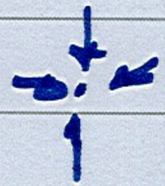
assume that the products of small quantities are negligible (linear acoustics)

$$\cancel{\rho_0}\frac{\partial s}{\partial t} + \cancel{\rho_0}\frac{\partial u_x}{\partial x} = 0$$

2-D $\boxed{\dfrac{\partial s}{\partial t} + \dfrac{\partial u_x}{\partial x} = 0}$

3-D $\boxed{\dfrac{\partial s}{\partial t} + \nabla\cdot\overline{u} = 0}$ (2)

$\nabla\cdot\overline{u}$ divergence of the particle velocity

$\nabla\cdot\overline{u}$ is +ve  is −ve 

$$\bar{u} = u_x \bar{i} + u_y \bar{j} + u_z \bar{k}$$

$$\nabla \cdot \bar{u} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}$$

Combine (1) & (2)

$$\boxed{\frac{1}{\beta} \frac{\partial p}{\partial t} + \nabla \cdot \bar{u} = 0}$$

linearized continuity eqn

note correction

- relates p to particle velocity